NORMAL SHOCK WAVES

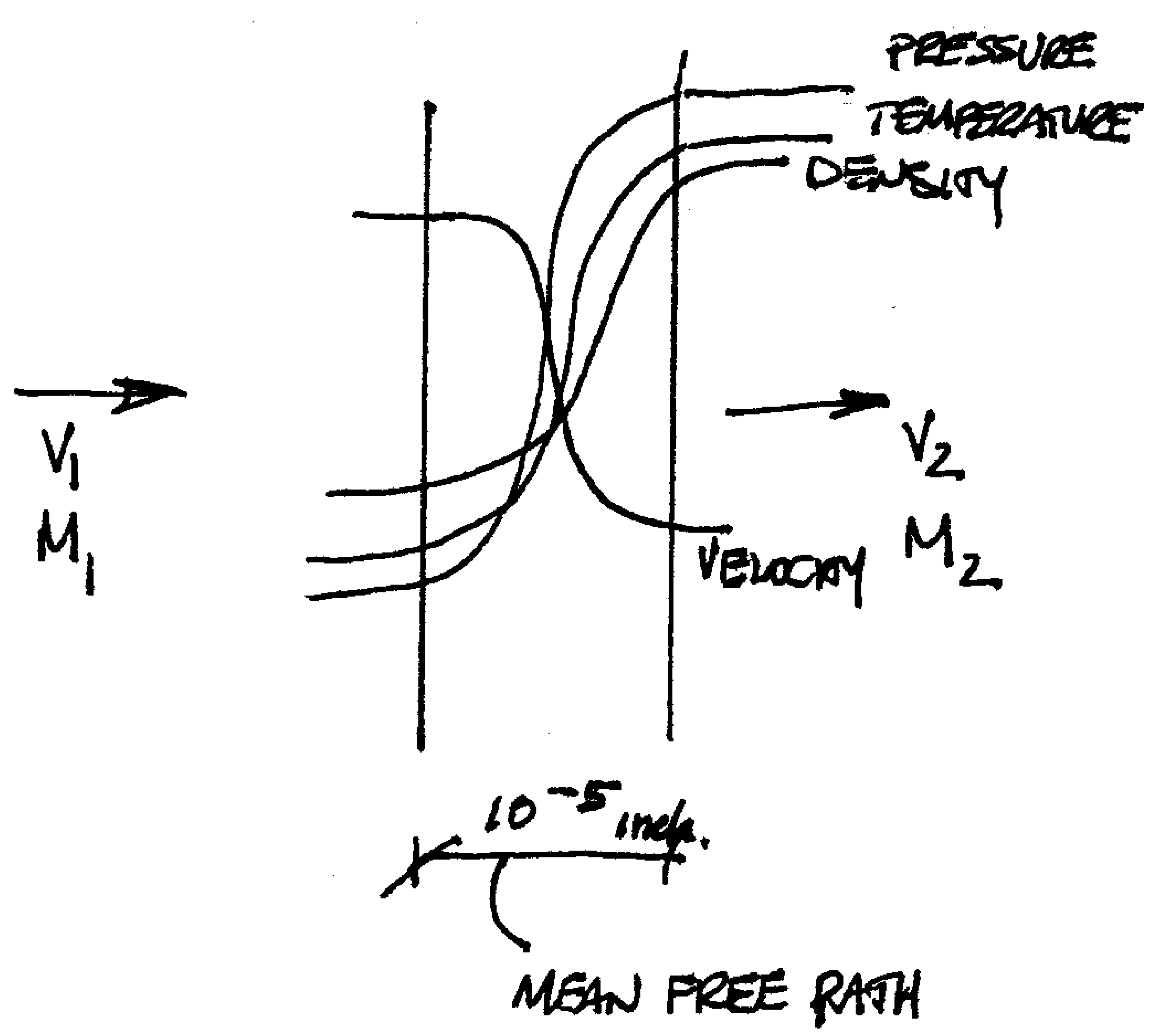
PROPERTIES OF SHOCK WAVES

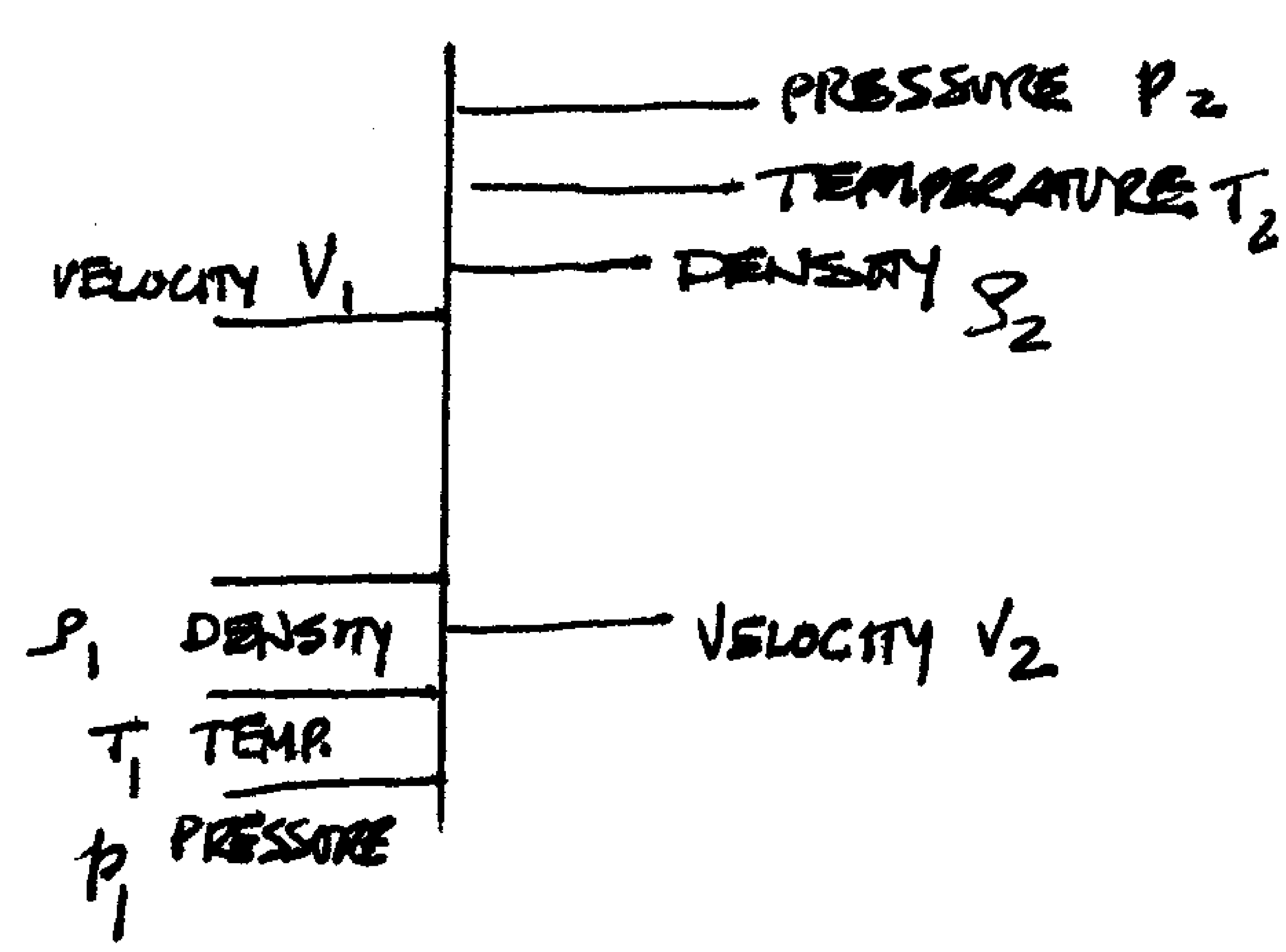
- FINITE VARIATIONS IN THERMODYNAMIC AND DYNAMIC VARIABLES OCCUR OVER A DISTANCE COMPARABLE TO A MEAN FREE PATH OF THE GAS ($\sim 10^{-5}$ inch.)

