LIFT AND DRAG

WE CONCLUDE OUR TREATMENT OF INCOMPRESSIBLE FLOW OVER AERODYNAMIC SHAPES BY PRESENTING A SUMMARY OF LIFT AND DRAG. THE SPECIFIC DETAILS OF VISCOUS FLOW OVER THREE-DIMENSIONAL BODIES ARE BEYOND THE SCOPE OF OUR CLASS. HENCE, WE WILL PRESENT A SUMMARY.

DEFINITIONS

LIFT COEFFICIENT, C_L

$$C_L \equiv \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 S}$$

$L =$ TOTAL LIFT
$S =$ CHARACTERISTIC AREA

DRAG COEFFICIENT, C_D

$$C_D = \frac{D}{\frac{1}{2}\rho_\infty U_\infty^2 S}$$

$D =$ TOTAL DRAG

THE TOTAL DRAG CONSISTS OF THE SUM OF PROFILE DRAG AND THE INDUCED DRAG. THE PROFILE DRAG IS THE RESISTIVE FORCE DUE TO VISCOUS FRICTION (BOUNDARY LAYER) PLUS THE RESISTIVE FORCE DUE TO SHAPE AND FRONTAL AREA (FORM DRAG). HENCE

PROFILE DRAG = FRICTION DRAG (VISCOUS BOUNDARY LAYER)
+ FORM DRAG (NET AFT PRESSURE FORCE)

THE PROFILE DRAG IS INDEPENDENT OF THE LIFT FORCE AND THEREFORE EXISTS AT ZERO LIFT. THE PROFILE DRAG IS APPROXIMATELY CONSTANT FOR LOW SPEED FLOW. LET'S DENOTE PROFILE DRAG BY D_o AND THE PROFILE DRAG COEFFICIENT BY C_{D_o}.

$$D_o \equiv \text{PROFILE DRAG}$$

$$C_{D_o} \equiv \text{PROFILE DRAG COEFFICIENT.}$$

PROFILE DRAG MAY BE REFERRED TO AS <u>ZERO LIFT DRAG</u>.

THE INDUCED DRAG IS THE RESISTIVE FORCE DUE TO LIFT OR DUE TO THE CIRCULATION OR INDUCED ANGLE OF ATTACK. LET'S DENOTE INDUCED DRAG BY D_i AND THE INDUCED DRAG COEFFICIENT BY C_{D_i}.

$$D_i \equiv \text{INDUCED DRAG}$$

$$C_{D_i} \equiv \text{INDUCED DRAG COEFFICIENT}$$

$$D_i = L\alpha_i$$

α_i = INDUCED ANGLE OF ATTACK

$$\alpha_i = \frac{C_L}{\pi \mathcal{R} e}$$

$\mathcal{R}$ = ASPECT RATIO

e = OSWALD'S EFFICIENCY FACTOR

$$C_{D_i} = \frac{C_L^2}{\pi \mathcal{R} e}$$

WE EXPRESS THE TOTAL DRAG COEFFICIENT AS:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi AR e}$$

$$C_D = K_1 + K_2 C_L^2$$

WHERE THE SECOND EQUATION ASSUMES CONSTANT C_{D_L}, CONSTANT AR, AND CONSTANT e.

THE TOTAL DRAG MAY BE WRITTEN AS:

$$D = C_D \left(\tfrac{1}{2}\rho_\infty U_\infty^2 S\right)$$

$$D = C_{D_0}\left(\tfrac{1}{2}\rho_\infty U_\infty^2 S\right) + \frac{C_L^2}{\pi AR e}\left(\tfrac{1}{2}\rho_\infty U_\infty^2 S\right)$$

