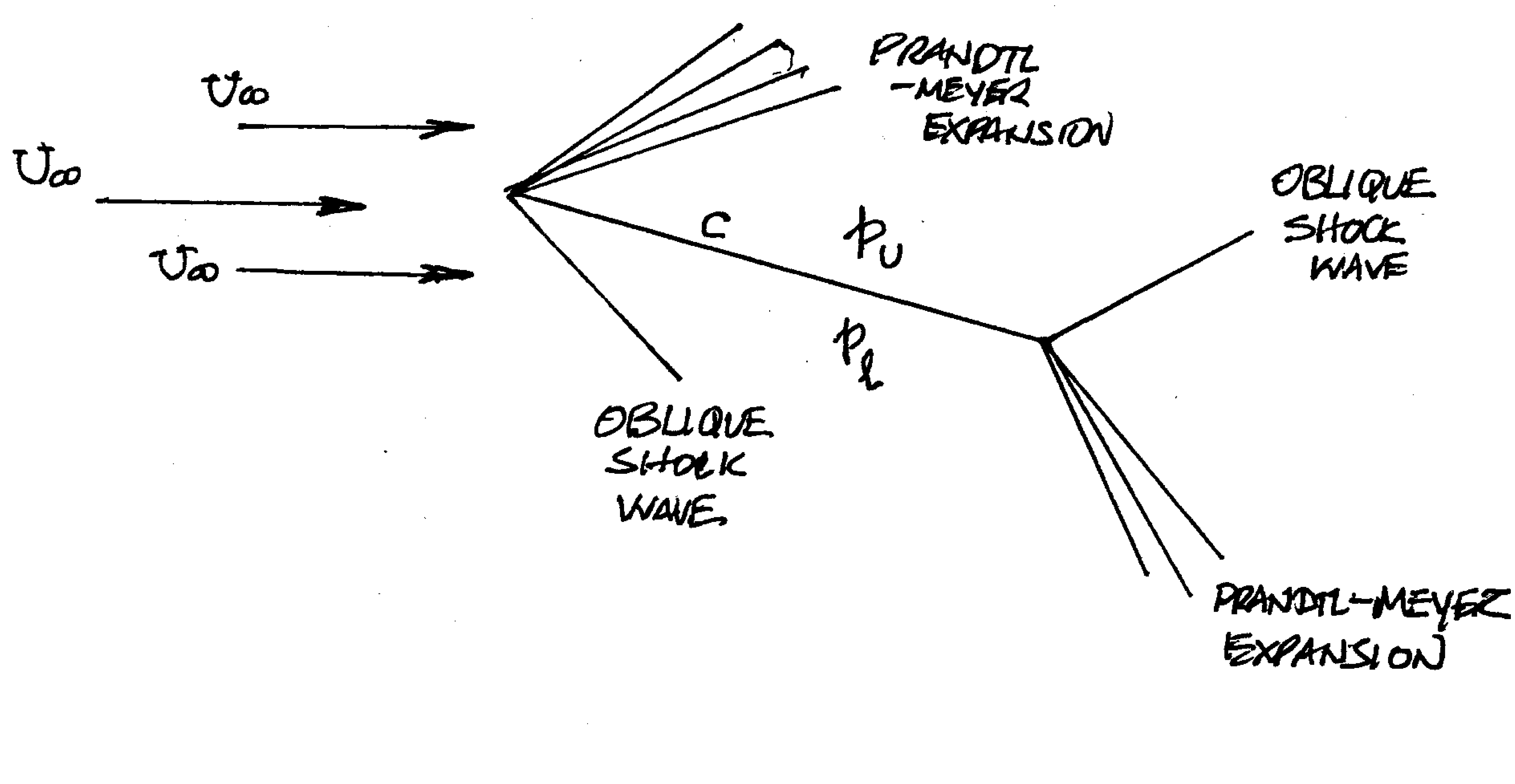
INVISCID COMPRESSIBLE FLOW

WE WILL CONSIDER SUBSONIC AND SUPERSONIC FLOWS IN DUCTS OR WIND TUNNELS AND OVER SLENDER TWO DIMENSIONAL AERODYNAMIC PROFILES. SOUND WAVES, NORMAL, AND OBLIQUE SHOCK WAVES WILL BE CONSIDERED. ISENTROPIC PRANDTL-MEYER EXPANSIONS WILL ALSO BE CONSIDERED.

