ONE DIMENSIONAL ISENTROPIC FLOWS
REFERENCE CONDITIONS

A. STAGNATION CONDITIONS

STAGNATION CONDITIONS DENOTES A STATE OF COMPRESSIBLE FLOW BROUGHT TO REST (ZERO SPEED) ISENTROPICALLY.

RECALL OUR CONSERVATION OF ENERGY EXPRESSION

$$d\left(h + \frac{1}{2}V^2\right) = 0$$

h = ENTHALPY PER UNIT MASS
V = LOCAL SPEED OF THE FLUID MASS

INTEGRATE

$$h + \frac{1}{2}V^2 = \text{CONSTANT} = C$$

DEFINE C TO BE THE TOTAL ENTHALPY

LET

$$h + \frac{1}{2}V^2 = C_0$$

$C_0 \equiv$ STAGNATION ENTHALPY
→ THE GAS IS BROUGHT TO REST ISENTROPICALLY.

FOR A CALORICALLY PERFECT GAS:

$$h = c_p T \quad ; \quad c_p = \frac{\gamma R}{\gamma - 1} \quad ; \quad a^2 = \gamma R T$$

ENERGY CONSERVATION EQUATION BECOMES:

$$\frac{T_0}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

SINCE THE FLOW PROCESS IS ISENTROPIC, ONE CAN SHOW:

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

THE MASS FLOW RATE MAY BE EXPRESSED IN TERMS OF MACH NUMBER AND STAGNATION CONDITIONS, VIZ.

$$\dot{m} = \rho V A = \frac{p_0 A M}{\sqrt{R T_0 / \gamma}} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{(\gamma + 1)}{2(1 - \gamma)}}$$

B. CRITICAL CONDITIONS

A USEFUL REFERENCE CONDITION IS THE STATE CORRESPONDING TO A LOCAL MACH NUMBER OF

$$M = M^* = 1$$

THIS STATE IS DEFINED AS THE CRITICAL CONDITION. IT IS ALSO REFERRED TO AS THE THROAT CONDITION.

WE PROCEED FROM OUR CONSERVATION OF ENERGY EQUATION:

$$\frac{T}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}$$

$$\left(\frac{T}{T_0}\right)_{M=M^*=1} = \frac{T^*}{T_0} = \left(1 + \frac{\gamma - 1}{2}(1)\right)^{-1}$$

$$\frac{T^*}{T_0} = \frac{2}{1+\gamma}$$

FOR ISENTROPIC PROCESSES:

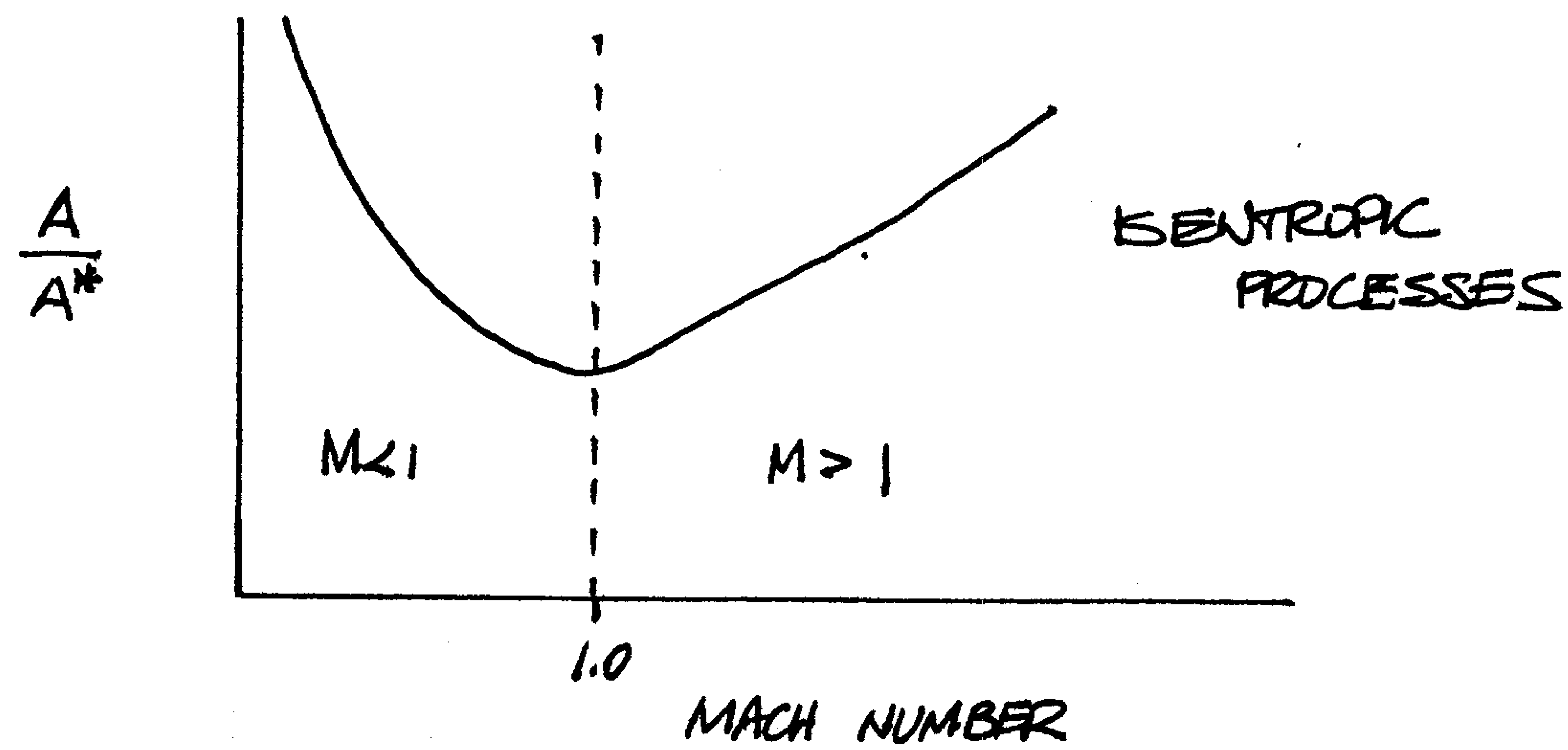
$$\frac{p^*}{p_0} = \left(\frac{2}{1+\gamma}\right)^{\frac{\gamma}{\gamma-1}} \quad ; \quad \frac{\rho^*}{\rho_0} = \left(\frac{2}{1+\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{a^*}{a_0} = \left(\frac{2}{1+\gamma}\right)^{1/2}$$

