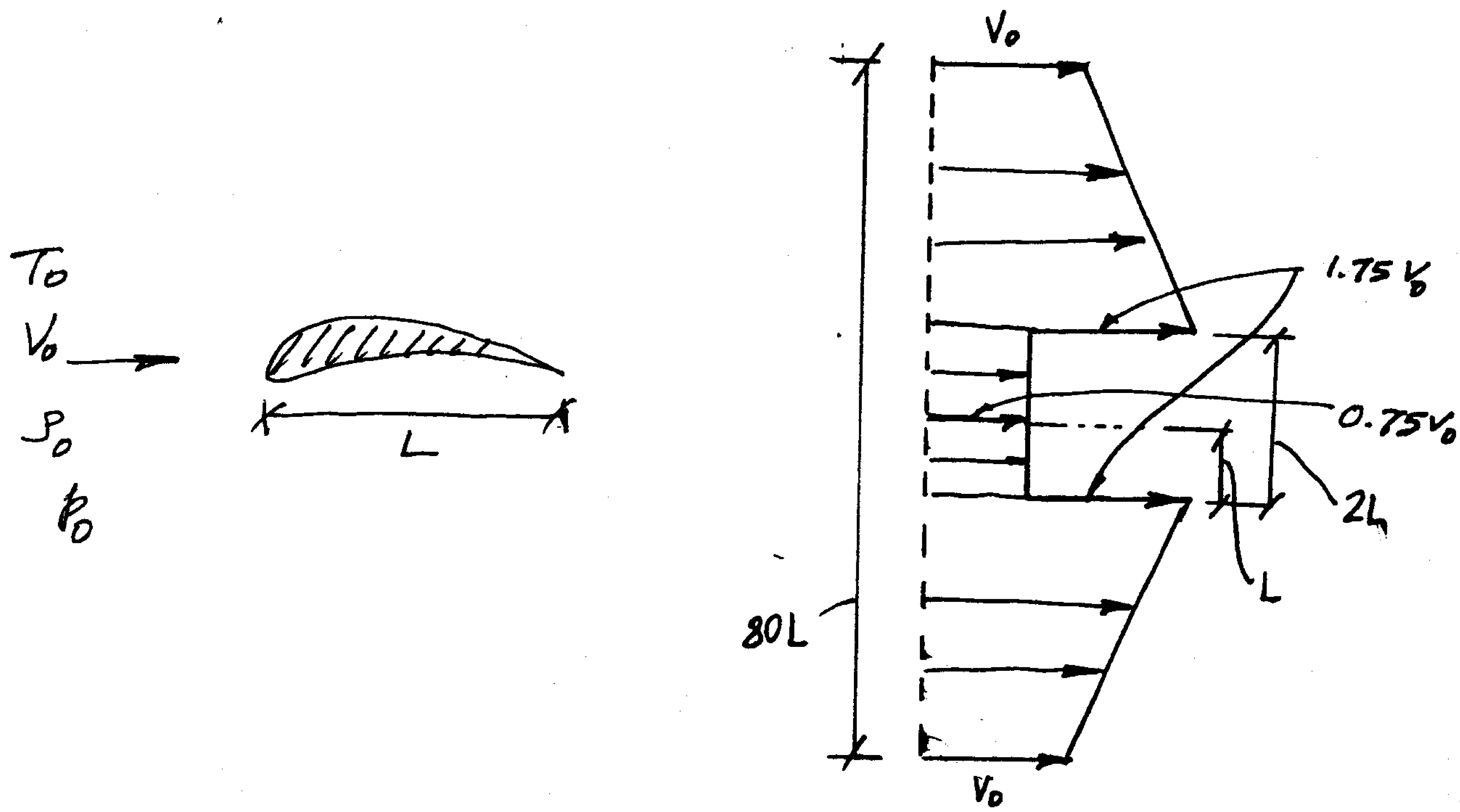
CONSERVATION OF LINEAR MOMENTUM (INTEGRAL FORM)