- MEANS BY WHICH A SUPERSONIC FLOW ADJUSTS TO THE PRESENCE OF A BODY/DISTURBANCE

- INTERNAL STRUCTURE
  - IRREVERSIBLE
  - HEAT CONDUCTION EFFECTS DUE TO LARGE TEMPERATURE GRADIENTS
  - VISCOUS DISSIPATION EFFECTS DUE TO LARGE VELOCITY GRADIENTS
  - NON-EQUILIBRIUM THERMODYNAMIC STATE

- OVER-ALL OR "AVERAGE" CHANGE (MODEL)
  - ONE DIMENSIONAL
  - STEADY
  - CONSTANT AREA
  - ADIABATIC
  - CALORICALLY PERFECT GAS UPSTREAM AND DOWNSTREAM

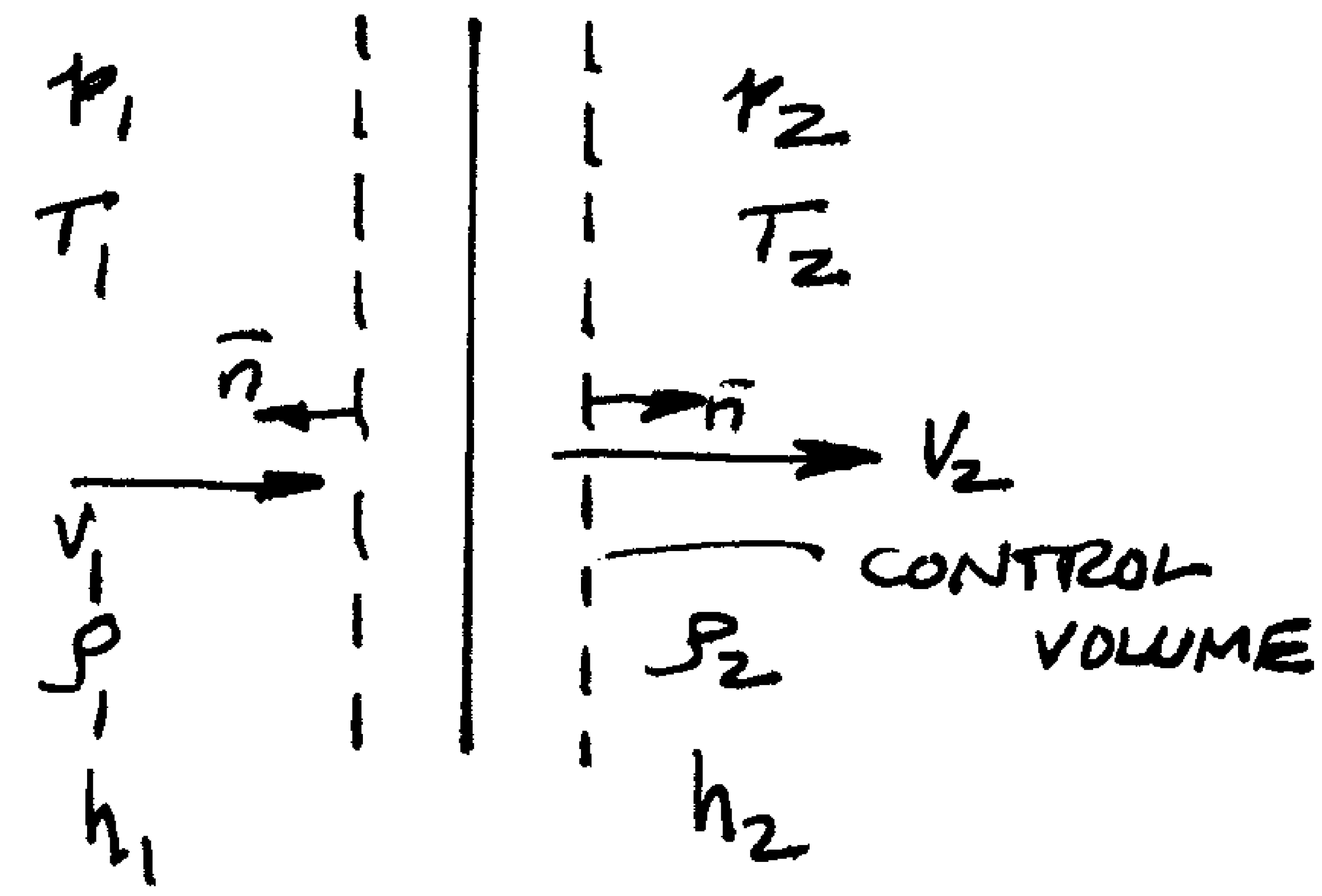




MODEL

EQUATION OF MOTION FOR A NORMAL SHOCK WAVE

NOW PLACE A CONTROL VOLUME AROUND THE NORMAL SHOCK WAVE. THE NORMAL SHOCK WAVE IS CONSIDERED TO BE FIXED IN SPACE (STATIONARY). HENCE, THE FOLLOWING MODEL



