CHOKED FLOWS

IN A CONVERGING NOZZLE, IF p_{BACK} IS REDUCED BELOW THE VALUE OF p^* AT THE THROAT NECESSARY TO ESTABLISH THE CRITICAL PRESSURE RATIO, THEN THE PRESSURE p_{BACK} CANNOT BE TRANSMITTED BACK INTO THE THROAT OF THE NOZZLE, BECAUSE $V_{THROAT} = V^* = a_{THROAT} = a^*$.

ON THE OTHER HAND, IF THE PRESSURE p_{BACK} IS ABOVE THAT REQUIRED BY THE CRITICAL PRESSURE RATIO, THE FLUID VELOCITY AT THE THROAT IS LESS THAN THE VELOCITY OF SOUND; IN THIS CASE p_{BACK} CAN BE COMMUNICATED BACK INTO THE THROAT AND $p_{THROAT} = p_{BACK}$.

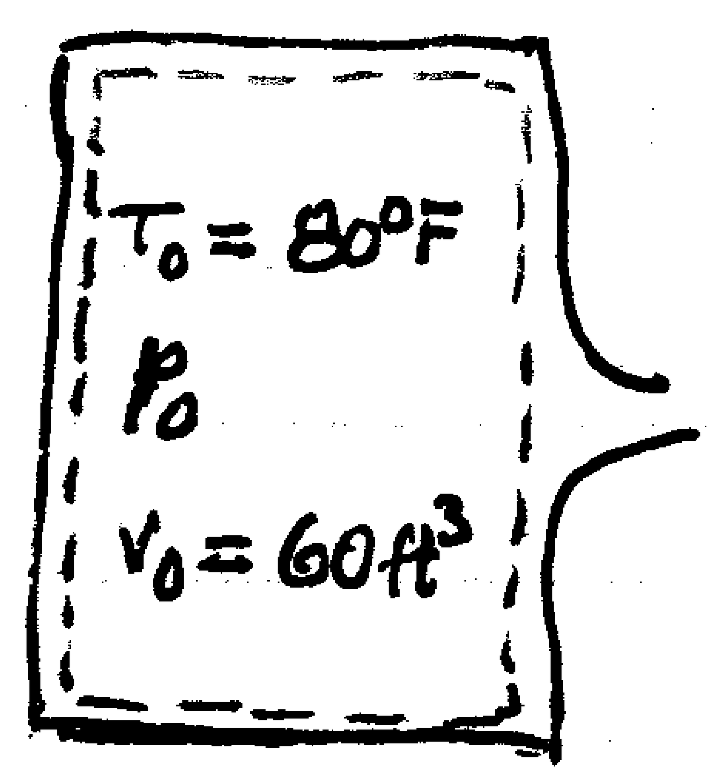
EXAMPLE PROBLEM B (JOHN)

NITROGEN IS STORED IN A TANK $60\,ft^3$ IN VOLUME AT A PRESSURE OF 500 psia AND A TEMPERATURE OF 80°F. THE GAS IS DISCHARGED THROUGH A CONVERGING NOZZLE OF EXIT AREA $2\,in^2$. FOR A BACK PRESSURE OF 14.7 psia, FIND THE TIME REQUIRED FOR THE TANK PRESSURE TO DROP TO 50 psia. ASSUME ISENTROPIC FLOW IN THE NOZZLE, WITH NITROGEN BEHAVING AS A PERFECT GAS WITH $\gamma = 1.4$ AND $R = 55.2\ ft\text{-}lb_f / lb_m \cdot {}^\circ R$. ASSUME QUASI-STEADY FLOW THROUGH THE NOZZLE, WITH THE FLOW EQUATIONS APPLICABLE AT EACH INSTANT OF TIME. ASSUME ALSO THAT T_0 IS CONSTANT.

SOLUTION

$$\dot{m} = \rho A V$$

$$\dot{m} = \frac{p}{RT} A V$$



AS THE RESERVOIR PRESSURE DROPS FROM 500 psia, THE RATIO p_{BACK} / p_0 REMAINS BELOW THE CRITICAL PRESSURE RATIO, AND THE NOZZLE-EXIT MACH NUMBER IS 1.

