REVIEW: THERMODYNAMICS

THERMALLY PERFECT GAS

$$p = \rho R T$$

$$R = \text{CONSTANT}$$

CALORICALLY PERFECT GAS

$$c_p \equiv \text{CONSTANT}$$

$$c_v \equiv \text{CONSTANT}$$

$$e = c_v T$$

$$h = c_p T$$

ENTHALPY

$$h = e + p/\rho$$

$$R = c_p - c_v \quad (\text{CONSTRAINTS?})$$

ISENTROPIC CONDITIONS

$$\frac{p}{\rho^{\gamma}} = \text{CONSTANT}$$

CHANGE IN ENTROPY

$$T\,ds = dh - \frac{1}{\rho}\,dp$$

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p}{p_1} \quad (\text{CONSTRAINTS?})$$

STEADY, 1-D GAS DYNAMICS

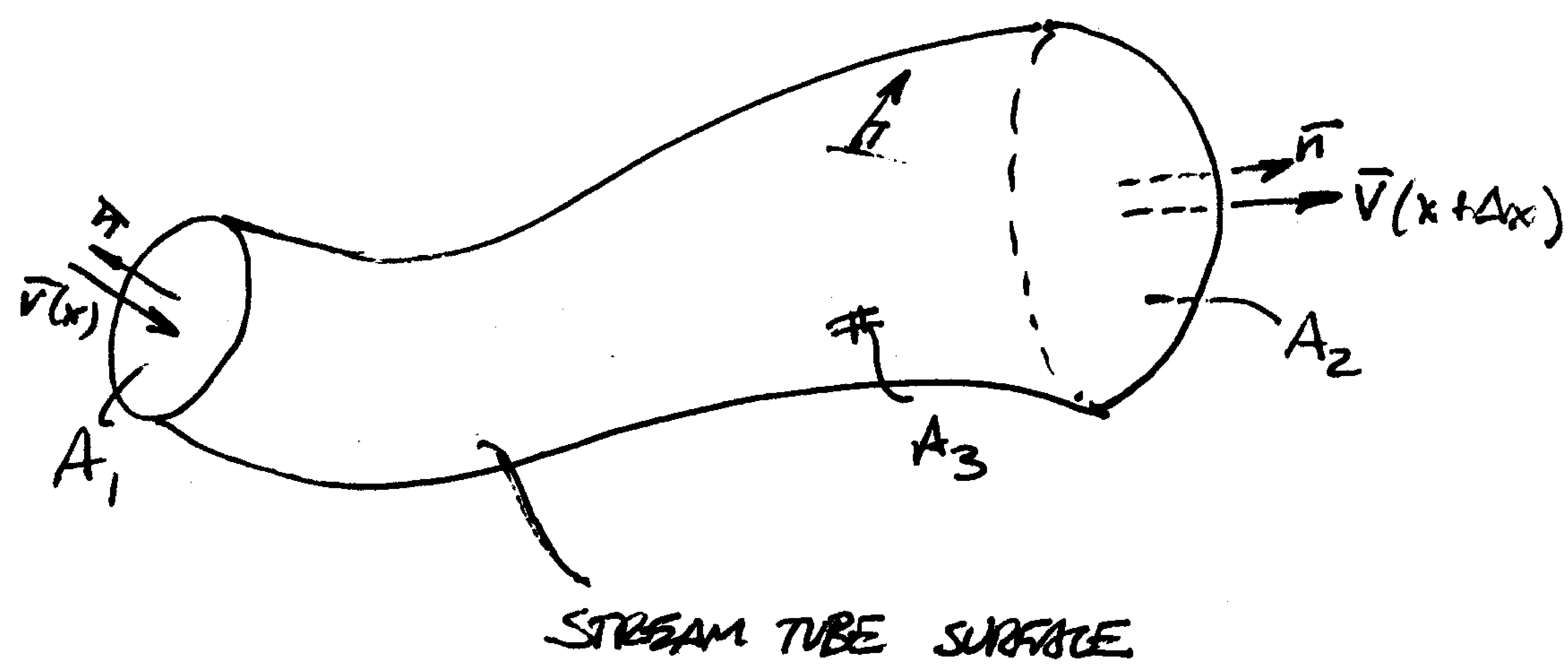
INTEGRAL FORM

ASSUMPTIONS

1. STEADY FLOW
2. INVISCID FLUID
3. ZERO BODY FORCES (ZERO GRAVITY)
4. ZERO SHAFT WORK

A. VELOCITY DEPENDS ON ONLY ONE COORDINATE, x

B. THERMODYNAMIC VARIABLES DEPEND ON ONLY ONE COORDINATE, x



ASSUMPTIONS 1, 2, 3, AND 4 APPLIED TO A STREAM TUBE YIELDS RESULTS A AND B.