AND FROM OUR EXPRESSION FOR MASS FLOW RATE:

$$\frac{A}{A^*} = \frac{\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{(2-2\gamma)}}}{M\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{(\gamma+1)}{(2-2\gamma)}}}$$

PLOT A/A^* vs M:



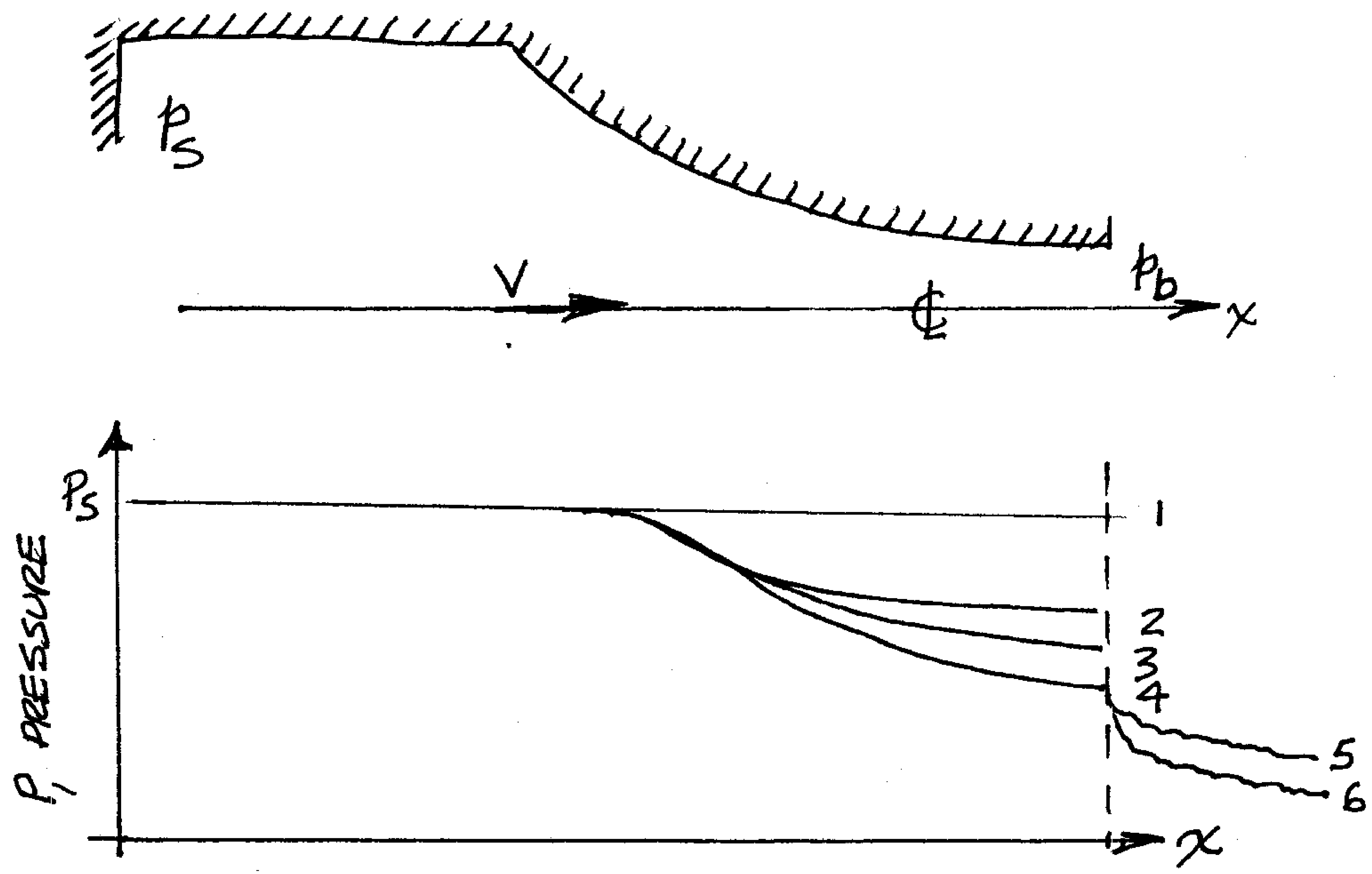
ISENTROPIC FLOW IN A CONVERGING NOZZLE

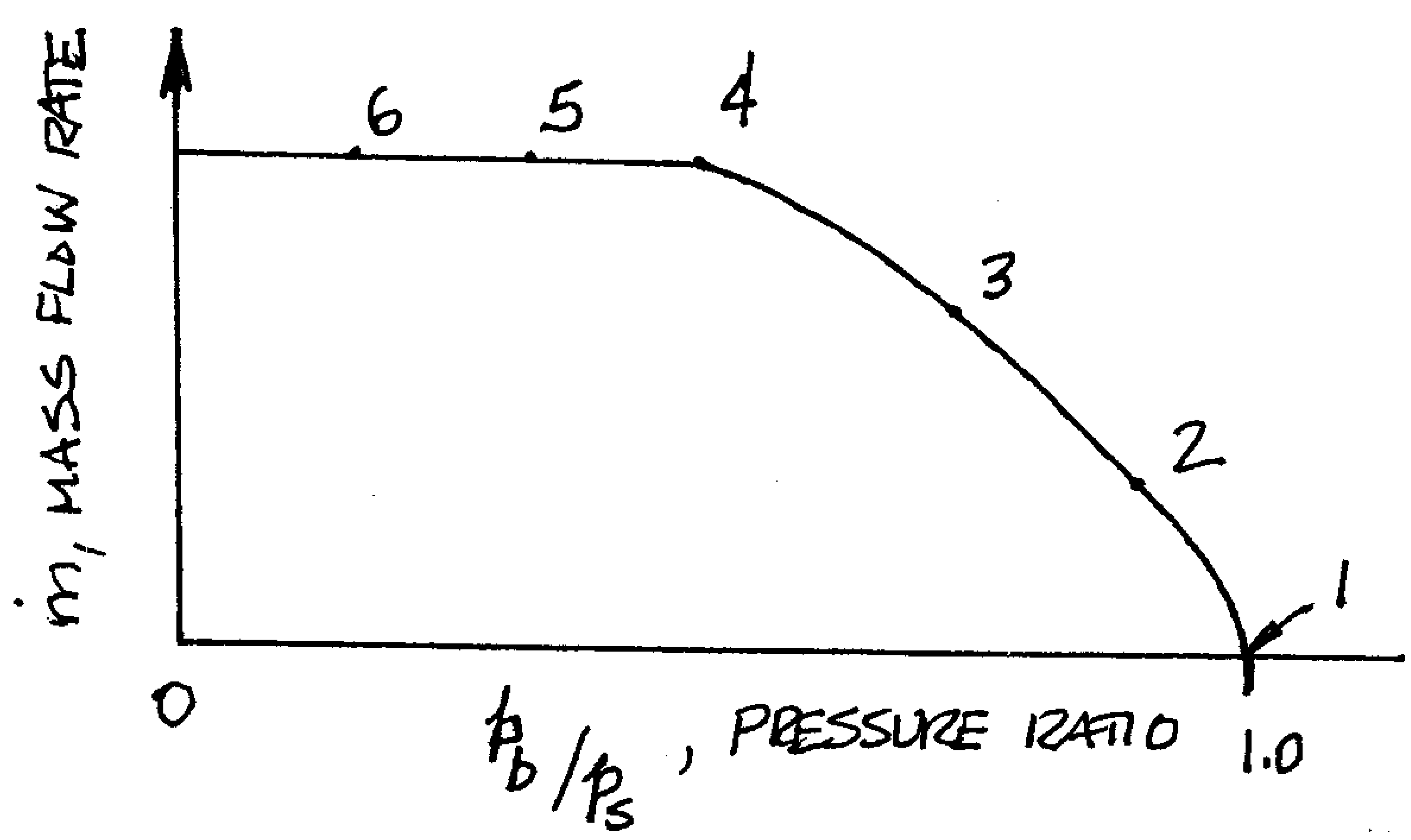
WE PLOT THE NOZZLE PRESSURE DISTRIBUTION AND MASS FLOW RATE AS A FUNCTION OF THE RATIO OF BACK PRESSURE TO STAGNATION PRESSURE. A FEW DEFINITIONS.