THEREFORE

$$\dot{m} = \frac{(0.528\, p_0 \times 144)}{55.2 \times 0.833 \times 540} \left(\frac{2}{144}\right) \sqrt{(1.4)(32.2)(55.2)(0.833)(540)}$$

$$\dot{m} = 4.5 \times 10^{-2}\, p_0 \quad \frac{lbm}{sec}$$

WITH p_{BACK} EXPRESSED IN psia

THE CONSERVATION OF MASS EQUATION STATES

$$\frac{\partial}{\partial t} \int_{VOL.} \rho \, dVol + \oint_{AREA} \rho \vec{V} \cdot \vec{n} \, dArea = 0$$

WHERE:

$$\int_{VOL.} \rho \, dVol = m \quad \left\{ \begin{array}{l} \text{WHERE } m \text{ IS THE MASS} \\ \text{INSIDE THE TANK AT} \\ \text{ANY INSTANT OF TIME} \end{array} \right.$$

SUBSTITUTING:

$$m = \frac{p_0 V_0}{R T_0}$$

$$\frac{V_0}{R T_0} \frac{dp_0}{dt} + 4.5 \times 10^{-2}\, p_0 = 0$$

$$2.91 \times 10^{-1} \frac{dp_0}{p_0} + 4.5 \times 10^{-2}\, dt = 0$$

$$t = -6.47 \int_{500}^{50} \frac{dp_0}{p_0}$$

$$= 6.47 \ln 10$$

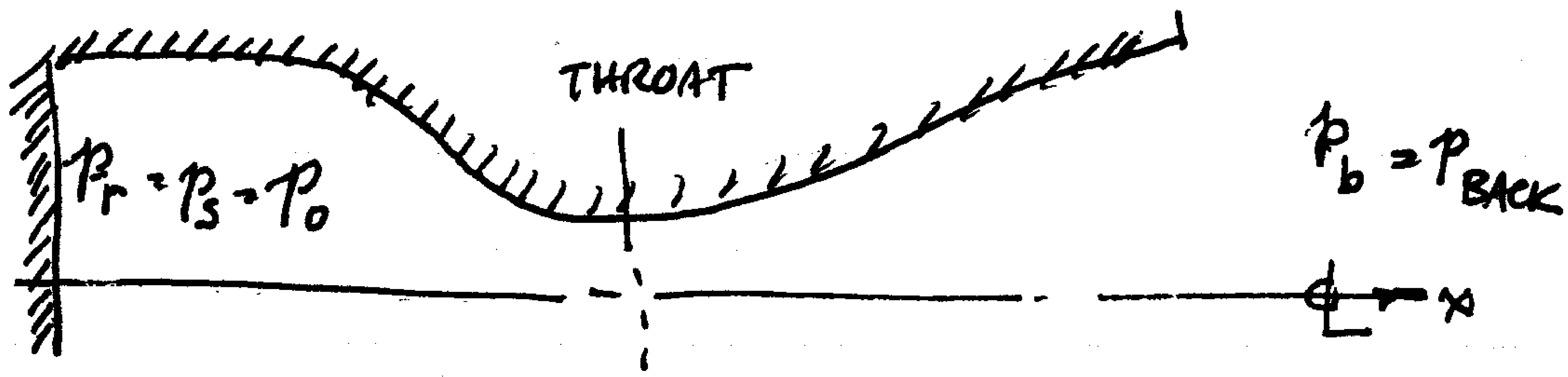
$$\underline{t = 14.9 \text{ sec.}}$$

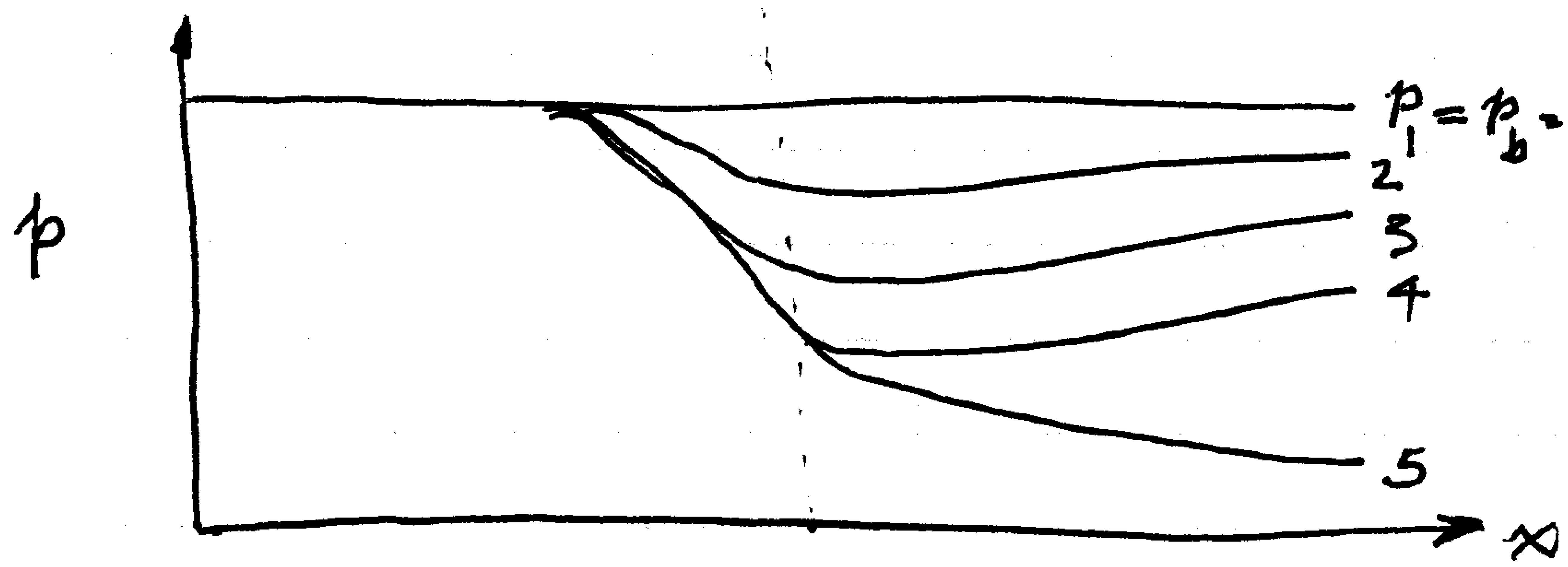
<u>OVER COFFEE OR TEA</u> (JOHN)

