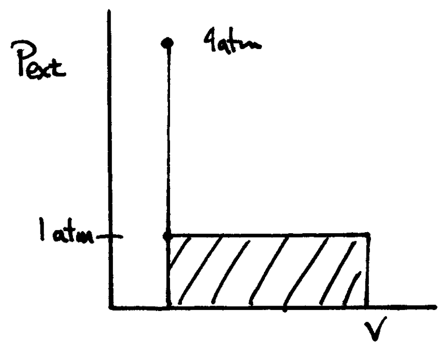
SOLUTIONS TO T5 BY WAITZ

a) All weights removed instantaneously



$P_{initial} = 4\,atm$, $T_i = 300K$

$$V_i = \frac{287 \cdot 300}{405.3\,kPa} = 0.21\,m^3/kg$$

WE KNOW $P_{final} = 1\,atm$ BUT WE DO NOT KNOW T_f OR V_f.

* NOTE THAT WE EXPECT $T_f < T_i$ SINCE IT IS A THERMALLY-INSULATED (ADIABATIC) CYLINDER AND WE ARE GETTING WORK OUT SO ΔU SHOULD BE NEGATIVE.

CONSIDER 1st LAW FOR ADIABATIC PROCESS:

$\Delta u = \cancel{q}^{\,0} - w$ so $C_v \Delta T = -P_{ext} \Delta V$

$C_v (T_f - T_i) = -P_{ext}(V_f - V_i)$ } 2 eqns

ideal gas: $P_f V_f = R T_f$ } 2 unknowns (V_f & T_f)

LET $V_f = \frac{RT_f}{P_f}$ $C_v(T_f - T_i) = -P_{ext}\left(\frac{RT_f}{P_f} - V_i\right)$

AT THE FINAL STATE $P_f = P_{ext}$ (it comes to thermodynamic equilibrium)

SO $C_v T_f - C_v T_i = -RT_f + P_{ext} V_i$

$(C_v + R) T_f = P_{ext} V_i + C_v T_i$

$$\boxed{T_f = \frac{P_{ext} V_i + C_v T_i}{C_v + R}} \;*$$

PLUG IN SOME NUMBERS

$$T_f = \frac{(101325)(0.21) + (716.5)(300)}{716.5 + 287} = 235.4\ K$$

$$P_f = P_{ext} = 101325\ N/m^2$$

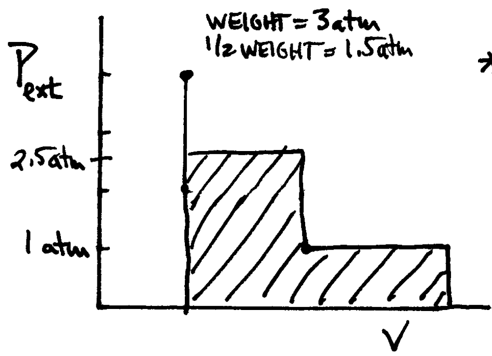
$$PV = RT \Longrightarrow V_f = 0.67\ m^3/kg$$

$$\Delta u = -\omega$$

$$\omega = -C_v(235.4 - 300) = +46.3\ kJ/kg$$

work by system

b) Two step process (expect more work out of system!)



* Define subscript "m" as middle condition (½ weight removed)

USE SAME EQUATIONS

$$T_m = \frac{P_{ext①} V_i + C_v T_i}{C_v + R} = \frac{253312 \cdot 0.21 + 716.5 \cdot 300}{716.5 + 287}$$

$$= 267.2K$$

$$P_m = 253312\ N/m^2$$

$$V_m = 0.30\ m^3/kg \qquad W_① = 23.5\ kJ/kg$$

$$T_f = \frac{P_{ext②} V_m + C_v T_m}{C_v + R} = \frac{101325 \cdot 0.30 + 716.5 \cdot 267.2}{716.5 + 287}$$

$T_f = 221\text{ K}$, $p_f = 101325\text{ Pa}$, $v_f = 0.63\text{ m}^3/\text{kg}$

$w_{②} = -716.5\,(221 - 267.2) = 33.1\text{ kJ/kg}$

$w_{total} = w_{①} + w_{②} = 56.6\text{ kJ/kg}$

c) Quasi-static process (expect the <u>most</u> work)
adiabatic

$\Rightarrow pv^{\gamma} = \text{const.}$ $\quad \dfrac{T_f}{T_i} = \left(\dfrac{p_f}{p_i}\right)^{\gamma-1/\gamma} = \left(\dfrac{1}{4}\right)^{0.4/1.4} = 0.673$

$\therefore T_f = 202\text{K}$

$p_f = 101325\text{ Pa}$

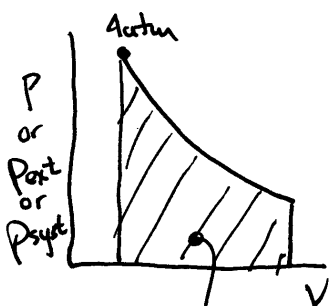
$v_f = 0.572\text{ m}^3/\text{kg}$

$\Delta u = \cancel{q}^{\,0} - w \qquad c_v\Delta T = -w$

$w = -716.5\,(202 - 300) = 70.2\text{ kJ/kg}$

MESSAGES:

1. THE AMOUNT OF WORK EXTRACTED DEPENDS ON PATH
2. IN CALCULATING WORK FOR <u>NON</u>-QUASI-STATIC PROCESSES, NEED TO USE p_{ext}.
3. THE CLOSER WE APPROACH A QUASI-STATIC PROCESS, THE MORE WORK WE GET OUT OF THE SYSTEM. THEREFORE IN DESIGNING AN ENGINE WE WANT TO LIMIT NON-QUASI-STATIC PROCESSES TO EVERY EXTENT POSSIBLE.



we get the most area under the curve