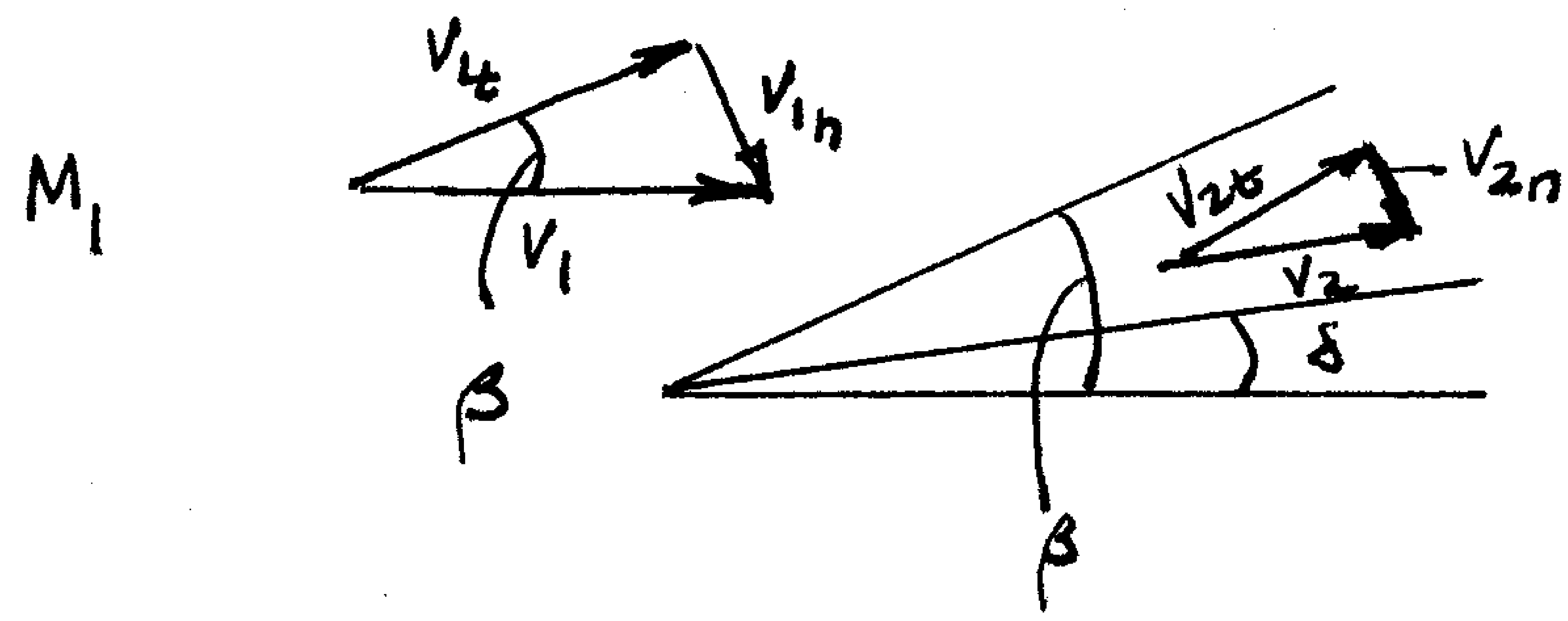
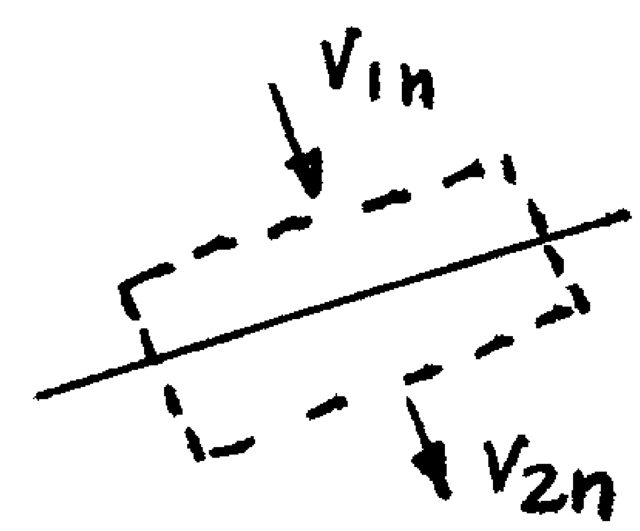
OBLIQUE SHOCK WAVES – EQUATIONS OF MOTION



