

Problem T4 (Unified Thermodynamics): SOLUTIONS

a) Describe the energy exchange processes in the device in terms of heat, work and various forms of energy. (LO's #1, #2)

External work is done on the upper chamber by the weight. The potential energy of the weight is reduced and the internal energy of the gas in the upper chamber is increased. The process is not quasi-static. Then heat is gradually transferred from the upper chamber to the lower chamber. During this process the internal energy of the upper chamber is decreased and the internal energy of the lower chamber is increased. As the heat is transferred from the upper chamber, work continues to be done on the upper chamber since the piston is free to move (and the potential energy of the weight continues to decrease). When the processes are over, the surroundings have provided energy (from the change in potential energy of the weight). The two chambers have received the energy, and it appears as increased internal energy of each of the chambers.

b) What processes will you use to model this system? Why?
(LO's #2, #4, #5)

The first process is not quasi-static but it is adiabatic because we expect pressure to equilibrate faster than the time it takes for appreciable heat transfer to occur. Therefore the upper chamber should be modeled as adiabatic with the work determined by considering the external pressure times the change in volume. After this first relatively fast process, we can assume that the heat transfer takes place more slowly. It is no longer adiabatic (both chambers have heat transfer with one another). But the upper chamber undergoes constant pressure cooling since the piston is free to move, but the weight remains on it. Since the cooling is relatively slow we can assume that the process in the upper chamber is quasi-static in terms of evaluating the work as $p dv$. The process in the lower chamber is constant volume heating (no work).

c) What is the temperature the gas in the upper chamber comes to shortly after the instantaneous dropping of the weight? (LO #4)

The process is an impulsive compression (not quasi-static). You are given the initial state. You are told it is an adiabatic process, and you know the external pressure that is applied. The first law with $q=0$ becomes:

$$\Delta u = -w = -p_{\text{ext}}(v_2 - v_1) \quad \text{or} \quad c_v(T_2 - T_1) = -p_{\text{ext}}(v_2 - v_1)$$

You know T_1, v_1 , but not T_2 and v_2 . However you know from the ideal gas law that

$$v_2 = \frac{RT_2}{p_2} = \frac{RT_2}{p_{\text{ext}}} \quad \text{since } p_2 = p_{\text{ext}} \text{ when the system comes to pressure equilibrium. Therefore,}$$

$$c_v(T_2 - T_1) = -p_{\text{ext}} \left(\frac{RT_2}{p_{\text{ext}}} - v_1 \right) = -p_{\text{ext}} \left(\frac{RT_2}{p_{\text{ext}}} - \frac{RT_1}{p_1} \right) = p_{\text{ext}} \frac{RT_1}{p_1} - RT_2$$

$$\text{so} \quad c_v T_2 - c_v T_1 = p_{\text{ext}} \frac{RT_1}{p_1} - RT_2 \quad \text{or} \quad (c_v + R)T_2 = p_{\text{ext}} \frac{RT_1}{p_1} + c_v T_1$$

$$(c_v + c_p - c_v)T_2 = T_1 \left(\frac{R p_{\text{ext}}}{p_1} + c_v \right) \quad \text{or} \quad T_2 = \frac{T_1}{c_p} \left(\frac{R p_{\text{ext}}}{p_1} + c_v \right)$$

$$T_2 = \frac{300\text{K}}{1003.5\text{J/kgK}} \left(\frac{287\text{J/kgK}(1000 \times 10^3 \text{Pa})}{100 \times 10^3 \text{Pa}} + 716.5\text{J/kgK} \right) = 1072\text{K}$$

d) What is the temperature the gas in the lower chamber comes to when the whole system eventually reaches thermodynamic equilibrium? (LO #4)

The upper chamber slowly cools in a constant pressure cooling process. The lower chamber slowly heats in a constant volume heating process. Other than the rigid copper wall separating the two chambers everything else is thermally-insulated, so the heat transferred from the upper chamber is equal to the heat transferred to the lower chamber.

The easiest way to do this is to use two different forms of the first law for the two chambers.

$du = \delta q - pdv$ is convenient for the constant volume process in the lower chamber

$dh = \delta q + vdp$ is convenient for the constant pressure process in the upper chamber

So for the lower chamber $c_v(T_{3_{\text{lower}}} - T_{2_{\text{lower}}}) = \delta q$ since volume is constant

and for the upper chamber $c_p(T_{3_{\text{upper}}} - T_{2_{\text{upper}}}) = \delta q$ since pressure is constant

Here I have assigned state 2 as the state right after the upper chamber comes to pressure equilibrium. For the lower chamber, this is the same as state 1 (for my model, I assumed that no appreciable heat is transferred to the lower chamber during the first process).

The heat that leaves the upper chamber is the same magnitude, but opposite in sign from that which is added to the lower chamber. So with the addition of a negative sign, we can equate the two first law expressions.

$$c_p(T_{3_{\text{upper}}} - T_{2_{\text{upper}}}) = -c_v(T_{3_{\text{lower}}} - T_{2_{\text{lower}}})$$

and we know that $T_{2_{\text{lower}}} = T_1 = 300\text{K}$, and $T_{2_{\text{upper}}} = 1072\text{K}$ from above, and $T_{3_{\text{upper}}} = T_{3_{\text{lower}}}$ since the system comes to thermal equilibrium!

$$1003.5\text{J/kgK}(T_{3_{\text{lower/upper}}} - 1072\text{K}) = -716.5\text{J/kgK}(T_{3_{\text{lower/upper}}} - 300\text{K})$$

So $T_{3_{\text{upper}}} = T_{3_{\text{lower}}} = 750.5\text{K}$