Lab (S/L4) – Water Rocket Trajectory Calculation

Reading: Lab 4 Notes — Water Rocket Calculations

Learning Objectives

- Application of Physics I principles and ideal gas behavior
- Derivation of relations for water rocket thrust prediction
- Numerical prediction of water rocket performance

Procedure

You will modify your spreadsheet program from Lab 2 for the case of a water rocket. Following the Lab 4 notes, you will first need to

- develop the relations for calculation of the initial velocity V_0, and
- develop the relations for calculation of the water exhaust velocity u_e.

In addition to the parameters from Lab 2, such as m_0, A, Δt, etc., you will now have a host of new parameters which affect the simulation, such as $\mathcal{V}_{\rm empty}$, A_e, $\rho_{\rm w}$, m_p, etc. As in Lab 2, all these should be easily changeable in your program.

Your program will then be applied to the calculation of water rocket performance for a baseline set of charge and operating conditions.

Reporting

1) Derive and present the necessary equations and expressions for h_0, V_0, m_0. You may simply quote any relations from the Lab 4 notes that you use (no need to copy the notes' derivations). Give numerical values for h_0, V_0, m_0 using the following baseline parameters:

Volume of bottle	$\mathcal{V}_{\rm empty}$	2.40×10^{-3}	$\rm m^3$
Exit area	A_e	3.5×10^{-4}	$\rm m^2$
Bottle mass	$m_{\rm empty}$	4.90×10^{-2}	kg
Launch rod length	ℓ	1.78×10^{-1}	m
Atm. pressure	$p_{\rm atm}$	1.01×10^{5}	Pa
Atm. temperature	$T_{\rm atm}$	288	K
Ideal gas constant	R	287	J/kg K
Water density	$\rho_{\rm w}$	1.0×10^{3}	$\rm kg/m^3$
Charge gauge pressure	$(p_c)_{\rm gauge}$	3.44×10^{5}	Pa
Charge gauge pressure	$(p_c)_{\rm gauge}$	50	psi
Water volume fraction	$f_{\mathcal{V}}$	0.6	
Gravity accel.	g	9.81	$\rm m/s^2$
Atm. density	$\rho_{\rm atm}$	1.225	$\rm kg/m^3$
Drag coefficient	C_D	0.15	
Bottle ref. area	A	9.9×10^{-3}	$\rm m^2$
Payload mass	m_p	5×10^{-2}	kg

Notes:
The charge "gauge pressure" $(p_c)_{\rm gauge}$ is what's displayed on the air pump's pressure gauge. The corresponding absolute charge pressure is $p_c = (p_c)_{\rm gauge} + p_{\rm atm}$.

The water volume fraction is $f_{\mathcal{V}} = \mathcal{V}_{\rm w}/\mathcal{V}_{\rm empty}$, where $\mathcal{V}_{\rm w}$ is the initial water volume.

2) Derive and present the necessary equations and expressions needed to compute the instantaneous rocket thrust T from some arbitrary rocket state h, V, m.

3) Modify your Lab 2 spreadsheet or code, incorporating your expressions from parts 1) and 2). Use the baseline parameters above to compute a rocket trajectory at least until the maximum height is reached. Choose a suitable time step Δt, to ensure adequate accuracy. To keep the number of time steps reasonable, it is suggested that you use two different Δt's: a small Δt for the thrust phase, and a significantly longer one for the ballistic phase. However, this is optional.

Plot your computed $h(t)$, $V(t)$, $m(t)$. State the maximum height $h_{\rm max}$ you predicted. Calculate also the maximum value for the product of height and payload mass, $(h * m_p)_{\rm max}$. This will be your figure of merit for the rocket competition.