IN NOT CONSIDERING TRANSONIC NOR HYPERSONIC FLOWS, WE WILL RETAIN LINEAR MODELS AND THE PRINCIPLE OF SUPERPOSITION. EXCEPT FOR THE "STRUCTURE" OF SHOCK WAVES OUR FLOWS WILL BE INVISCID.

A CRITICALLY IMPORTANT AND CONSISTENT FEATURE OF OUR STUDY OF INVISCID COMPRESSIBLE FLOWS IS THE PRESSURE DISTRIBUTION. IN THE CASE OF COMPRESSIBLE FLOWS THROUGH DUCTS, STREAMTUBES, AND WIND TUNNELS, IT IS THE STAGNATION OR SETTLING CHAMBER PRESSURE, THE BACK-PRESSURE AT THE EXIT FROM THE DUCT, AND THE PRESSURE PROFILE ALONG THE DIRECTION OF THE FLOW. FOR FLOW OVER AERODYNAMIC PROFILES, WE WILL BE CALCULATING THE PRESSURE DISTRIBUTION ON THE SURFACE OF THE AERODYNAMIC PROFILE. THIS LATER CASE WILL BE DEMONSTRATED IN THE SUPERSONIC THIN AIRFOIL THEORY (STAT) FOR THIN BODIES AT SMALL ANGLE OF ATTACK, $M > 1$. SUCH A FLOW IS SHOWN BELOW (FLAT PLATE AT ANGLE OF ATTACK, α).



WHAT DO WE MEAN BY COMPRESSIBILITY? IN A COMPRESSIBLE FLUID FLOW, CHANGES IN PRESSURE DRIVES CHANGES IN DENSITY

COMPRESSIBLE FLOWS: $\left|\frac{d\rho}{dp}\right| = \text{FINITE}$; $\frac{d\rho}{dp} > 0$

INCOMPRESSIBLE FLOWS: $\left|\frac{d\rho}{dp}\right| \ll 1$; $\frac{d\rho}{dp} \ll 1$

THE FORMULATION OF COMPRESSIBLE FLUID FLOW PROBLEMS IS FUNDAMENTALLY DIFFERENT FROM THAT OF INCOMPRESSIBLE FLUID FLOWS. CONSIDER THE FOLLOWING TABLE/COMPARISON.

	INCOMPRESSIBLE FLUID FLOWS $\lvert d\rho/dp\rvert \ll 1$	COMPRESSIBLE FLUID FLOWS $\lvert d\rho/dp\rvert = \text{FINITE}$
GOVERNING EQUATION	CONSERVATION OF MASS, LINEAR MOMENTUM	CONSERVATION OF MASS, LINEAR MOMENTUM, ENERGY, SECOND LAW, EQUATION OF STATE
UNKNOWNS. (DEPENDENT VARIABLES)	p, u, v, w	p, u, v, w, ρ, T, s