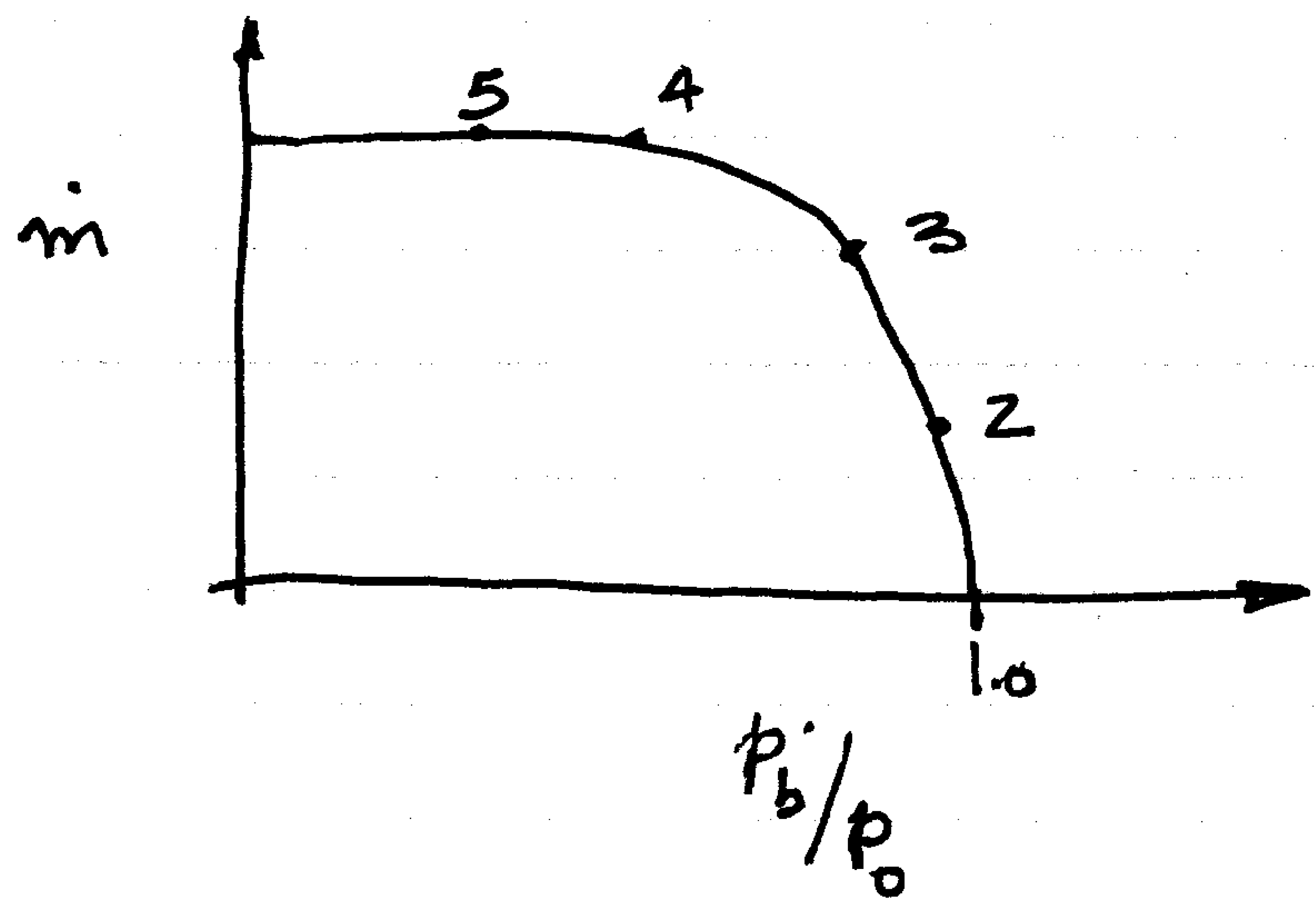
AN AIR STREAM FLOWS IN A CONVERGING DUCT FROM A CROSS-SECTIONAL AREA $A_1 = 15 \text{ in}^2$ TO A CROSS-SECTIONAL AREA $A_2 = 10 \text{ in}^2$. IF $T_1 = 500°R$, $p_1 = 15$ psia, $V_1 = 300$ ft/sec, <u>FIND</u>: M_2, p_2, AND T_2. ASSUME STEADY, ONE-DIMENSIONAL, ISENTROPIC FLOW.

<u>ANS</u>: $M_2 = 0.44$; $p_2 = 13.8$ psia ; $T_2 = 489°R$

ISENTROPIC FLOW IN A CONVERGING-DIVERGING NOZZLE







ISENTROPIC FLOW IN A CONVERGING-DIVERGING NOZZLE

EXAMPLE PROBLEM A (JOHN)

