

Lect #7

Note Title

10/26/2008

Next:

- Source Transformation
- Thévenin and Norton Equivalents

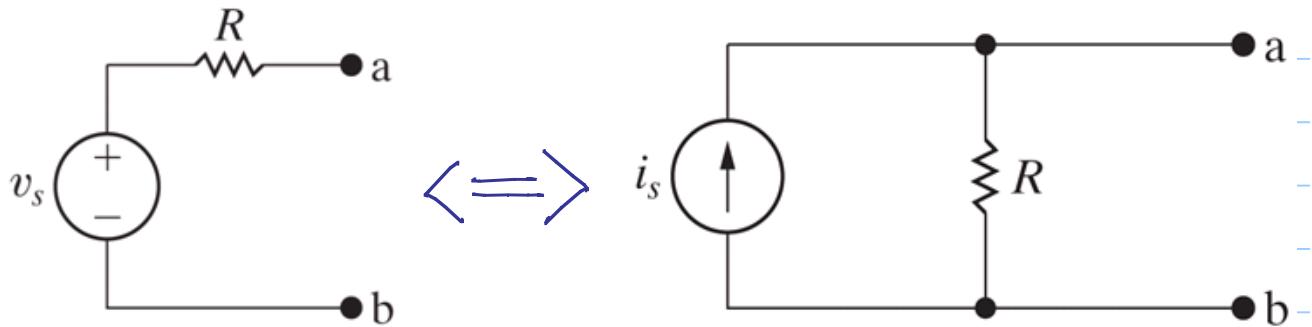
Objectives:

Understand source transformation and be able to use it to solve a circuit

Understand the concept of Thévenin and Norton equivalent circuits and be able to construct a Thévenin or Norton equivalent for a circuit.

= Source Transformation

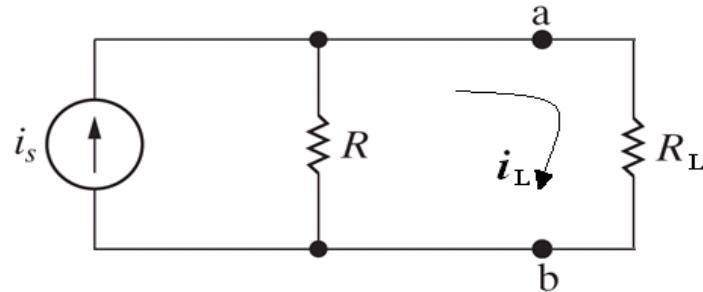
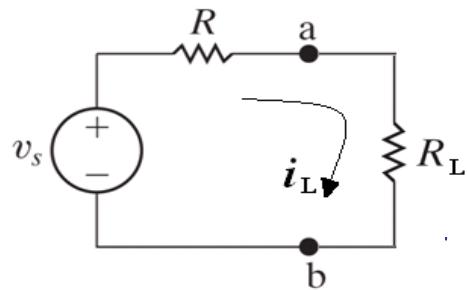
- . allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor (or vice versa)



- . Question: Under what conditions the two circuits are equivalent?
"equivalent" means that if you connect any load resistor R_L to the terminals a and b , it would have the same current, and hence the same voltage and the same power.

In other words: What is the relationship

between v_s and i_s for any value of R_L ?