CONSERVATION OF MASS

$$\int_{VOL} \frac{\partial \rho}{\partial t}\, dVol + \oint_{AREA} \rho \bar{V}\cdot\bar{n}\, dArea = 0$$

$$\int_{VOL} \frac{\partial \rho}{\partial t}\, dVol + \int_{A_1+A_2+A_3} \rho \bar{V}\cdot\bar{n}\, dArea = 0$$

WHERE

$$\rho = \rho(x,y,z,t) = \rho(x) \quad \text{ONLY}$$

$$\therefore \quad \frac{\partial \rho}{\partial t} = 0$$

$$\int_{\text{VOL.}} \frac{\partial \rho}{\partial t}\, dVol = 0$$

$$\bar{V} \cdot \bar{n} = 0 \quad \text{ON } A_3 ,$$

BY DEFINITION OF A STREAM TUBE.

HENCE THE CONSERVATION OF MASS EQUATION REDUCES TO:

$$\int_{A_1 + A_2} \rho \bar{V} \cdot \bar{n}\, dArea = 0$$

$$\int_{A_1} \rho \bar{V} \cdot \bar{n}\, dArea + \int_{A_2} \rho \bar{V} \cdot \bar{n}\, dArea = 0$$

EVALUATING THESE INTEGRALS:

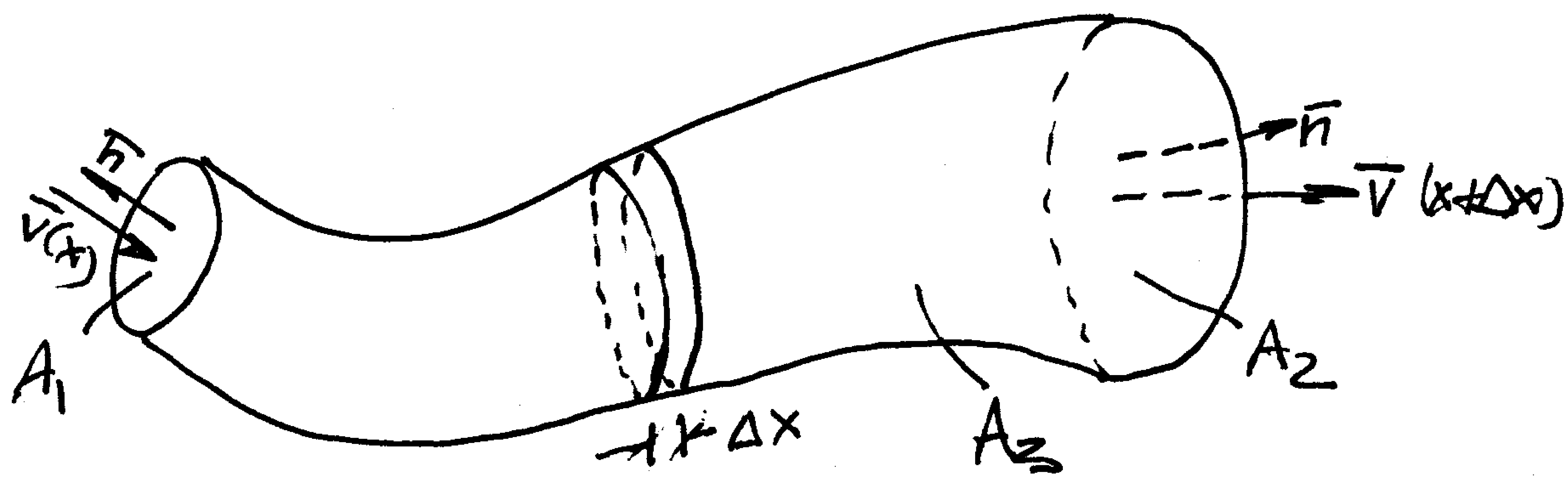
$$-\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \quad \checkmark$$