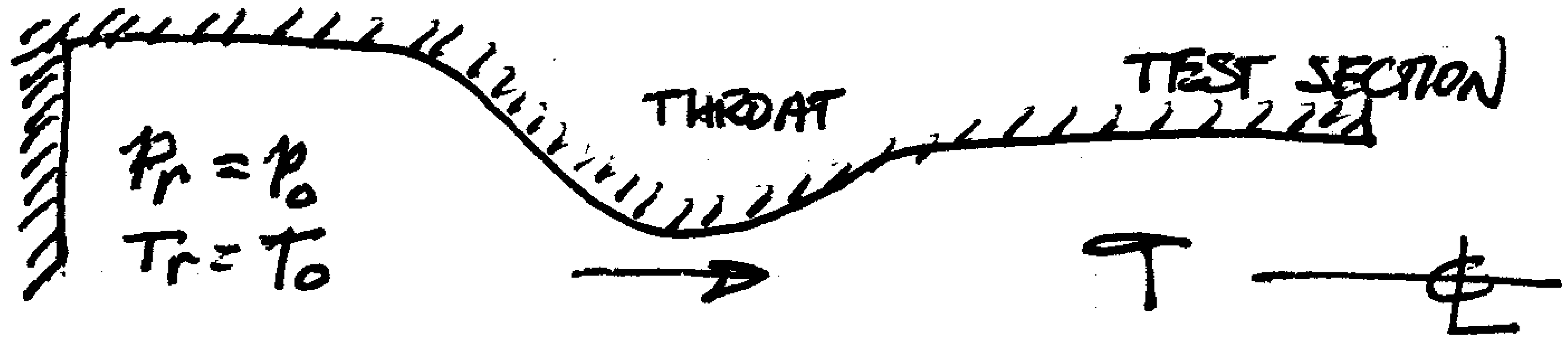
A NOZZLE IS TO BE DESIGNED FOR A SUPERSONIC WIND TUNNEL. TEST SECTION SPECIFICATIONS ARE:

DIAMETER = 6.0 in

MACH NUMBER = 3.0

$\left.\begin{array}{l}\text{STATIC PRESSURE} = 4.36 \text{ psia} \\ \text{STATIC TEMPERATURE} = 412°R\end{array}\right\}$ 30,000 ft. ALTITUDE

DETERMINE THE MASS FLOW RATE, THE TUNNEL THROAT AREA, AND THE RESERVOIR (STAGNATION) TEMPERATURE AND PRESSURE.



SOLUTION

THE TEST SECTION MASS FLOW RATE:

$$\dot{m}_{TS} = (\rho A V)_{TS} = \left(\frac{p}{RT}\right)_{TS}\left(\frac{\pi}{4}D^2\right)_{TS} M_{TS}\sqrt{\gamma R T_{TS}}$$

$$\dot{m}_{TS} = (0.0286)\,\frac{lb_m}{ft^3}\,(0.196)\,ft^2\,(2990)\,\frac{ft}{sec.}$$