Ohm's Law \Rightarrow

$$i_L = \frac{v_s}{R + R_L}$$

Current Division Eq \Rightarrow

$$i_L = \frac{R}{R + R_L} i_s$$

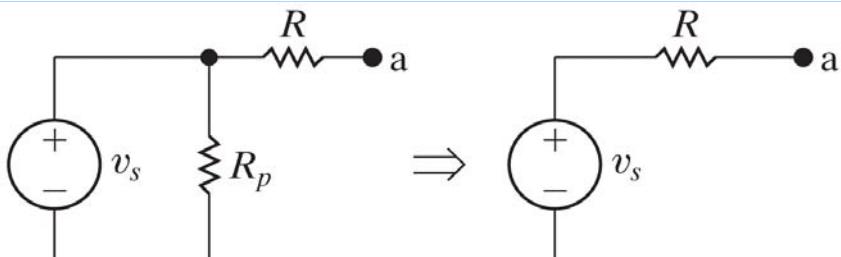
Thus

$$v_s = i_s R$$

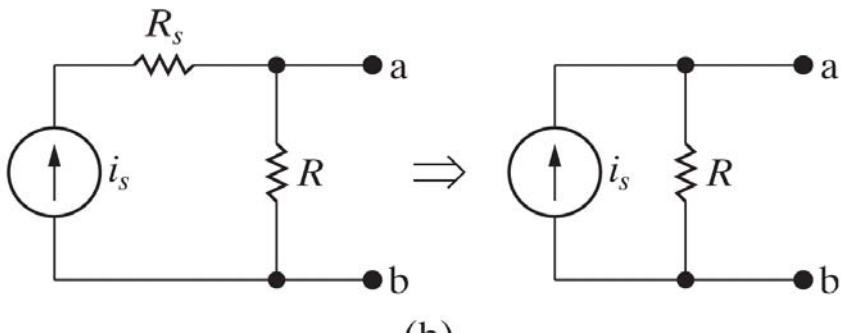
$$i_s = \frac{v_s}{R}$$

Note: If the polarity of v_s is reversed, then the direction of i_s must be reversed to maintain equivalence.

- What happens if there is a
 - . resistance R_p in parallel with the voltage source
 - . resistance R_s in series with the current source



(a)



(b)

Figure: 04-39a,b

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- No effect on equivalent circuit since the same voltage and current in any resistance R_L connected between nodes a and b.

Source transformation can be very useful to simplify circuit analysis problem.

Example: Find the power associated with 6 V source.

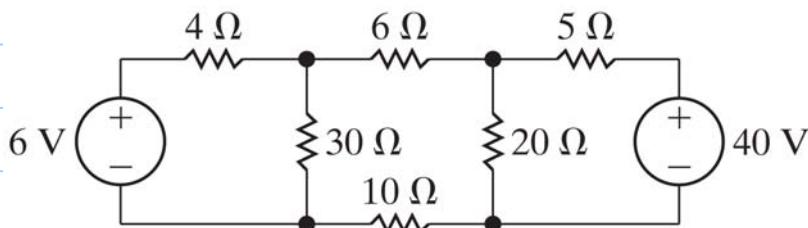
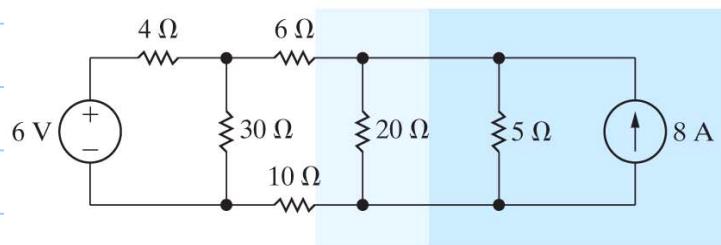
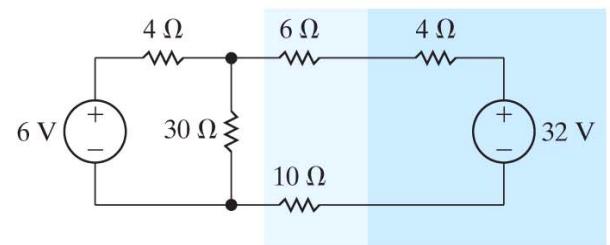