DEFINITIONS

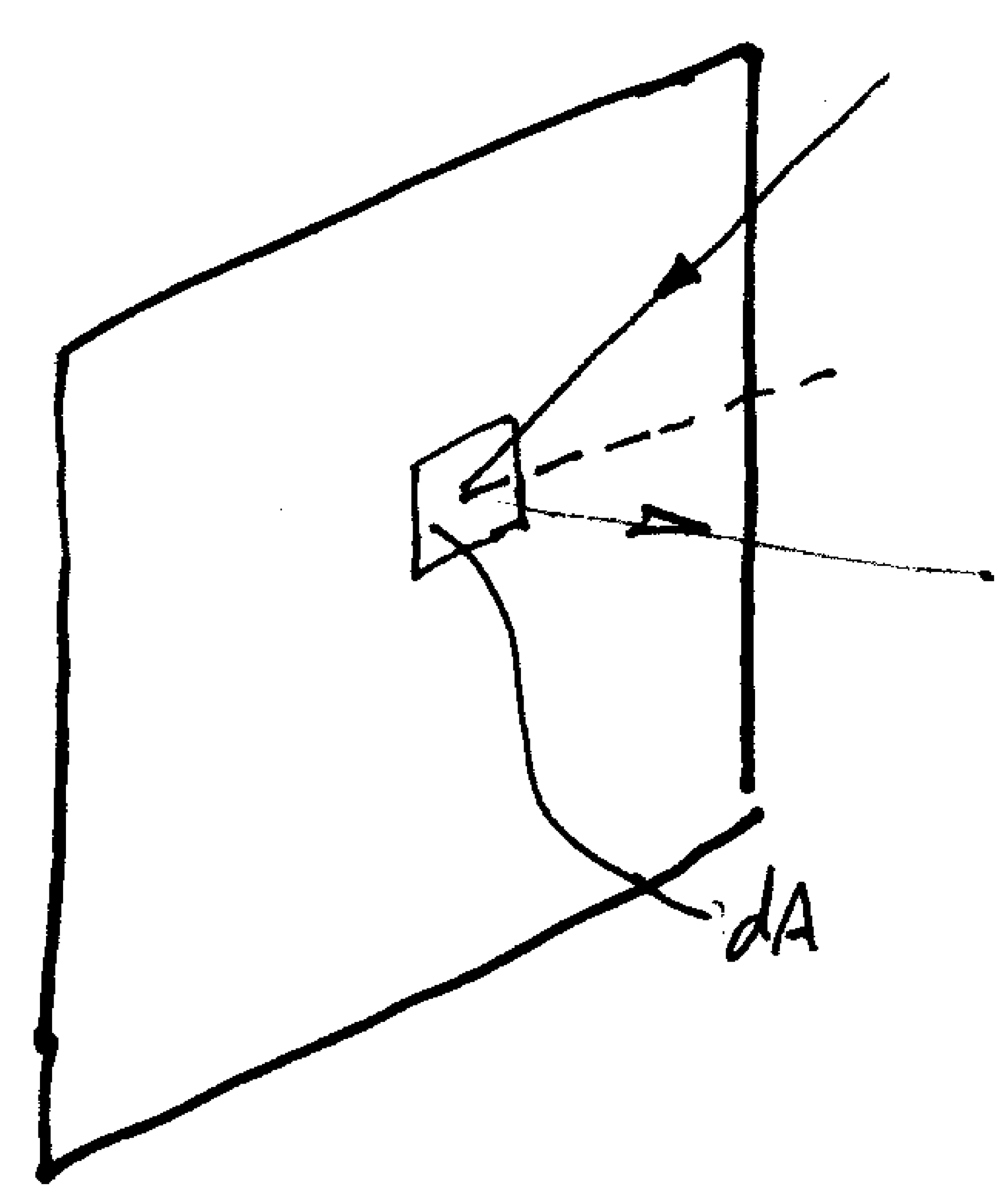
CONTINUUM : THE FUNDAMENTAL QUESTION IS THE RESOLUTION OF THE CONSERVATION PRINCIPLES IN AN EULERIAN SYSTEM. THERE IS NO DISPUTE NOR DISCOMFORT IN EXPRESSING THE CONSERVATION PRINCIPLES FOR INDIVIDUAL PARTICLES. BUT, HOW DO WE MANAGE 10^7 PARTICLES.