CONSERVATION OF MASS

$$\rho_1 V_{1n} = \rho_2 V_{2n}$$



CONSERVATION OF LINEAR MOMENTUM

$$p_1 + \rho_1 V_{1n}^2 = p_2 + \rho_2 V_{2n}^2$$

$$\rho_1 V_{1n} V_{1t} = \rho_2 V_{2n} V_{2t}$$

CONSERVATION OF ENERGY

$$\frac{1}{2}\left(V_{1n}^2 + V_{1t}^2\right) + \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} = \frac{1}{2}\left(V_{2n}^2 + V_{2t}^2\right) + \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2}$$

WHERE:

$$V_{1n} = V_1 \sin\beta \qquad V_{1t} = V_1 \cos\beta$$

$$V_{2n} = V_{2t} \tan(\beta - \theta) \qquad V_{2t} = V_2 \cos(\beta - \theta)$$

RESULTING EQUATIONS

$$V_{1_t} = V_{2_t}$$

$$V_{2n} = \frac{V_1}{\sin\beta}\left(\frac{\gamma-1}{\gamma+1}\sin^2\beta + \frac{2}{\gamma+1}\frac{1}{M_1^2}\right)$$

$$\tan(\beta-\theta) = \frac{1}{\sin\beta\cos\beta}\left[\left(\frac{\gamma-1}{\gamma+1}\right)\sin^2\beta + \frac{2}{\gamma+1}\frac{1}{M_1^2}\right]$$

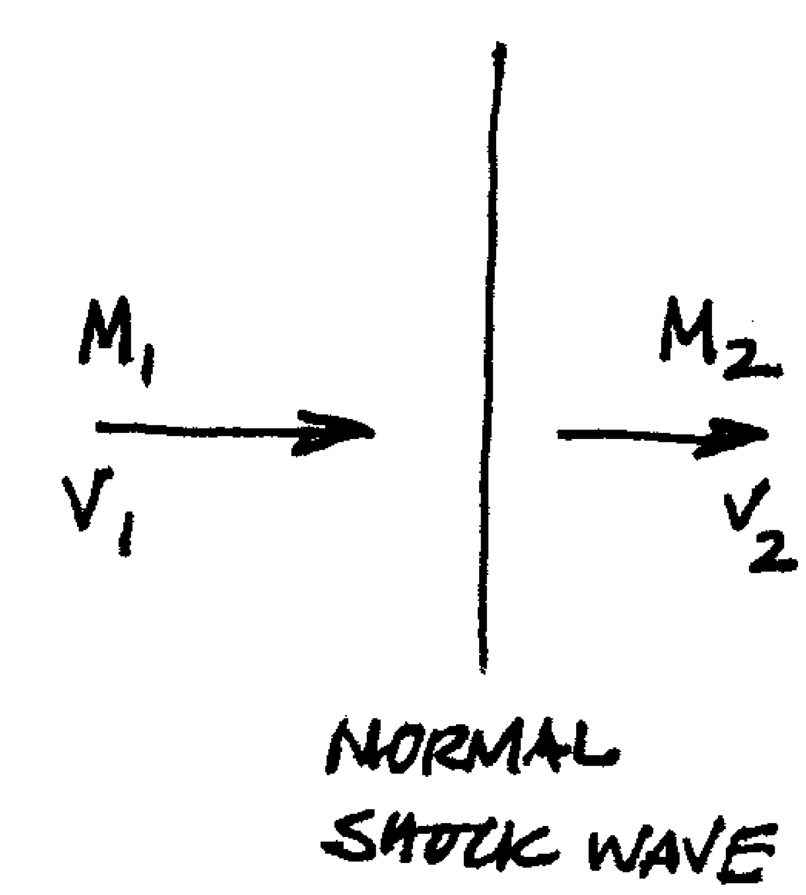
$$\rho_2/\rho_1 = \frac{(\gamma+1)M_1^2\sin^2\beta}{(\gamma-1)M_1^2\sin^2\beta+2} = \frac{(\gamma+1)M_{1_n}^2}{(\gamma-1)M_{1_n}^2+2}$$