NOW APPLY CONSERVATION PRINCIPLES:

MASS

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

WHERE

$$A_1 = A_2$$

LINEAR MOMENTUM

$$-\rho_1 V_1^2 A_1 + \rho_2 V_2^2 A_2 = p_1 A_1 - p_2 A_2$$

ENERGY

$$h_1 + \tfrac{1}{2} V_1^2 = h_2 + \tfrac{1}{2} V_2^2$$

THE FOLLOWING RELATIONSHIP/EXPRESSIONS MAY BE DERIVED FROM ABOVE EQUATION FOR A CALORICALLY PERFECT GAS:

$$V_1 V_2 = a_0^2 \left(\frac{2}{\gamma+1}\right) = a^{*2}$$

$$\frac{V_1}{a^*} \frac{V_2}{a^*} = 1$$

$$M_2^* = 1/M_1^*$$

$$\frac{V_1}{V_2} = \frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2}$$

$$\lim_{M_1^2 \to \infty} \left(\frac{V_1}{V_2}\right) = \frac{\gamma+1}{\gamma-1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{(\gamma+1)} (M_1^2 - 1)$$

$$\frac{T_2}{T_1} = 1 + \frac{2(\gamma-1)}{(\gamma+1)^2} \frac{\gamma M_1^2 + 1}{M_1^2} (M_1^2 - 1)$$

$$\frac{S_2 - S_1}{R} = \ln \left\{ \left[1 + \frac{2\gamma}{(\gamma+1)} (M_1^2 - 1)\right]^{\frac{1}{\gamma-1}} \left[\frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2}\right]^{\frac{-\gamma}{\gamma-1}} \right\}$$

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$$

$$\frac{p_{o_2}}{p_{o_1}} = ?$$

$$\frac{T_{o_2}}{T_{o_1}} = ?$$

$$\frac{p_{o_2}}{p_{o_1}} = \left[\frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \cdot \left[\frac{(\gamma+1)}{2\gamma M_1^2 + (1-\gamma)}\right]^{\frac{1}{\gamma-1}} \quad ; \quad \frac{T_{o_2}}{T_{o_1}} = 1$$

EXAMPLE PROBLEM

AIR AT 14.0 psia, 60°F, WITH A MACH NUMBER OF 2.5 ENTERS A NORMAL SHOCK. FIND p_2, T_2 V_2, AND $(S_2 - S_1)$.

SOLUTION

FIRST, COMPUTE V_1:

$$\gamma = 1.4 \ , \ R = 53.35 \frac{lb_f - ft}{lb_m - {}^\circ R} \ , \ T_1 = 60°F = 520°R$$

$$V_1 = a_1 M_1 = \sqrt{\gamma R T_1}\, M_1 = \sqrt{(1.4)(53.35)(32.2)(520)}\ (2.5)$$

$$V_1 = 2795.7\ ft/sec.$$

NOW USE THE NORMAL SHOCK TABLES IN NACA 1135. FOR $\gamma = 1.4$ AND $M_1 = 2.5$, WE OBTAIN:

$$\left(\frac{p_2}{p_1}\right)_{M_1=2.5} = 7.125 \qquad \left(\frac{T_2}{T_1}\right)_{M_1=2.5} = 2.137 \qquad \text{AND}$$

$$\left(\frac{V_2}{V_1}\right)_{M_1} = \left(\frac{\rho_1}{\rho_2}\right)_{M_1} = \frac{1}{3.33} \qquad \left(\frac{p_{02}}{p_{01}}\right)_{M_1=2.5} = 0.49902$$

COMPUTING

$$p_2 = (14.0)(7.125) = 99.75\ psia$$

$$T_2 = (520)(2.137) = 1111.5°R$$

$$V_2 = (2795.7)\left(\frac{1}{3.33}\right) = 839.5\ \frac{ft}{sec}$$

$$S_2 - S_1 = -R \ln(p_{02}/p_{01}) = -(53.35)\frac{lb_f - ft}{lb_m \cdot {}^\circ R}\left(\frac{1}{778.2}\frac{BTU}{lb_f\text{-}ft}\right)\ln(0.49902)$$

$$S_2 - S_1 = 0.0476\ \frac{BTU}{lb_m \cdot {}^\circ R}$$