FLUID ELEMENT BECOMES THE CENTER OF ATTENTION

— CONSISTS OF MANY PARTICLES (MOLECULES) $\sim 10^7$

— SMOOTH, CONTINUOUS DISTRIBUTION OF THERMODYNAMIC AND DYNAMIC VARIABLES $T, \rho, p, \overline{V}$

PRESSURE : NET TRANSFER OF LINEAR MOMENTUM TO A SURFACE DUE TO THE COLLISION OF FLUID PARTICLES (MOLECULES) WITH THE SURFACE



DISTRIBUTION OF RANDOM, THERMAL SPEEDS

SUPERIMPOSE A MEAN SPEED → RESULTS ??

FRICTIONLESS: WE WILL NEGLECT THE PRESENCE OF BOUNDARY LAYERS ON THE SURFACE OF BODIES OVER WHICH A FLUID FLOWS. FOR REYNOLDS NUMBER OF THE ORDER OF 10^6, THIS IS A GOOD APPROXIMATION.

WE WILL ALSO NEGLECT VISCOUS EFFECTS IN THE FREE STREAM.

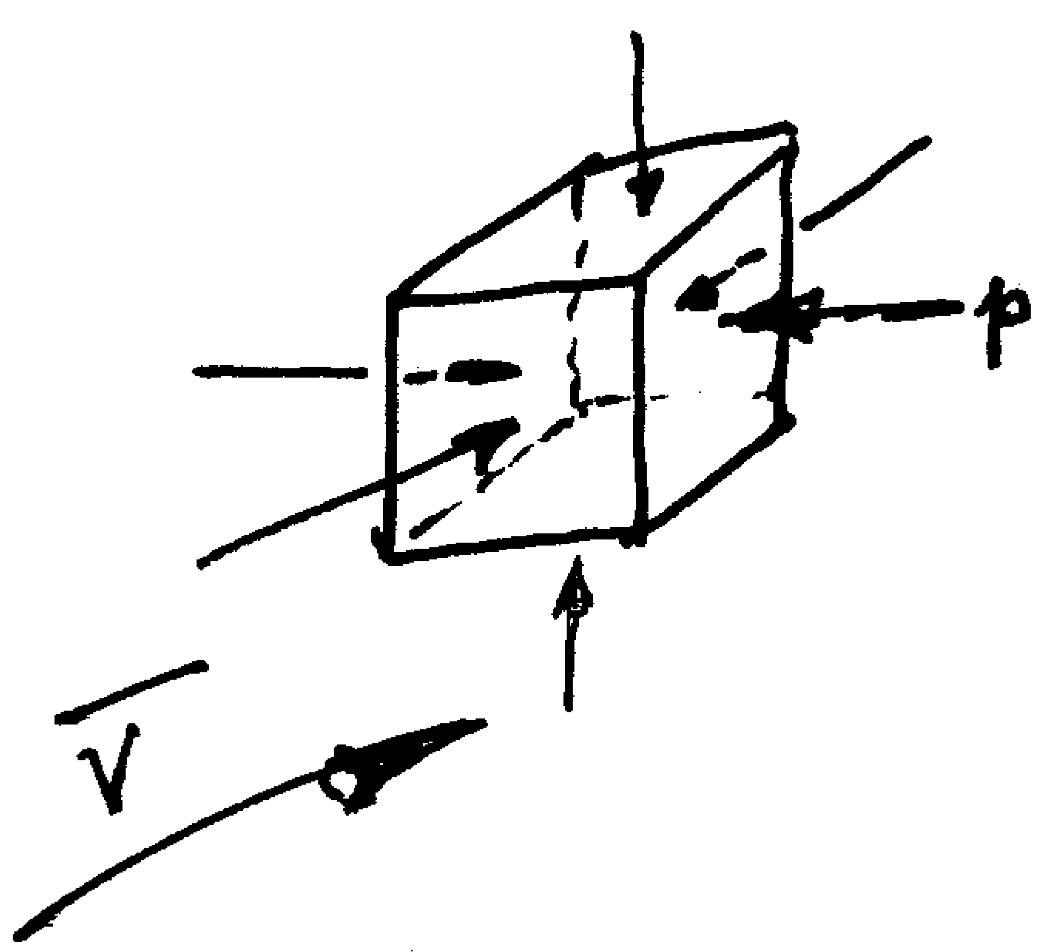
COMPRESSIBILITY:

$$\tau = \frac{d\rho}{\rho\, dp} = \frac{\text{PERCENT CHANGE IN DENSITY}}{\text{UNIT PRESSURE CHANGE}}$$

$$(dp)(\tau) = \frac{d\rho}{\rho}$$

$> 5\% \rightarrow$ COMPRESSIBLE FLUID FLOW

FROM BERNOULLI'S EQUATION



$$\frac{d\rho}{\rho} \sim M^2$$

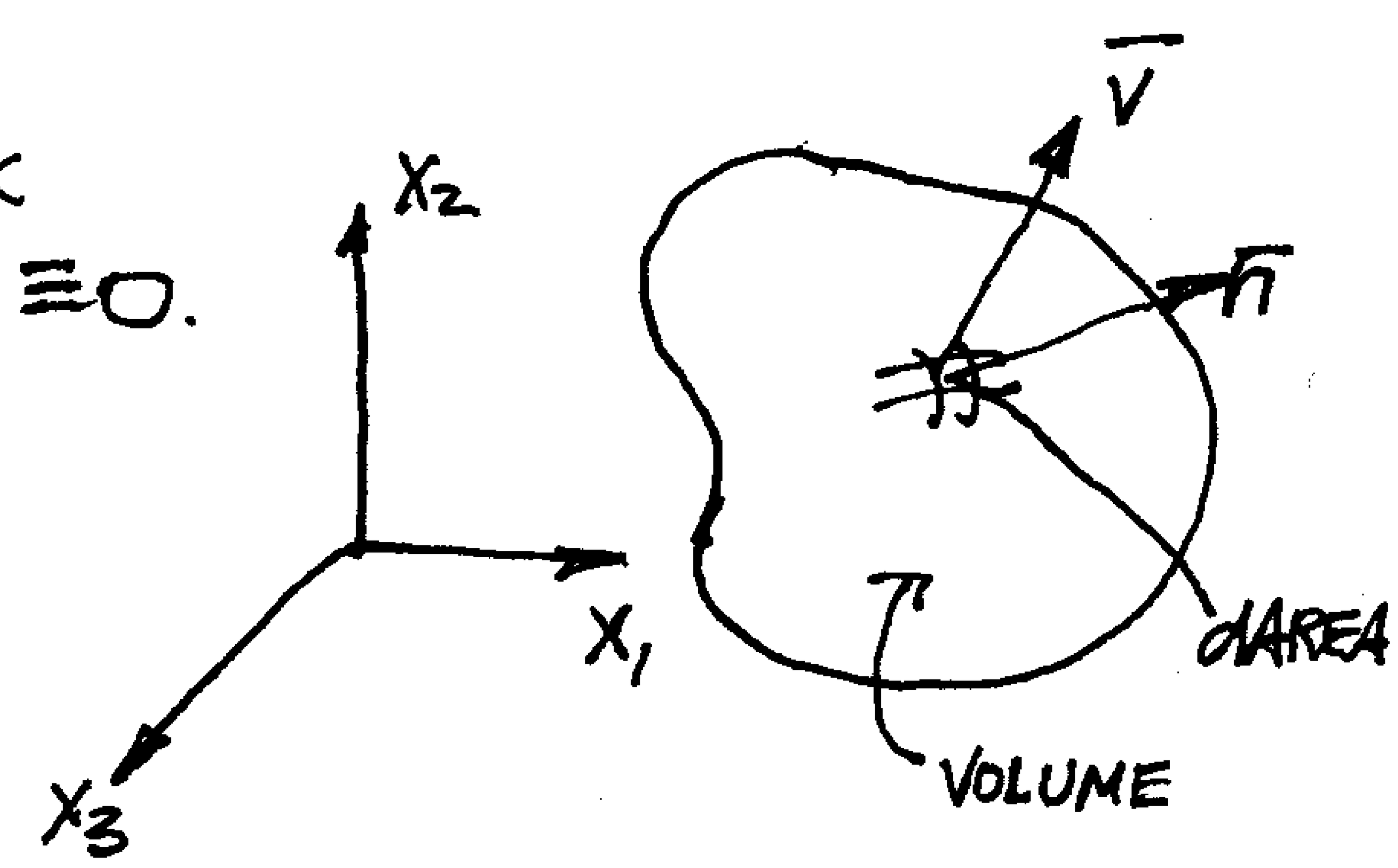
FLOW REGIMES

MACH NUMBER	FLOW REGIME	PDE	FEATURE
$0 \le M < 0.3$	INCOMPRESSIBLE	ELLIPTIC	CONTINUOUS PRESSURE PROFILE
$0.3 < M < 0.8$	SUBSONIC	ELLIPTIC	LINEAR
$0.8 < M < 1.15$	TRANSONIC	PARABOLIC	NON-LINEAR SHOCK WAVES C_p DIVERGENCE
$1.15 < M < 4$	SUPERSONIC	HYPERBOLIC	LINEAR SHOCK WAVES
$4 < M < 10-15$	HYPERSONIC	HYPERBOLIC	NON-LINEAR VERY STRONG SHOCK WAVES

CONSERVATION PRINCIPLES

TO ANALYZE COMPRESSIBLE FLOW PROBLEMS, WE MUST CONSERVE MASS, LINEAR MOMENTUM, AND ENERGY. WE WILL ALSO NEED AN EQUATION OF STATE, BOUNDARY CONDITIONS, AND INITIAL CONDITIONS. WE WILL DEVELOP THE CONSERVATION PRINCIPLES AND RELATED GOVERNING EQUATIONS IN INTEGRAL EQUATION FORMAT BASED ON THE EULERIAN FORMULATION.

MASS: THE RATE OF CHANGE OF MASS IN VOLUME VOL PLUS THE NET OUTWARD FLUX OF MASS THROUGH AREA ENCLOSING VOL $\equiv 0$.