p_b = PRESSURE AT NOZZLE EXIT, BACK PRESSURE

p_s = STAGNATION PRESSURE IN NOZZLE SETTLING CHAMBER

$\dot{m}$ = MASS FLOW RATE IN NOZZLE





FLOW STATE 4

$M_b = 1$ (CHOKED FLOW)

$\gamma = 1.4$ (AIR)

$$p_b/p_s = \left(1+\frac{\gamma-1}{2}\right)^{-\frac{\gamma}{(\gamma-1)}} = 0.528$$

ISENTROPIC FLOW IN A CONVERGING NOZZLE
EXAMPLE PROBLEM A (JOHN)

AIR IS ALLOWED TO FLOW FROM A LARGE RESERVOIR THROUGH A CONVERGING NOZZLE OF EXIT AREA 10 in^2. THE RESERVOIR IS LARGE ENOUGH SO THAT NEGLIGIBLE CHANGES IN RESERVOIR PRESSURE AND TEMPERATURE OCCUR AS FLUID IS EXHAUSTED THROUGH THE NOZZLE. ASSUME ISENTROPIC FLOW IN THE NOZZLE, WITH $p_0 = 100$ psia, $T_0 = 1000^\circ R$; ASSUME ALSO THAT AIR BEHAVES AS A PERFECT GAS WITH CONSTANT SPECIFIC HEATS, $\gamma = 1.4$.

DETERMINE THE MASS FLOW THROUGH THE NOZZLE FOR BACK PRESSURES OF 0, 25, 50, AND 75 psia.

SOLUTION

FOR $\gamma = 1.4$,
$$\left(\frac{p_{BACK}}{p_0}\right)_{CRIT.} = \left(\frac{p_{BACK}}{p_0}\right)_{M_{BACK}=1}$$
$$= \left(1 + \frac{\gamma-1}{2}\right)^{\frac{-\gamma}{\gamma-1}}$$
$$= 0.528$$

THEREFORE FOR ALL BACK PRESSURES BELOW 52.8 psia, THE NOZZLE IS CHOKED. UNDER THESE CONDITIONS, THE MACH NUMBER AT THE EXIT PLANE IS 1, THE PRESSURE

AT THE EXIT PLANE IS 52.8 psia (NOT EQUAL TO THE BACK PRESSURE), AND THE TEMPERATURE AT THE EXIT PLANE IS

$$T_E = \left(\frac{T}{T_0}\right)_{M=1} T_0 = (0.833)(1000°R) = 833°R$$

THUS THE MASS FLOW RATE FOR BACK PRESSURES OF 0, 25, AND 50 psia IS:

$$\dot{m} = \rho A V = \rho_E A_E V_E = \frac{p_E}{R T_E} A_E M_E \sqrt{\gamma R T_E}$$

$$\dot{m} = \frac{52.8 \times 10}{53.3 \times 833} \times 1 \times \sqrt{1.4 \times 322 \times 53.3 \times 833}$$

$$\underline{\dot{m} = 16.9 \text{ lbm/sec.}}$$

FOR A BACK PRESSURE OF 75 psia, THE NOZZLE IS NOT CHOKED, AND THE EXIT PLANE PRESSURE IS EQUAL TO THE BACK PRESSURE. FOR

$$\frac{p_{BACK}}{p_0} = \frac{75}{100} = 0.75$$

THE MACH NUMBER AT THE EXIT PLANE IS

$$M_E = 0.65$$

$$\therefore \left(\frac{T_E}{T_0}\right)_{M_E = 0.65} = 0.922$$

NOW COMPUTE $\dot{m}_{E_{M_E=0.65}}$:

$$\dot{m}_{E_{M_E=0.65}} = \frac{75 \times 10}{53.3 \times 922} \times 0.65 \times \sqrt{1.4 \times 32.2 \times 53.3 \times 922}$$

$$\dot{m}_{E_{M_E=0.65}} = 14.7 \text{ lbm/sec.}$$

WHAT DOES THE $\dot{m}$ VS. p_{BACK} PROFILE SHOW?