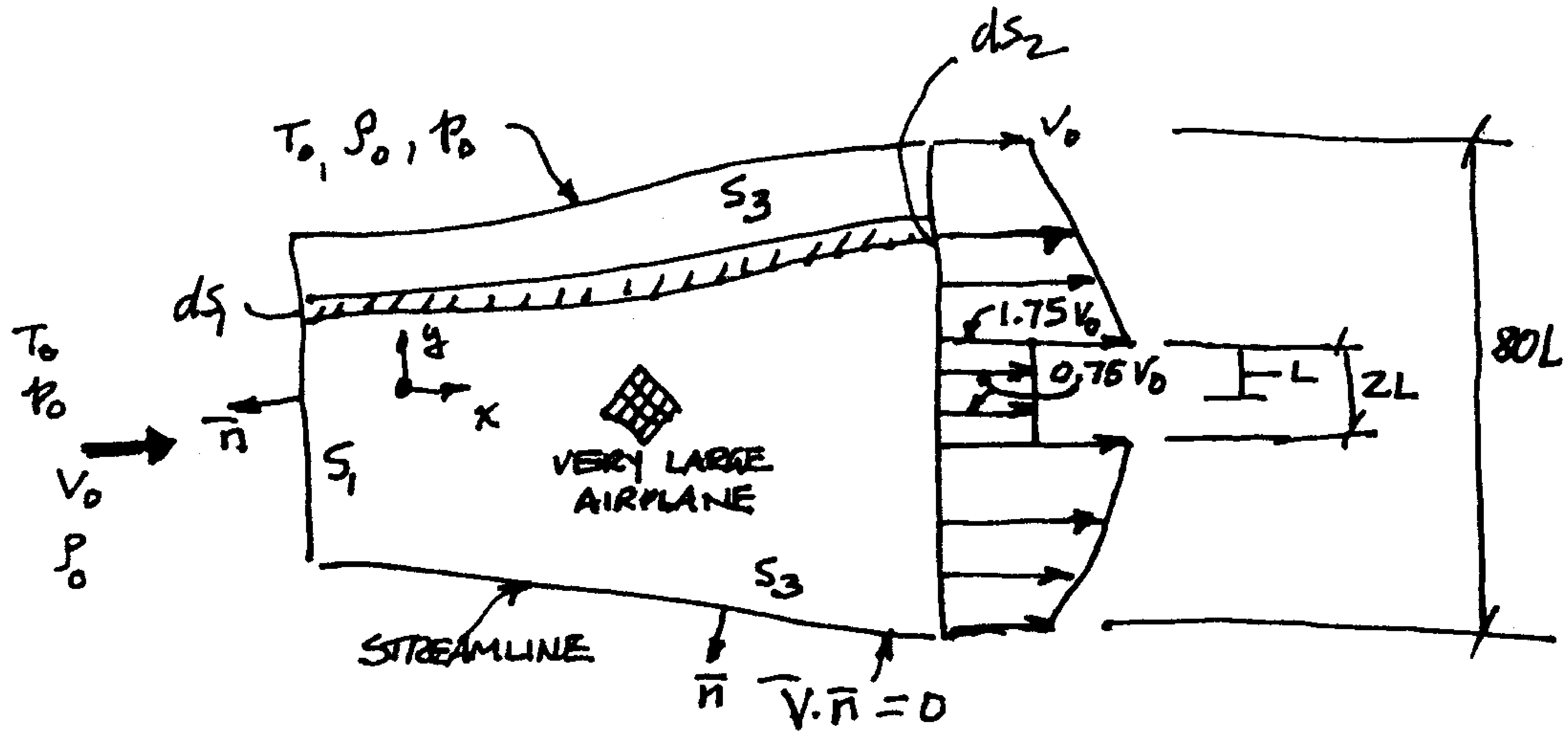
DRAG CALCULATION

AN INTERNATIONAL CONSORTIUM OF AIRCRAFT COMPANIES HAS AGREED TO BUILD A LOW SPEED VERY LARGE AIRPLANE (VLA). THE VLA CAN TRANSPORT A MAXIMUM OF 800 PASSENGERS A DISTANCE OF 5,000 NAUTICAL MILES. THE WAKE PROFILE (VELOCITY PROFILE) PRODUCED BY A TWO-DIMENSIONAL AIRFOIL MODEL OF A WING SECTION OF THE VLA HAS BEEN SHOWN TO BE:



COMPUTE/CALCULATE AND DISCUSS FULLY THE NET FORCE ACTING ON THE AIRCRAFT AIRFOIL IN THE DIRECTION OF THE FREE STREAM VELOCITY.

SOLUTION



ASSUMPTIONS:

1. STEADY FLOW
2. INCOMPRESSIBLE FLUID
3. VERY SMALL VISCOUS EFFECTS
4. UNBOUNDED FLUID
5. UNIFORM UPSTREAM CONDITIONS: T_0, p_0, V_0, ρ_0
6. 2-D FLOW
7. ON S_3: $\vec{V} \cdot \vec{n} = 0$, $\rho = \rho_0$, $p = p_0$
8. ON S_2: $\vec{V_2} = V_2(y)\,\vec{i}$, $\rho = \rho_0$, $p = p_0$
9. ZERO GRAVITY EFFECTS
10. WAKE PROFILE IS SYMMETRIC W.R.T. THE X-AXIS

FROM CLASS NOTES:

$$DRAG = -\iint_{SURF} (p - p_0)\,\vec{n} \cdot \vec{i}\; dSURF - \iint_{SURF} \rho \vec{V} \cdot \vec{i}\, (\vec{V} \cdot \vec{n})\, dSURF$$

FROM ASSUMPTIONS 5, 7, AND 8: $\iint_{SURF} (p - p_0)\,\vec{n} \cdot \vec{i}\; dSURF = 0$

$$\therefore\; DRAG = -\iint_{SURF} \rho \vec{V} \cdot \vec{i}\, (\vec{V} \cdot \vec{n})\, dSURF = \iint_{S_1} \rho_0 V_0^2\, dS_1 - \iint_{S_2} \rho_0 V_0^2(y)\, dS_2$$

APPLY CONSERVATION OF MASS PRINCIPLE TO ARBITRARY STRIP BETWEEN TWO STREAMLINES CONNECTING S_1 AND S_2, AND OBTAIN:

$$\rho_0 V_0 \, dS_1 = \rho_0 V_2(y) \, dS_2$$

SUBSTITUTE AND CONTINUE AS:

$$DRAG = \iint_{S_2} \rho_0 V_0 V_2(y) \, dS_2 - \iint_{S_2} \rho_0 V_0^2(y) \, dS_2 = \iint_{S_2} \rho_0 V_2(y) (V_0 - V_2(y)) \, dS_2$$

USING ASSUMPTION 10:

$$DRAG = 2 \int_0^{40L} \rho_0 V_2(y) (V_0 - V_2(y)) \, dy$$

FROM WAKE PROFILE

$$V_2(y) = \begin{cases} 0.75 V_0 \, , & 0 < y < L \\ 1.75 V_0 + m^* (L - y) \, , & L < y < 40L \\ & m^* \equiv \dfrac{V_0}{52.0 L} \end{cases}$$

SUBSTITUTE AND EVALUATE INTEGRALS:

$$\boxed{DRAG = -197.25 \rho_0 V_0^2 L}$$

THIS VERY LARGE AIRPLANE PRODUCES A NET THRUST!!