Kelvin's Theorem: $$\frac{d\Gamma}{dt} = 0 \qquad \text{(for a drifting circuit)}$$

Vertical velocity distribution of vortex sheet $\gamma(\xi)$ on x-axis:

$$w(x) = \frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)}{\xi - x}\, d\xi$$

Trigonometric coordinate substitutions:

$$x = \frac{c}{2}\,(1-\cos\theta_o) \qquad \xi = \frac{c}{2}\,(1-\cos\theta) \qquad d\xi = \frac{c}{2}\,\sin\theta\, d\theta$$

Fundamental equation of Thin Airfoil Theory:

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)}{x-\xi}\, d\xi = V_\infty\left(\alpha - \frac{dZ}{dx}\right) \qquad \text{(for } 0 < x < c\text{)}$$

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\,\sin\theta}{\cos\theta - \cos\theta_o}\, d\theta = V_\infty\left(\alpha - \frac{dZ}{dx}\right) \equiv V_\infty\, f(\theta_o) \qquad \text{(for } 0 < \theta_o < \pi\text{)}$$

Cosine-series function representation and Fourier Analysis:

$$f(\theta_o) = A_0 - \sum_{n=1}^{N} A_n \cos n\theta_o$$

$$A_0 = \frac{1}{\pi}\int_0^\pi f(\theta)\, d\theta = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dZ}{dx}\, d\theta$$

$$A_n = -\frac{2}{\pi}\int_0^\pi f(\theta)\,\cos n\theta\, d\theta = \frac{2}{\pi}\int_0^\pi \frac{dZ}{dx}\,\cos n\theta\, d\theta$$

Series solution to Fundamental equation of T.A.T.:

$$\frac{\gamma(\theta)}{2V_\infty} = \frac{1}{4}\Delta C_p(\theta) = A_0\,\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{N} A_n \sin n\theta$$

Lift and moment coefficients:

$$c_\ell = \int_0^c \Delta C_p\, dx = \pi\,(2A_0 + A_1) = 2\pi(\alpha - \alpha_{L=0})$$

$$c_{m,c/4} = \int_0^c \left(\frac{1}{4} - \frac{x}{c}\right)\Delta C_p\, dx = \frac{\pi}{4}\,(A_2 - A_1)$$

T.A.T. summary:

$$\alpha\,,\ \frac{dZ}{dx}(\theta_o) \ \xrightarrow{\text{Fourier analysis}}\ A_0\,,\ A_1 \ldots A_N \ \xrightarrow{\text{series summing}}\ \gamma(\theta) \ \xrightarrow{\text{chordwise integration}}\ c_\ell\,,\ c_m$$

Orthogonality properties of sine and cosine functions:

$$\int_0^\pi \sin n\theta\,\sin m\theta\, d\theta = \begin{cases} \pi/2 & \text{(if } n = m\text{)} \\ 0 & \text{(if } n \neq m\text{)} \end{cases}$$

$$\int_0^\pi \cos n\theta\,\cos m\theta\, d\theta = \begin{cases} \pi & \text{(if } n = m = 0\text{)} \\ \pi/2 & \text{(if } n = m \neq 0\text{)} \\ 0 & \text{(if } n \neq m\text{)} \end{cases}$$

Glauert integral:

$$\int_0^{\pi} \frac{\cos n\theta}{\cos\theta - \cos\theta_o}\, d\theta \;=\; \frac{\pi\, \sin n\theta_o}{\sin\theta_o}$$

Biot Savart Law:

$$\vec{V}(x,y,z) \;=\; \frac{\Gamma}{4\pi} \int_{-\infty}^{+\infty} \frac{d\vec{\ell} \times \vec{r}}{|\vec{r}|^3}$$

Wing-wake vortex sheet strength:

$$\gamma(y) \;=\; -\frac{d\Gamma}{dy}$$

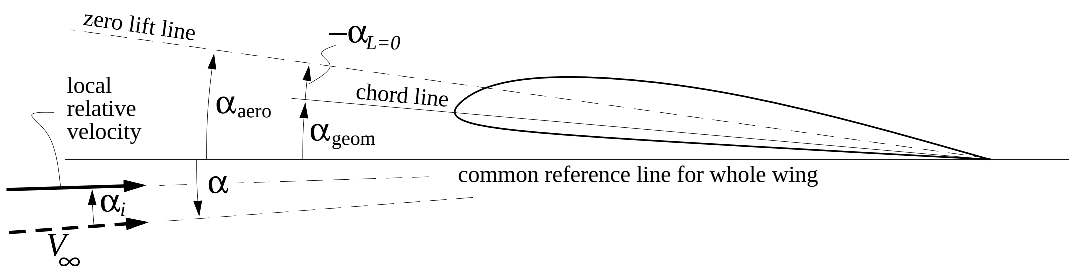
Trigonometric coordinate substitutions:

$$y_o \;=\; \frac{b}{2}\, \cos\theta_o \qquad\qquad y \;=\; \frac{b}{2}\, \cos\theta \qquad\qquad dy \;=\; -\frac{b}{2}\, \sin\theta\, d\theta$$

Induced angle distribution along wing:

$$\alpha_i(y_o) \;=\; \frac{-w(y_o)}{V_\infty} \;=\; \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \gamma(y)\, \frac{dy}{y - y_o} \;=\; \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \frac{d\Gamma}{dy}\, \frac{dy}{y_o - y}$$

Angle relations at a spanwise station:



Circulation/chord/angle relations:

$$\Gamma(y) \;=\; \frac{1}{2}\, V_\infty\, c(y)\, c_\ell(y) \;=\; \frac{1}{2}\, V_\infty\, c(y)\, a_o \left[\alpha + \alpha_{\rm aero}(y) - \alpha_i(y)\right]$$

Sine-series circulation representation and associated induced angle:

$$\Gamma(\theta) \;=\; 2bV_\infty \sum_{n=1}^{N} A_n\, \sin n\theta$$

$$\alpha_i(\theta_o) \;=\; \sum_{n=1}^{N} nA_n\, \frac{\sin n\theta_o}{\sin\theta_o}$$

Overall Lift and Induced Drag results:

$$L \;=\; \int_{-b/2}^{b/2} \rho V_\infty \Gamma\, dy \;=\; \frac{\pi}{2}\, \rho\, V_\infty^2 b^2\, A_1$$

$$D_i \;=\; \int_{-b/2}^{b/2} \rho\, V_\infty \Gamma(y)\, \alpha_i\, dy \;=\; \frac{\pi}{2}\, \rho\, V_\infty^2 b^2\, A_1^2 \left[1 \;+\; \sum_{n=2}^{N} n \left(\frac{A_n}{A_1}\right)^2\right] \;=\; \frac{(L/b)^2}{\frac{1}{2}\rho\, V_\infty^2\, \pi}\, [1 \;+\; \delta]$$

$$C_{Di} \;=\; \frac{C_L^2}{\pi\, e\, AR}$$

Level flight relations:

$$V \;=\; \left(\frac{2\, W/S}{\rho\, C_L}\right)^{1/2} \qquad\qquad P \;=\; \frac{1}{\eta_{\rm p}} \left(\frac{2\, W/S}{\rho}\right)^{1/2} W\, \frac{C_D}{C_L^{3/2}}$$