$$M_{1_n} \equiv M_1\sin\beta$$

NORMAL AND OBLIQUE SHOCK WAVES

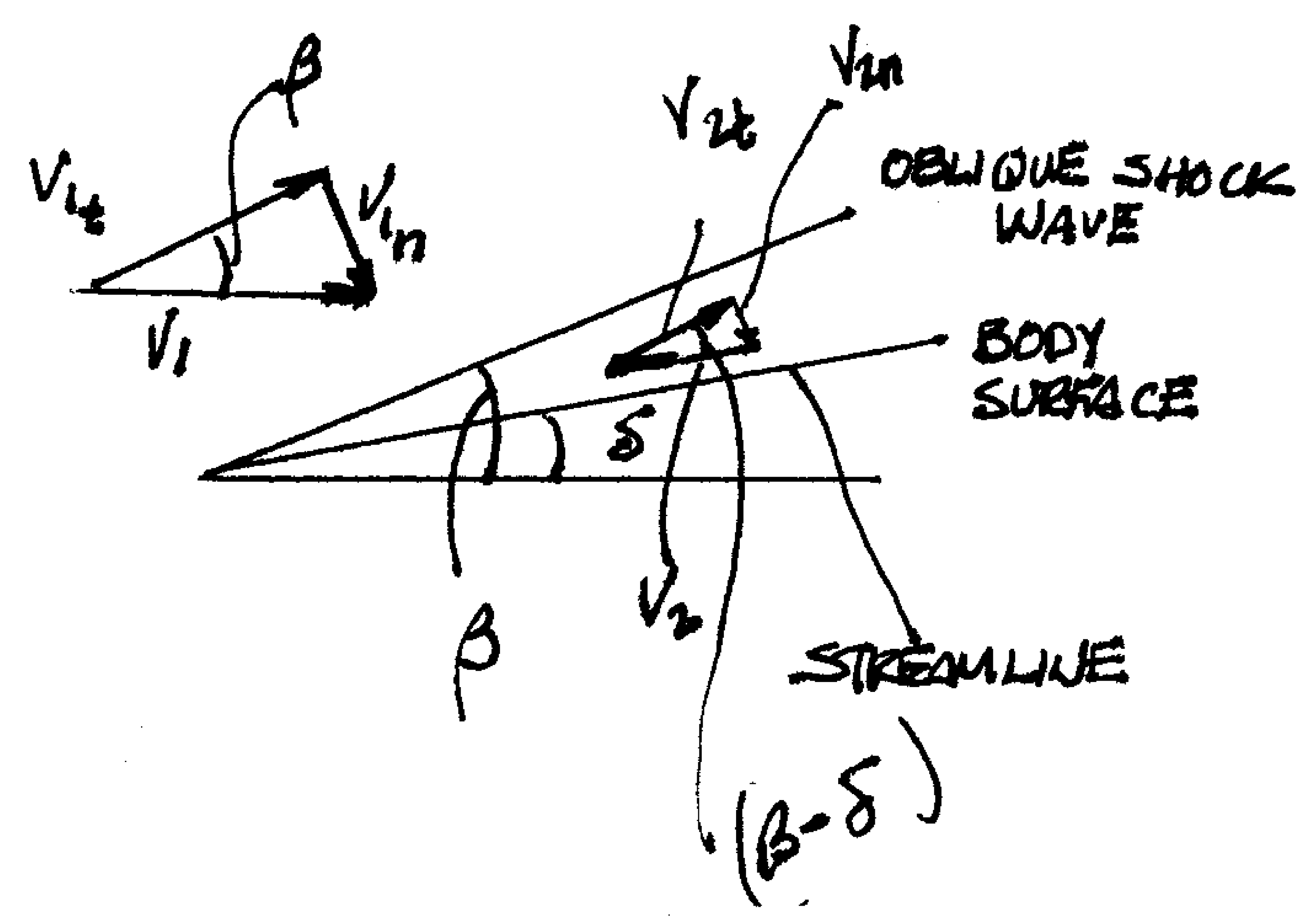
NORMAL SHOCK WAVES — ASSUMPTIONS

- ONE DIMENSIONAL FLOW
- STEADY FLOW
- EQUILIBRIUM STATES ON EITHER SIDE OF THE NORMAL SHOCK
- DISSIPATIVE MECHANISMS OF HEAT CONDUCTION AND VISCOSITY ARE EFFECTIVE <u>WITHIN</u> THE SHOCK
- NO HEAT IS ADDED WITHIN THE FLOW
- CONSERVATION PRINCIPLES APPLY
- CONSTANT AREA
- CALORICALLY PERFECT GAS
- INVISCID FLOW ON EITHER SIDE OF THE NORMAL SHOCK



OBLIQUE SHOCK WAVES — ASSUMPTIONS

- SAME AS FOR NORMAL SHOCKWAVE
- RESOLVE ALL VELOCITIES INTO COMPONENTS PARALLEL AND NORMAL TO THE OBLIQUE SHOCK
- CONSERVATION OF LINEAR MOMENTUM CONSISTS OF TWO SCALAR EQUATIONS



RATIO OF THERMODYNAMIC VARIABLES ACROSS SHOCK WAVES

RATIO	NORMAL SHOCK	OBLIQUE SHOCK
EFFECTIVE MACH NUMBER	M_1	$M_1 \sin\beta \equiv M_{1n}$
ρ_2/ρ_1	$\dfrac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2}$	$\dfrac{(\gamma+1) M_{1n}^2}{(\gamma-1) M_{1n}^2 + 2}$