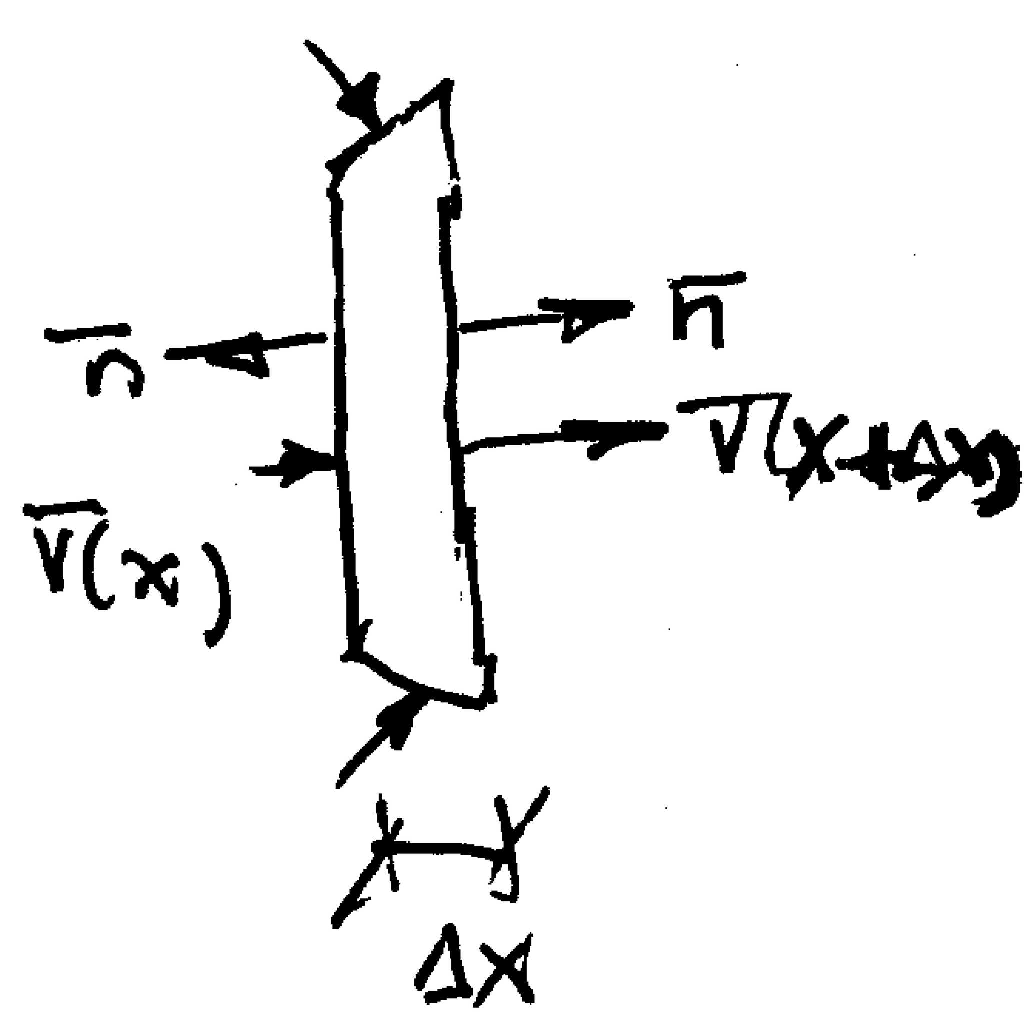
WHAT IS x IN THE ABOVE PRESENTATION?

CONSERVATION OF LINEAR MOMENTUM

$$\int_{VOL} \frac{\partial(\rho\bar{V})}{\partial t}\, dVol + \oint_{AREA} \rho\bar{V}\bar{V}\cdot\bar{n}\, dArea = -\oint_{AREA} p\bar{n}\, dArea$$



CONSIDER THE THIN SLICE Δx :



$$\left.\begin{array}{l} \rho = \rho(x) \\ V = V(x) \end{array}\right\} \Longrightarrow \frac{\partial(\rho\bar{V})}{\partial t} = 0$$

$$\oint_{AREA} \rho\bar{V}\bar{V}\cdot\bar{n}\, dArea = -\oint_{AREA} p\bar{n}\, dArea$$

OR IN TERMS OF ENTHALPY:

$$h = e + \frac{p}{\rho}$$

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2$$

SUMMARY OF GOVERNING EQUATIONS

MASS

1.A $$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

1.B $$d(\rho V A) = 0$$

1.C $$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

LINEAR MOMENTUM

2.A $$p_1 A_1 + \rho_1 V_1^2 A_1 - (p_2 A_2 + \rho_2 V_2^2 A_2) = \frac{p_1 + p_2}{2}(A_2 - A_1)$$

IN THE LIMIT OF $\Delta x \to 0$:

2.B $$d(pA + \rho V^2 A) = p\,dA$$

2.C $$dp + \rho V dV = 0$$

ENERGY

3.A. $$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2$$

3.B $$d(h + \frac{1}{2}V^2) = 0$$

SECOND LAW OF THERMODYNAMICS

4.A $$Tds = dh - \frac{dp}{\rho}$$

FOR ISENTROPIC FLOWS

4.B $$Tds = dh - \frac{dp}{\rho} = 0$$

4.C $$dh = \frac{dp}{\rho}$$

WORKING RELATIONSHIPS FOR ONE-DIMENSIONAL FLOWS

COMBINE (1.C) AND (2.C)

$$dp + \rho V^2 \left(- \frac{d\rho}{\rho} - \frac{dA}{A} \right) = 0$$

FOR ISENTROPIC FLOW PROCESSES, WE DEFINE THE LOCAL SPEED OF SOUND AS:

$$a^2 \equiv \left(\frac{\partial p}{\partial \rho} \right)_s$$

USE THESE TWO EQUATIONS TO ELIMINATE $d\rho$!

$$\left(1 - \frac{V^2}{a^2}\right) dp = \rho V^2 \frac{dA}{A}$$

$$(1 - M^2)\, dp = \rho V^2 \frac{dA}{A}$$

USING (2.C) AGAIN, THIS EQUATION BECOMES:

$$\frac{dV}{V} = \frac{-1}{(1-M^2)} \frac{dA}{A}$$