$$\dot{m}_{TS} = 16.8 \ lbm/sec.$$

FROM NACA 1135 TABLES,

$$M = 3.0$$

$$\left(\frac{A}{A^*}\right)_{M=3.0} = 4.235$$

$$\left(\frac{p}{p_0}\right)_{M=3.0} = 0.0272$$

$$\left(\frac{T}{T_0}\right)_{M=3.0} = 0.357$$

$$\therefore \quad A^* = \frac{0.196}{4.235} = 0.0464 \text{ ft}^2$$

$$p_0 = \frac{4.36}{0.0272} = 160 \text{ psia}$$

$$T_0 = \frac{412}{0.357} = 1150\,^\circ R$$

WAVE PROPAGATION IN COMPRESSIBLE MEDIA

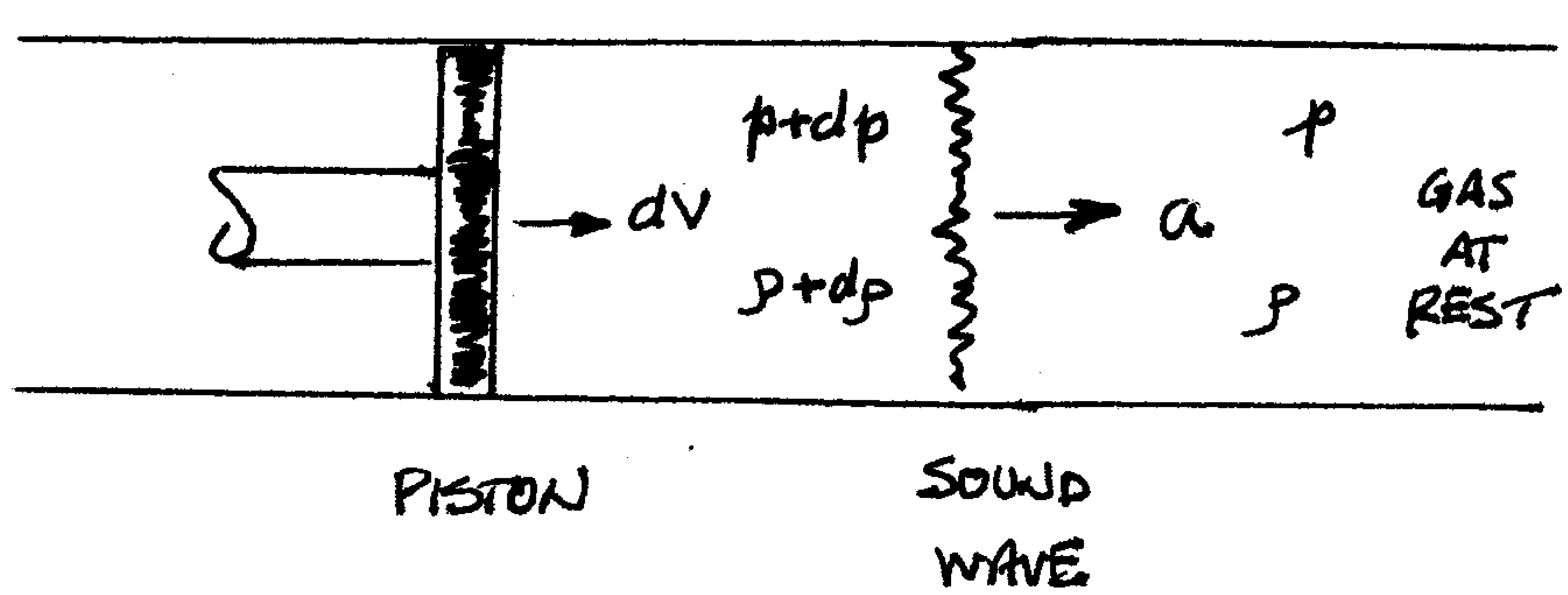
SPEED OF SOUND

IN THE CASE OF INCOMPRESSIBLE FLOW, WE KNOW THAT THE SPEED OF SOUND IS INFINITE. THIS MEANS THAT THE MICROSCOPIC LEVEL AND THE MARCOSCOPIC LEVEL "COMMUNICATE" INSTANTANEOUSLY. THE STREALINE PATTERNS ARE ALWAYS SMOOTH IN INCOMPRESSIBLE FLOW.

IN THE CASE OF COMPRESSIBLE FLOW, THE SPEED OF SOUND IS FINITE. THE MICROSCOPIC LEVEL DOES NOT "COMMUNICATE" WITH THE MARCOSCOPIC LEVEL INSTANTANEOUSLY. STREAMLINE PATTERNS MAY NOT BE SMOOTH.

THE SPEED OF SOUND IS THE SPEED AT WHICH AN INFINITESIMAL DISTURBANCE (ISENTROPIC) PROPAGATES THROUGH A COMPRESSIBLE GAS. THIS INFINITESIMAL DISTURBANCE PROPAGATES AS A LONGITUDINAL WAVE.