$(p_2 - p_1)/p_1$	$2\gamma \dfrac{(M_1^2 - 1)}{(\gamma+1)}$	$2\gamma \dfrac{(M_{1n}^2 - 1)}{(\gamma+1)}$
T_2/T_1	$1 + \dfrac{2(\gamma-1)(\gamma M_1^2+1)(M_1^2-1)}{(\gamma+1)^2 M_1^2}$	$1 + \dfrac{2(\gamma-1)(\gamma M_{1n}^2+1)(M_{1n}^2-1)}{(\gamma+1) M_{1n}^2}$
$e^{(S_2-S_1)/R}$	$\dfrac{\left[1 + \dfrac{2\gamma}{\gamma+1}(M_1^2-1)\right]^{\frac{1}{(\gamma-1)}}}{\left[\dfrac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2}\right]^{\frac{\gamma}{(\gamma-1)}}}$	$\dfrac{\left[1 + \dfrac{2\gamma}{\gamma+1}(M_{1n}^2-1)\right]^{\frac{1}{(\gamma-1)}}}{\left[\dfrac{(\gamma+1) M_{1n}^2}{(\gamma-1) M_{1n}^2 + 2}\right]^{\frac{\gamma}{(\gamma-1)}}}$

OBLIQUE SHOCK WAVES

RELATIONSHIP BETWEEN OBLIQUE SHOCK WAVE ANGLE β AND FLOW DEFLECTION ANGLE δ.

$$\tan\beta = \frac{V_{1n}}{V_{1t}}, \quad \tan(\beta-\theta) = \frac{V_{2n}}{V_{2t}}, \quad V_{1t} = V_{2t}$$

$$\frac{\tan(\beta-\delta)}{\tan\beta} = \frac{V_{2n}}{V_{1n}} = \frac{\rho_1}{\rho_2} = \frac{(\gamma-1)M_{1n}^2+2}{(\gamma+1)M_{1n}^2}$$

