

Equation of state for ideal gas:

$$p = \rho RT = \frac{\gamma-1}{\gamma} \rho h \quad , \quad h = c_p T$$

Isentropic relations:

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1} \right)^{1/(\gamma-1)} \quad \frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)} = \left(\frac{\rho_2}{\rho_1} \right)^\gamma$$

Total enthalpy definition:

$$h_o \equiv h + \frac{1}{2} V^2 = e + \frac{p}{\rho} + \frac{1}{2} V^2 = e_o + \frac{p}{\rho}$$

Integral Enthalpy Equation (steady):

$$\oint \rho (\vec{V} \cdot \hat{n}) h_o dA = \iiint \rho \dot{q} dV + \iiint \rho \vec{g} \cdot \vec{V} dV$$

Convective form of enthalpy equation:

$$\begin{aligned} \frac{Dh_o}{Dt} &= \frac{1}{\rho} \frac{\partial p}{\partial t} + \dot{q} + \vec{g} \cdot \vec{V} \\ h_o &= \text{constant} \quad (\text{steady, adiabatic, no gravity}) \end{aligned}$$

Rankine-Hugoniot shock jump relations:

$$\begin{aligned} \rho_1 u_1 &= \rho_2 u_2 \\ \rho_1 u_1^2 + p_1 &= \rho_2 u_2^2 + p_2 \\ w_1 &= w_2 \\ h_1 + \frac{1}{2} u_1^2 &= h_2 + \frac{1}{2} u_2^2 \end{aligned}$$

Speed of sound, Mach number:

$$a^2 = \gamma p / \rho = \gamma RT = (\gamma-1) h \quad M \equiv V/a$$

Total pressure relation:

$$\frac{p_o}{p} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)} = \left(1 - \frac{V^2}{2h_o} \right)^{-\gamma/(\gamma-1)}$$

Shock jump functions:

$$\begin{aligned} M_{n_2}^2 &= \frac{1 + \frac{\gamma-1}{2} M_{n_1}^2}{\gamma M_{n_1}^2 - \frac{\gamma-1}{2}} \\ \frac{\rho_2}{\rho_1} &= \frac{(\gamma+1) M_{n_1}^2}{2 + (\gamma-1) M_{n_1}^2} \\ \frac{p_2}{p_1} &= 1 + \frac{2\gamma}{\gamma+1} (M_{n_1}^2 - 1) \\ \frac{T_2}{T_1} &= \frac{p_2 \rho_1}{p_1 \rho_2} \\ \frac{p_{o_2}}{p_{o_1}} &= \frac{p_2}{p_1} \left(\frac{1 + \frac{\gamma-1}{2} M_{n_2}^2}{1 + \frac{\gamma-1}{2} M_{n_1}^2} \right)^{\gamma/(\gamma-1)} = f(M_{n_1}) \end{aligned}$$

Rayleigh Pitot formula:

$$\frac{p_{o2}}{p_1} = \frac{p_{o2} p_2}{p_2 p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)} \right)^{\gamma/(\gamma-1)} \frac{1 - \gamma + 2\gamma M_1^2}{\gamma+1}$$

Oblique-shock relations:

$$\begin{aligned} M_{n1} &= M_1 \sin \beta \\ M_{n2} &= M_2 \sin(\beta - \theta) \\ \tan \theta &= \frac{2}{\tan \beta} \frac{M_1^2 \sin^2 \beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2} \end{aligned}$$

Prandtl-Meyer expansion-wave Mach/angle relation:

$$\begin{aligned} \theta_2 - \theta_1 &= \nu(M_2) - \nu(M_1) \\ \nu(M) &\equiv \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan \sqrt{\frac{\gamma-1}{\gamma+1} (M^2 - 1)} - \arctan \sqrt{M^2 - 1} \end{aligned}$$

Channel mass flow relations:

$$\dot{m} = \rho u A = \rho^* a^* A^* = \text{constant}$$

Sonic-flow quantities:

$$\begin{aligned} \rho^* &= \rho_o \left(1 + \frac{\gamma-1}{2} \right)^{-\frac{1}{\gamma-1}} \\ p^* &= p_o \left(1 + \frac{\gamma-1}{2} \right)^{-\frac{\gamma}{\gamma-1}} \\ u^* &= a^* = a_o \left(1 + \frac{\gamma-1}{2} \right)^{-\frac{1}{2}} \end{aligned}$$

Area-Mach relation:

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Derived channel mass flow relations:

$$\begin{aligned} \dot{m} &= \frac{\gamma p_o}{\sqrt{(\gamma-1)h_o}} M \left(1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma+1}{2(\gamma-1)}} A \\ \dot{m} &= \frac{\gamma p}{\sqrt{(\gamma-1)h_o}} M \left(1 + \frac{\gamma-1}{2} M^2 \right)^{1/2} A \end{aligned}$$