HENCE, FOR A GIVEN AIRCRAFT CONFIGURATION AND A SPECIFIED ALTITUDE, THE DRAG "SCALES" ON THE SQUARE OF THE SPEED. THUS

$$D = K_1' V^2 + K_2' C_L^2 V^2$$

FOR LEVEL, UNACCELERATED FLIGHT WHERE LIFT EQUAL WEIGHT, WE MAY WRITE

$$D = K_1' V^2 + \frac{2W^2}{\rho V^2 S \pi AR e}$$

$$D = K_1' V^2 + K_3 \cdot \frac{1}{V^2}$$

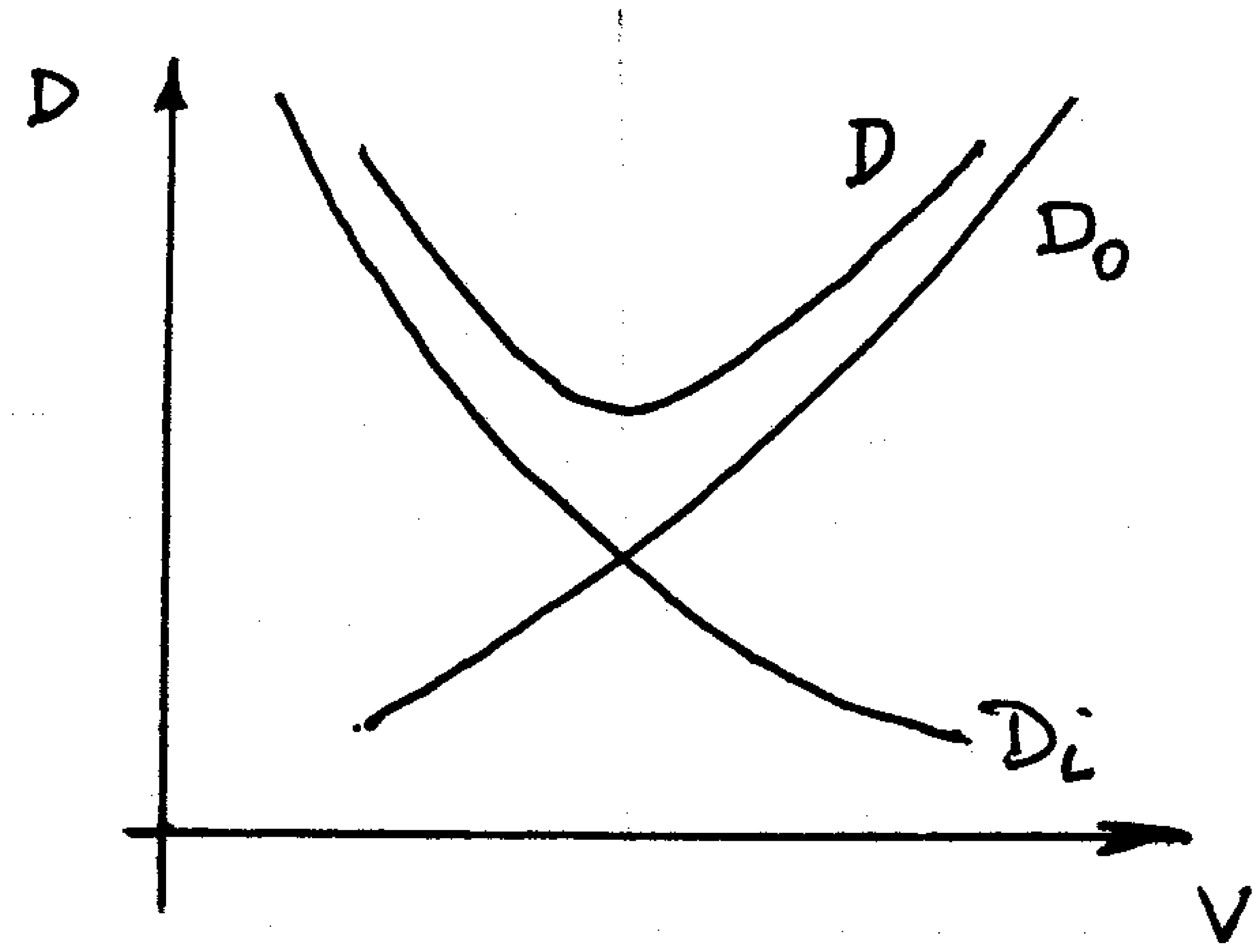
THE PROFILE DRAG IS PROPORTIONAL TO THE SQUARE OF THE SPEED AND THE INDUCED DRAG IS PROPORTIONAL TO THE INVERSE SQUARE OF THE SPEED.