OBTAIN

$$\tan\delta = 2\cot\beta\left\{\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos^2\beta)+2}\right\}$$

β, M_1 GIVEN
FIND δ

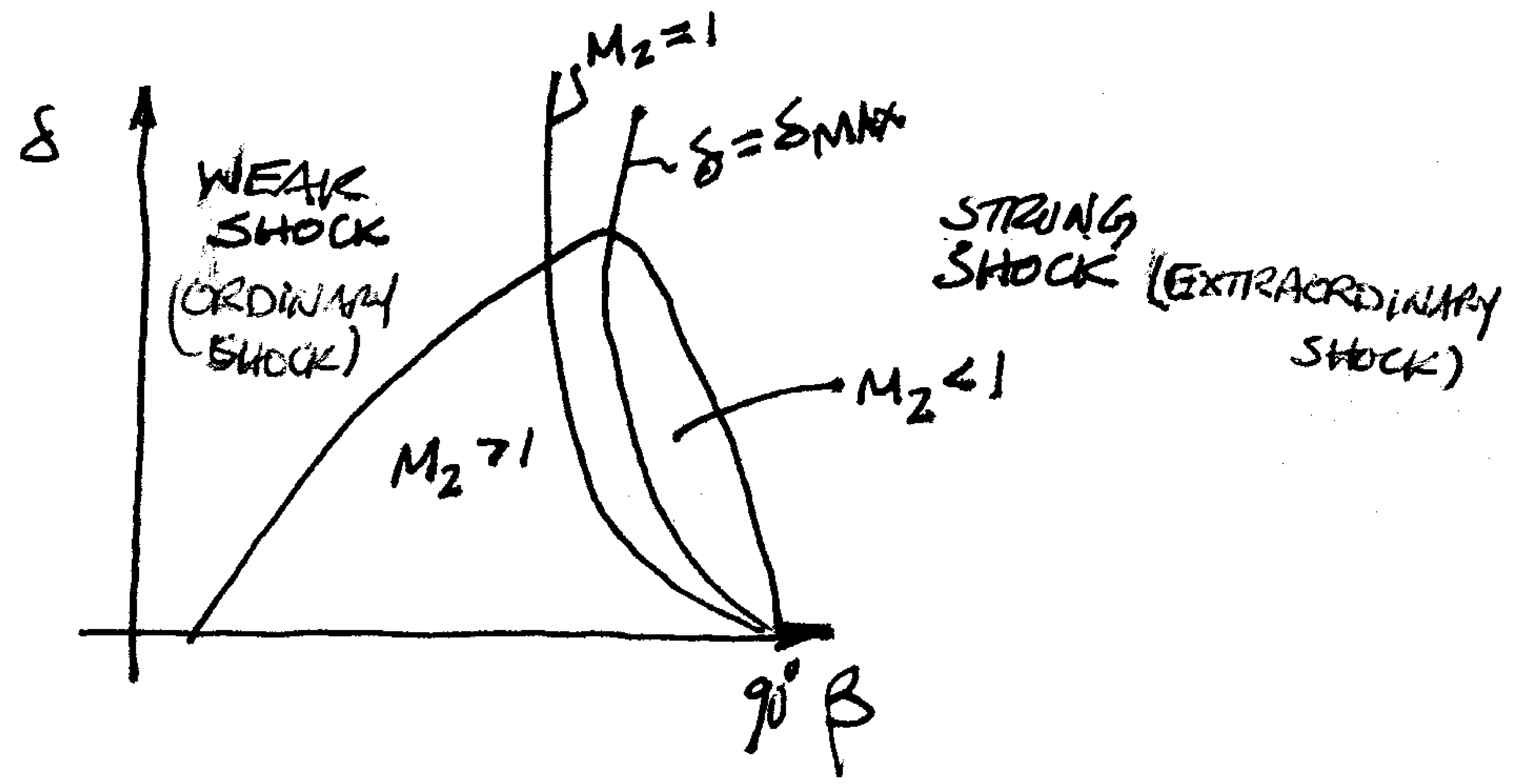
NOTE THAT ABOVE EQUATIONS YEILDS :