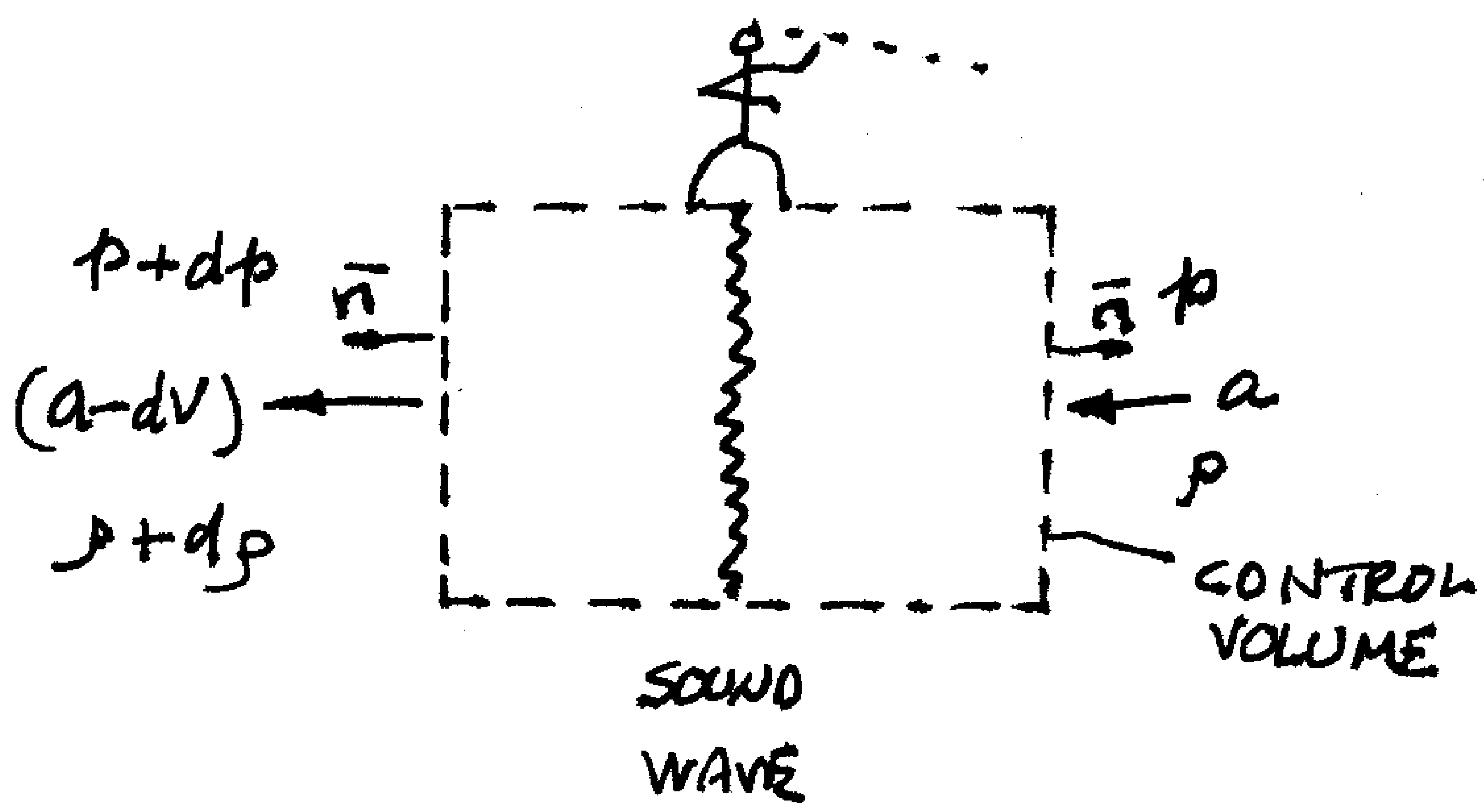
CONSIDER THE FOLLOWING "SOUND WAVE":



a = SPEED OF SOUND

dV = PISTON SPEED

NOW PLACE A CONTROL VOLUME AROUND THE SOUND WAVE AND EXPRESS ALL VELOCITIES RELATIVE TO THE CONTROL VOLUME. WE HAVE THE FOLLOWING MODEL:



NOW APPLY CONSERVATION PRINCIPLES:

MASS

$$(\rho+d\rho)(a-dV)A - a\rho A = 0$$

LINEAR MOMENTUM

$$(\rho+d\rho)(a-dV)^2 A - \rho a^2 A = -(p+dp)A + pA$$

DROPPING HIGHER ORDER TERMS, THESE EQUATIONS BECOME:

$$a\,d\rho - \rho\,dV = 0$$

$$-2\rho a\,dV + a^2 d\rho = -dp$$

COMBINING THESE EQUATIONS, WE OBTAIN

$$a^2 d\rho = dp$$

OR $$a^2 = \frac{dp}{d\rho}$$ QED