$$\int_{VOL.} \frac{\partial \rho}{\partial t}\, dVOL + \oint_{AREA} \rho \bar{V} \cdot \bar{n}\, dAREA = 0$$

LINEAR MOMENTUM: THE RATE OF CHANGE OF LINEAR MOMENTUM IN VOLUME VOL PLUS THE NET OUTWARD FLUX OF LINEAR MOMENTUM THROUGH AREA ENCLOSING VOL EQUALS NET FORCE ON THE FLUID.

$$\int_{VOL.} \frac{\partial (\rho \bar{V})}{\partial t}\, dVOL + \oint_{AREA} (\rho \bar{V}) \bar{V} \cdot \bar{n}\, dAREA = \int_{VOL} \rho \bar{g}\, dVOL + \oint_{AREA} -p \bar{n}\, dA$$

ENERGY: THE RATE OF CHANGE OF ENERGY IN VOL PLUS NET OUTWARD FLUX OF ENERGY THROUGH AREA ENCLOSING VOL EQUAL RATE OF WORK DONE BY BODY FORCES PLUS RATE OF WORK DONE BY SURFACE FORCES PLUS RATE OF SHAFT WORK ADDED PLUS RATE OF HEAT CONDUCTED PLUS RATE OF INTERNAL HEAT GENERATED.

$$\int_{VOL} \frac{\partial}{\partial t}\left[\rho\left(e+\frac{1}{2}V^2\right)\right] dVOL + \oint_{AREA} \rho\left[e+\frac{p}{\rho}+\frac{1}{2}V^2\right]\bar{V}\cdot\bar{n}\, dAREA$$

$$= \dot{Q} + \oint_{AREA} -k\nabla T\cdot\bar{n}\, dAREA$$

WHERE

$Q \equiv$ INTERNAL HEAT GENERATED

$-k\nabla T \equiv$ HEAT CONDUCTED PER UNIT AREA