$$\beta = 90^\circ = \pi/2 \longrightarrow \delta = 0$$

$$\beta = \sin^{-1}(1/M_1) \longrightarrow \delta = 0$$

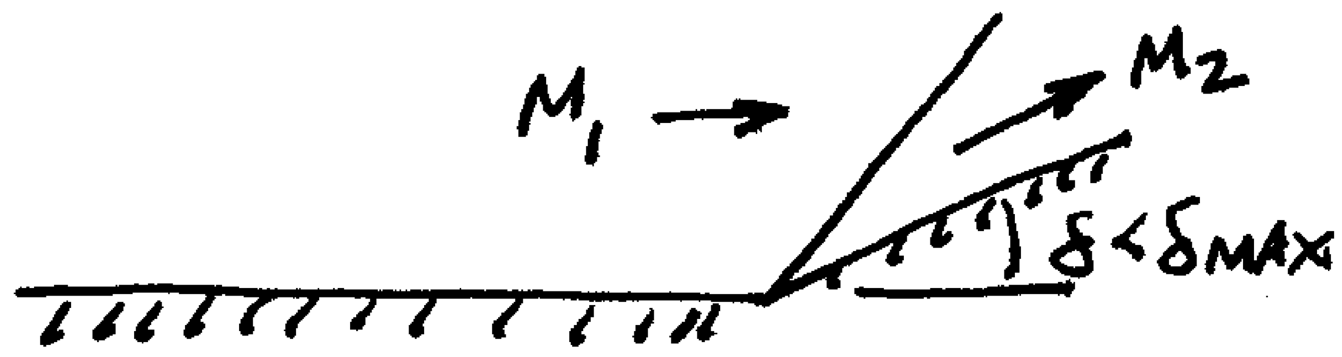
FOR EACH M_1, δ HAS A MAXIMUM, $\delta = \delta_{MAX}$