MINIMUM DRAG

FOR LEVEL, UNACCELERATED FLIGHT,

$$D = D(V)$$

THE DRAG PLOTS AS FOLLOWS :



$$\frac{dD}{dV} = 0$$ DETERMINES THE MINIMUM DRAG POINT.

$$D = K_1' V^2 + \frac{K_3}{V^2}$$

$$\frac{dD}{dV} = 2K_1' V - 2\frac{K_3}{V^3} = 0$$

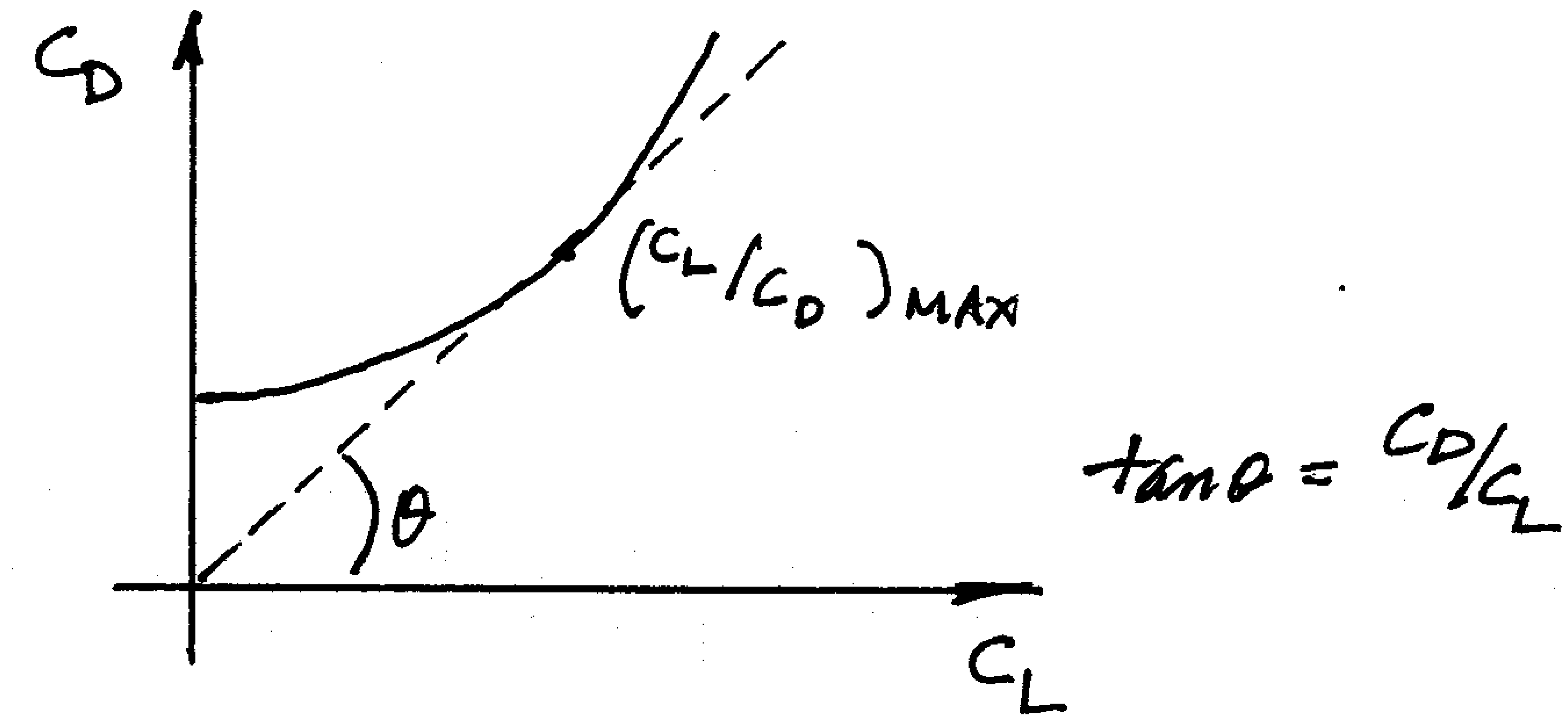
$$K_1' V^2 = \frac{K_3}{V^2}$$

$$D_o = D_i$$

$$C_{D_o} = C_{D_i}$$

HENCE, MINIMUM DRAG OCCURS WHERE THE PROFILE DRAG EQUALS THE INDUCED DRAG AND WHERE THE PROFILE DRAG COEFFICIENT EQUALS THE INDUCED DRAG COEFFICIENT.

BUT WHAT ABOUT THE LIFT? TO MINIMIZE DRAG, ONE MUST MAXIMIZE THE LIFT COEFFICIENT TO DRAG COEFFICIENT RATIO. A <u>DRAG POLAR PLOT</u> IS USEFUL.



A LINE PASSING THROUGH THE ORIGIN AND INTERCEPTING THE DRAG POLAR DEFINES AN ANGLE TO THE ORDINATE AXIS WHOSE TANGENT IS C_D/C_L. THE MINIMUM ANGLE IS AT THE MAXIMUM C_L/C_D RATIO OR MINIMUM C_D/C_L.