Figure: 04-37Ex4.8

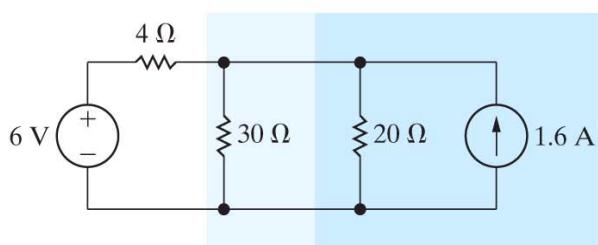
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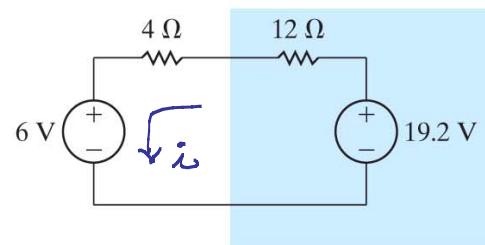
(a) First step



(b) Second step



(c) Third step



(d) Fourth step

Figure: 04-38a-dEx4.8

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a) Step 1.

$$i_s = \frac{40}{5} = 8 \text{ A}$$

b) Step 2.

$$20 \parallel 5 = \frac{20 \cdot 5}{20 + 5} = 4 \Omega$$

$$U_s = 8 \cdot 4 = 32 \text{ V}$$

c) Step 3

$$R = 6 + 4 + 10 = 20 \Omega$$

$$i_s = \frac{32}{20} = \frac{8}{5} = 1.6 \text{ A}$$

d) Step 4

$$30 \parallel 20 = \frac{30 \cdot 20}{30 + 20} = 12 \Omega$$

$$V_s = \frac{8}{5} \cdot 12 = \frac{96}{5} = 19.2 \text{ V}$$

e) current in the source

$$i = \frac{19.2 - 6}{12 + 4} = \frac{13.2}{16} \text{ Amp}$$

$$P = 6 \cdot \frac{13.2}{16} = \frac{39.6}{8}$$

$$= 4.95 \text{ Watts.}$$

= Thévenin and Norton Equivalents

At times in circuit analysis, we want to concentrate on what happens at the terminals.

eg. When we plug in the hair dryer into the wall outlet, we have little or no interest in the effect that the hair dryer has on the voltages and currents elsewhere in the circuit supplying the outlet.

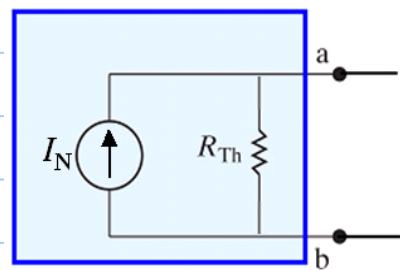
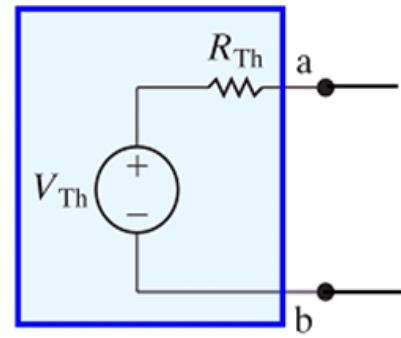
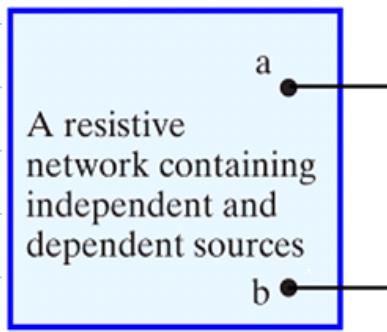
We are more interested in terminal behavior to the set of appliances, each requiring a different amount of power.

In other words, we are then interested in how the voltage and current delivered at the outlet changes as we change appliances.

- Thevenin and Norton Equivalents are circuit simplification techniques that focus on terminal behavior.

Thevenin and Norton Equivalent Circuits

- Thevenin equivalent circuit is an independent voltage source V_{Th} in series with resistance R_{Th}



- Equivalent to the original circuit in a sense that if we connect any load resistance in terminals a and b of each circuit, we get the same voltage and current at the terminals of the load.