PLOTING

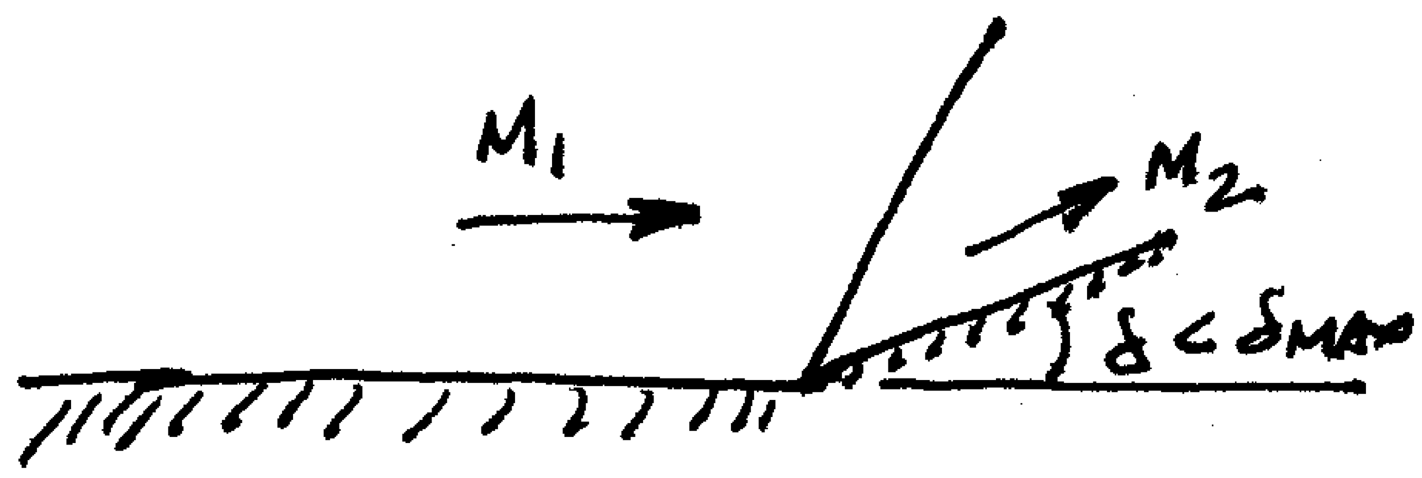


SUPERSONIC FLOW OVER WEDGES

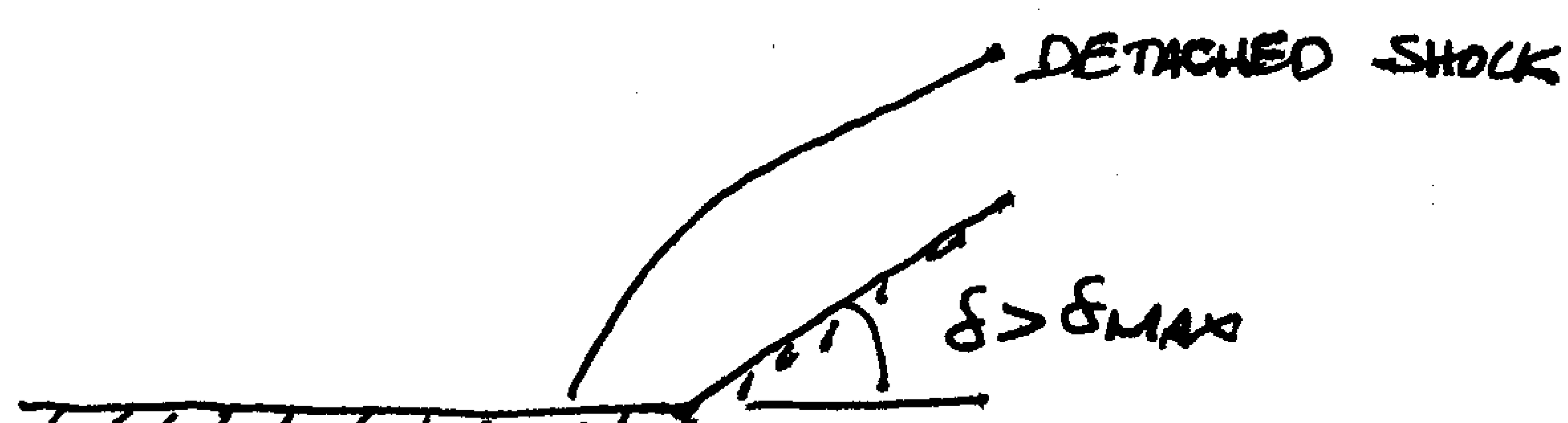
A. CORNER FLOW



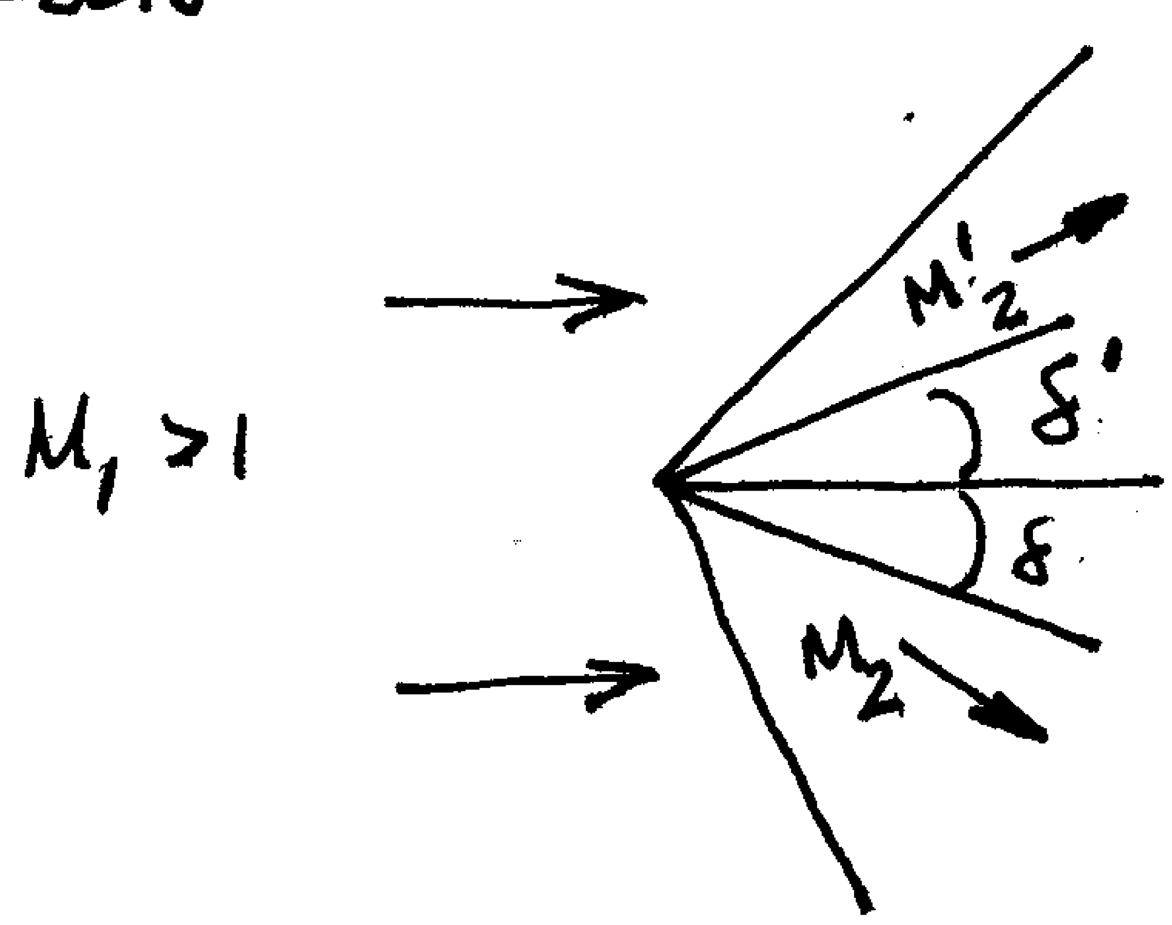
ORDINARY SOLUTION
(STRONG SHOCK)
$M_2 \geq 1$



EXTRAORDINARY SOLUTION
(WEAK SHOCK)
$M_2 < 1$



B. WEDGE FLOW



$\delta' < \delta < \delta_{MAX}$

OBLIQUE SHOCK WAVE PROBLEMS

A. A WEDGE IS INTRODUCED INTO A FLOW HAVING A MACH NUMBER $M_1 = 1.7$. (a) WHAT ARE THE POSSIBLE WAVE ANGLES (β) IF THE FLOW IS TO BE DEFLECTED BY 13°? (b) WHAT IS THE MAXIMUM ANGLE AT WHICH THE FLOW CAN TURN WITHOUT BECOMING DETACHED FROM THE WEDGE? (54°, 77°, 17°)

B. AIR FLOWING AT A MACH NUMBER OF 2.5 PASSES OVER A TWO-DIMENSIONAL WEDGE WITH A HALF-ANGLE OF 12°. FIND β, M_2, p_2/p_1, T_2/T_1, AND p_{02}/p_{01} FOR BOTH THE ORDINARY AND EXTRAORDINARY SOLUTIONS.

	EXTRA-ORDINARY SOLUTION (STRONG SHOCK)	ORDINARY SOLUTION (WEAK SHOCK)
β	85°	34°
M_2	0.505	2.0
p_2/p_1	7.0	2.1
T_2/T_1	2.12	1.25
p_{02}/p_{01}	0.505	0.96