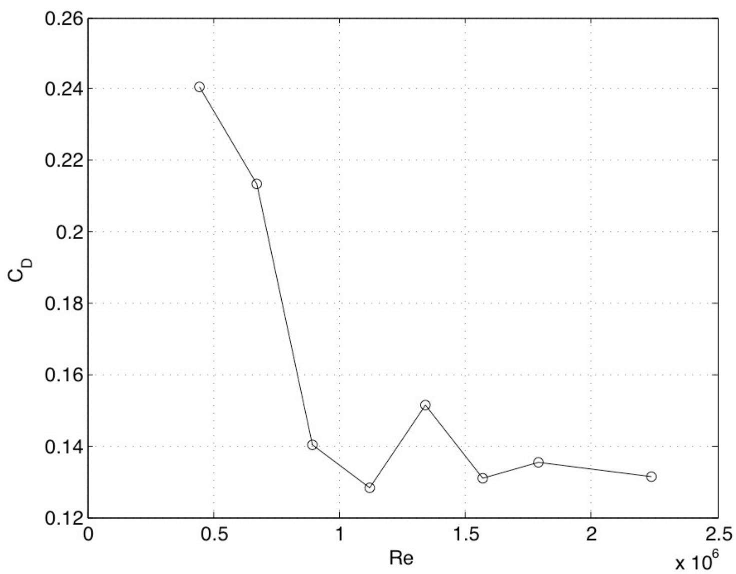
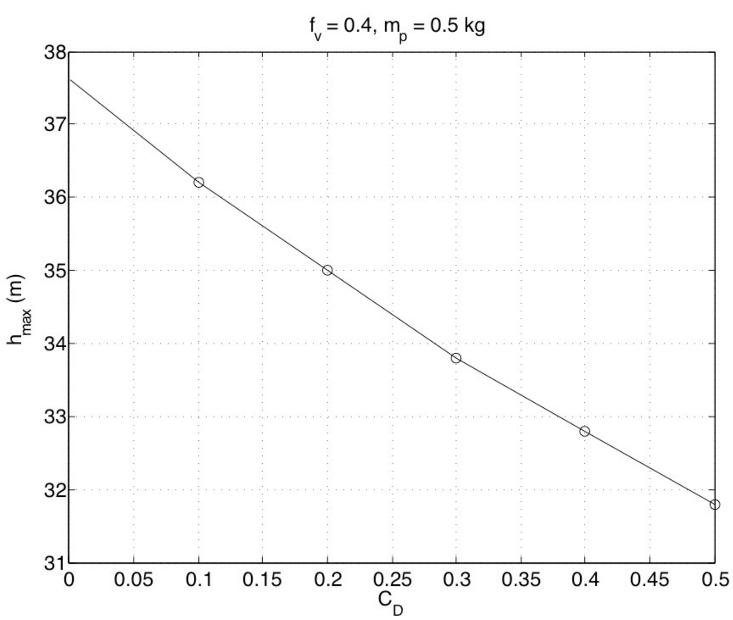
Solutions SL/5

1) The Reynolds number is defined as: $\mathrm{Re} = \dfrac{\rho V L}{\mu}$.

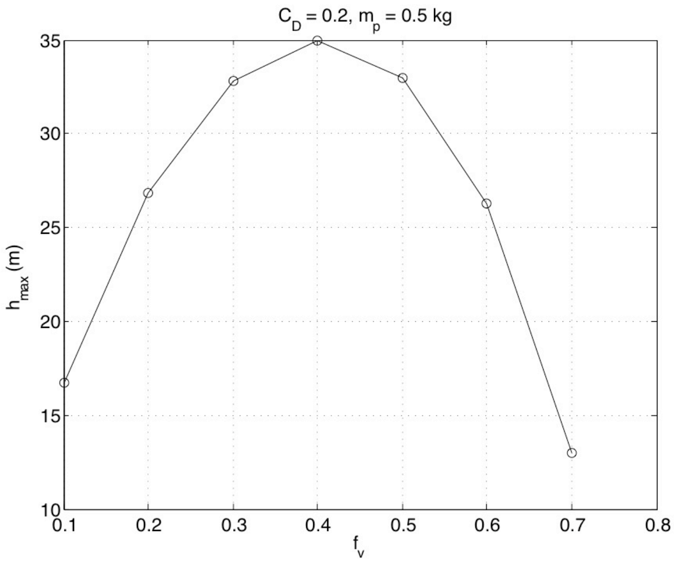
The following plot is constructed using data for air and our rocket:
$\rho = 1.22\ \mathrm{kg \cdot m^{-3}}$, $\mu = 1.8 \times 10^{-5}\ \mathrm{Pa \cdot s}$ and $L = 30$ cm