In words:

$$V_{Th} = \text{Thévenin equivalent voltage}$$
$$= \text{"open circuit voltage"}$$

$$I_N = \text{Norton equivalent current}$$
$$= \text{"short circuit current"}$$

$$R_{Th} = \text{Thévenin equivalent resistance}$$
$$= \text{"output resistance"}$$

- When R_L is infinitely large, we have an open circuit. In this case

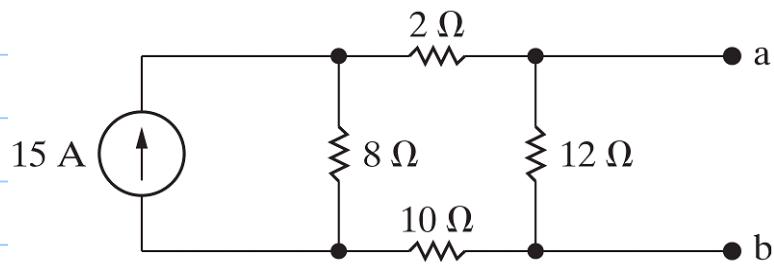
$$V_{ab} = V_{oc} = V_{Th}$$

- When $R_L = \infty$, we have a short circuit

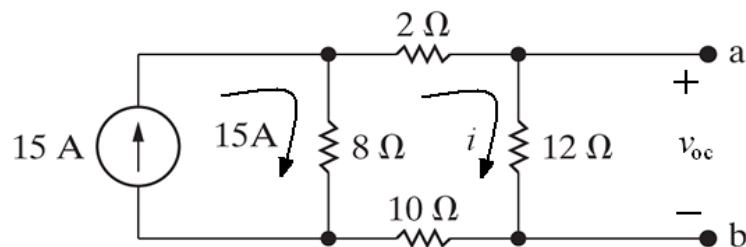
$$i_{sc} = \frac{V_{Th}}{R_{Th}} = I_N$$

$$R_{Th} = \frac{V_{oc}}{i_{sc}}$$

Example: Find the Thévenin equivalent of the circuit below to the left of the terminals a, b



Step 1. Find v_{oc}



Using the mesh current method:

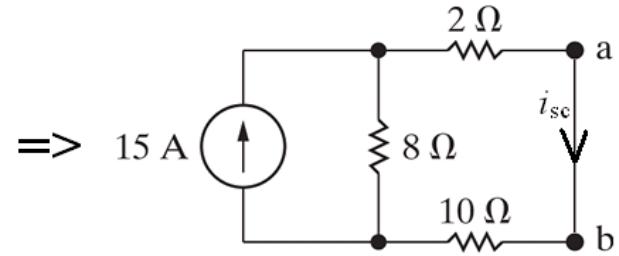
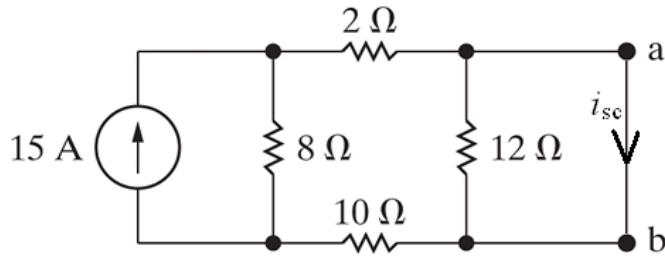
$$i_{\text{mesh}} : 2i + 12i + 10i + 8(i - 15) = 0$$

$$32i = 8(15)$$

$$i = 3.75 \text{ A}$$

$$v_{oc} = 12i = 45 \text{ V}$$

Step 2: Find i_{sc}



(note: $12\ \Omega$ resistor can be removed when analyzing the circuit since all the current will bypass the $12\ \Omega$ resistor when "short-circuited")

Using current division

$$i_{sc} = \frac{8}{8+12} \cdot 15$$

$$= 6\text{ A}$$

Step 3. Calculate R_{Th} and draw the Thévenin equivalent circuit

$$R_{Th} = \frac{V_{oc}}{i_{sc}} = \frac{45}{6} = 7.5 \Omega$$