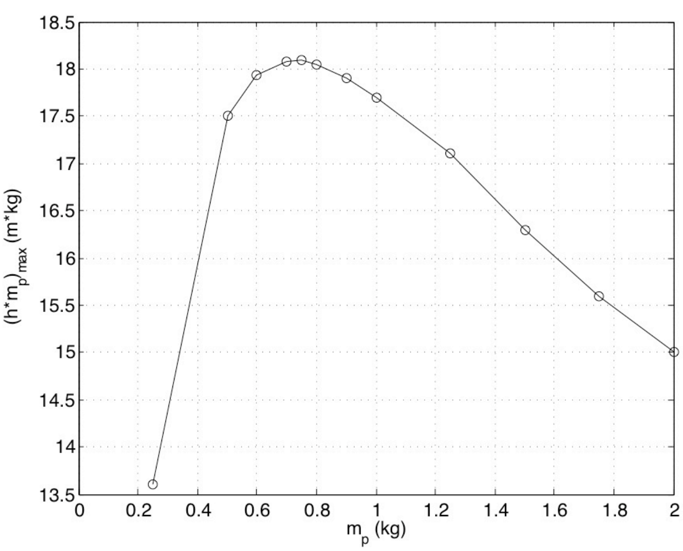
2a) It is clear that lower C_D's lead to higher altitudes. This is not as dramatic in our case with a relatively heavy payload mass, since the rocket velocity is not as high. But the effect is still clearly noticeable.



2b) There is a maximum in the curve since there is a trade-off between the amount of water in the rocket and the amount of pressurized air. Too much water means a heavy rocket and low air expansion capacity, while too little means that water will be exhausted while air is still over-pressurized.



2c) Again we have a trade-off, now between the payload mass and the rocket altitude. If the payload mass is too low, the product $h*m_p$ will also be low, even though the altitude will be the highest. If the payload mass is too large, the rocket will not climb as much and again the product $h*m_p$ will be low. There is a unique selection of payload mass that maximizes this product.



Sample of MATLAB code.
This version leaves left-over water inside the rocket if the air pressure becomes atmospheric.

```
% operating conditions
rho = 1.225;
rhow = 1000;
patm = 101325;
Cd = 0.2;
g = 9.8;
gamma = 1.4;

% Initial masses and volumes
me = 0.049;
mp = .9;
Ve = 0.0024;
fv = 0.4;
lrod = 0.178;
Ae = 0.00035;
A = 9.9e-3;
Vrod = Ae*lrod;
Vc = Ve*(1-fv) - Vrod;
V0 = Ve*(1-fv);
m0 = me + mp + Ve*fv*rhow;

% Charging pressure
pgauge = 450000;
pc = pgauge + patm;
p0 = pc*(Vc/V0)^gamma;

% mass of water that remains in the rocket
mw = rhow*Ve*(1-V0/Ve*(p0/patm)^(1/gamma));
if mw <= 0
    mw = 0;
end

% initial conditions
W = (pc*Vc - p0*V0)/(gamma-1) - patm*(V0-Vc);
v0 = sqrt(2*(W/m0 - g*lrod));
ue0 = sqrt(2*(p0-patm)/rhow);
mdot0 = rhow*ue0*Ae;
T0 = mdot0*ue0;
v(1) = v0;
h(1) = lrod;
m(1) = m0;
mdot(1) = mdot0;
T(1) = T0;
t(1) = 0;
```

```
Vol(1) = V0;
ue(1) = ue0;

% time step
dt = 0.001;

% initial sign of the velocity vector
vect = 1;

j = 1;
pressflag = 0;
while h(j)>=0,
    j = j+1;
    v(j) = v(j-1) + dt*(vect*mdot(j-1)*ue(j-1)/m(j-1) -
0.5*Cd*A*rho*v(j-1)*abs(v(j-1))/m(j-1) - g);
    h(j) = h(j-1) + v(j-1)*dt;
    Vol(j) = Vol(j-1) + mdot(j-1)/rhow*dt;
    m(j) = m(j-1) - mdot(j-1)*dt;
    t(j) = t(j-1)+dt;
    if m(j) <= me+mp+mw
        pressflag = 1;
    end
    if pressflag == 0
        p = p0*(V0/Vol(j))^gamma;
    else
        p=patm;
        dt = 0.01;
    end
    ue(j) = sqrt(2*(p-patm)/rhow);
    mdot(j) = rhow*ue(j)*Ae;
    T(j) = vect*mdot(j)*ue(j);
    if h(j)<h(j-1)
        vect = -1;
    end
end
```