SUMMARY

WE HAVE DISCUSSED THE FUNDAMENTALS OF FLUID MECHANICS. WE HAVE INTRODUCED A MODEL OF A FLUID, DEVELOPED KINEMATICAL RELATIONS FOR FLUIDS, AND DERIVED THE CONSERVATION PRINCIPLES FOR A FLUID-BODY INTERACTION. WE HAVE PERFORMED SOME USEFUL CALCULATIONS OF FLUID FLOW IN DUCTS, CALCULATED DRAG AND LIFT IN SOME VERY SPECIAL CASES, AND GENERALLY BEGAN TO DEVELOP THAT "GUT FEELING" FOR FLUIDS. THAT "GUT FEELING" SERVES US QUITE WELL IN ENHANCING OUR UNDERSTANDING OF FLUID-BODY INTERACTION.

WE HAVE LIKEWISE INTRODUCED TWO "NEW" CONCEPTS, NAMELY: VORTICITY AND CIRCULATION. IN THE PROCESS OF DEFINING THE CHARACTER OF VORTICITY AND CIRCULATION, WE ARRIVED AT THE FOLLOWING:

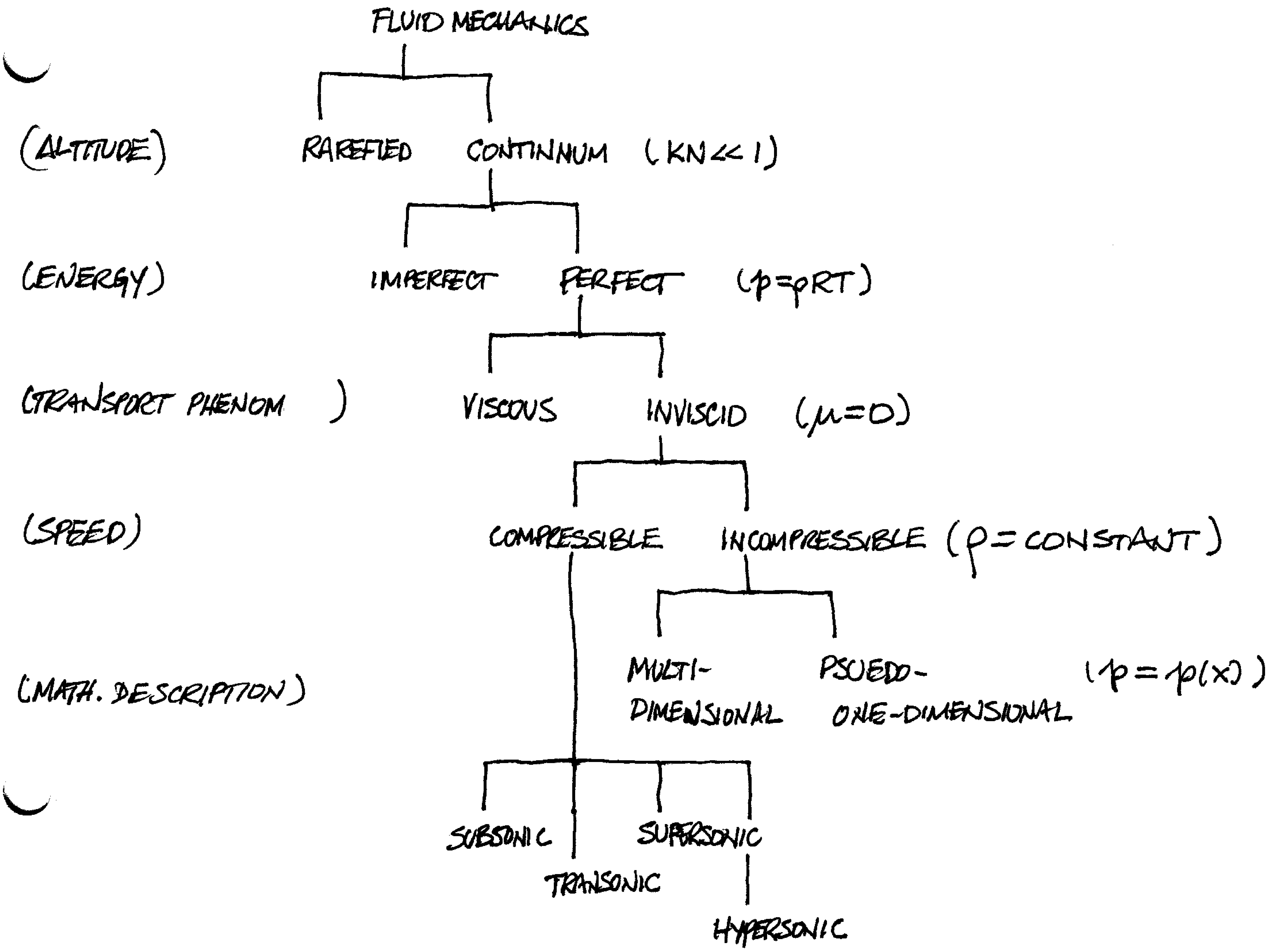
a. KELVIN'S THEOREM $\left[\frac{D\Gamma}{Dt} = 0 \right]$

b. HELMHOLTZ'S LAW $\left[\frac{D\bar{\Omega}}{Dt} = (\bar{\Omega} \cdot \bar{\nabla})\bar{V} \right]$

c. KUTTA-JOUKOWSKI THEOREM $\left[L = -\rho U_\infty \Gamma \,;\, D = 0 \right]$

THE KUTTA-JOUKOWSKI THEOREM IS VALID FOR IRROTATIONAL MOTION OF AN INCOMPRESSIBLE FLUID OVER A RIGHT CIRCULAR CYLINDER WITH CIRCULATION

THE FOLLOWING "CONCEPT" MAP PROVIDES ANOTHER VIEW OF OUR WORLDLINE THROUGH FLUID MECHANICS.



FLUID MECHANICS
(ALTITUDE)
RAREFIED
CONTINNUM
(KN<<1)
(ENERGY)
IMPERFECT
PERFECT
($p=\rho RT$)
(TRANSPORT PHENOM)
VISCOUS
INVISCID
($\mu=0$)
(SPEED)
COMPRESSIBLE
INCOMPRESSIBLE
(ρ = CONSTANT)
(MATH. DESCRIPTION)
MULTI-DIMENSIONAL
PSUEDO-ONE-DIMENSIONAL
($p=p(x)$)
SUBSONIC
TRANSONIC
SUPERSONIC
HYPERSONIC

THICK AIRFOILS

A RIGHT CIRCULAR CYLINDER IS A VERY INTERESTING GEOMETRIC SHAPE BUT NOT A VERY USEFUL AERODYNAMIC PROFILE. THE CYLINDER HAS HELPED US UNDERSTAND HOW LIFT IS GENERATED IN HYDRODYNAMICS. IT DID NOT ANSWER ALL OF OUR QUESTIONS REGARDING FORCES AND MOMENTS ON AERODYNAMIC BODIES MOVING THROUGH A FLUID. FOR EXAMPLE, WHAT DO WE DO ABOUT THE DRAG FORCE WHICH WAS SHOWN TO BE ZERO IN THE HYDRODYNAMIC FLOW OVER A CYLINDER WITH AND WITHOUT CIRCULATION.

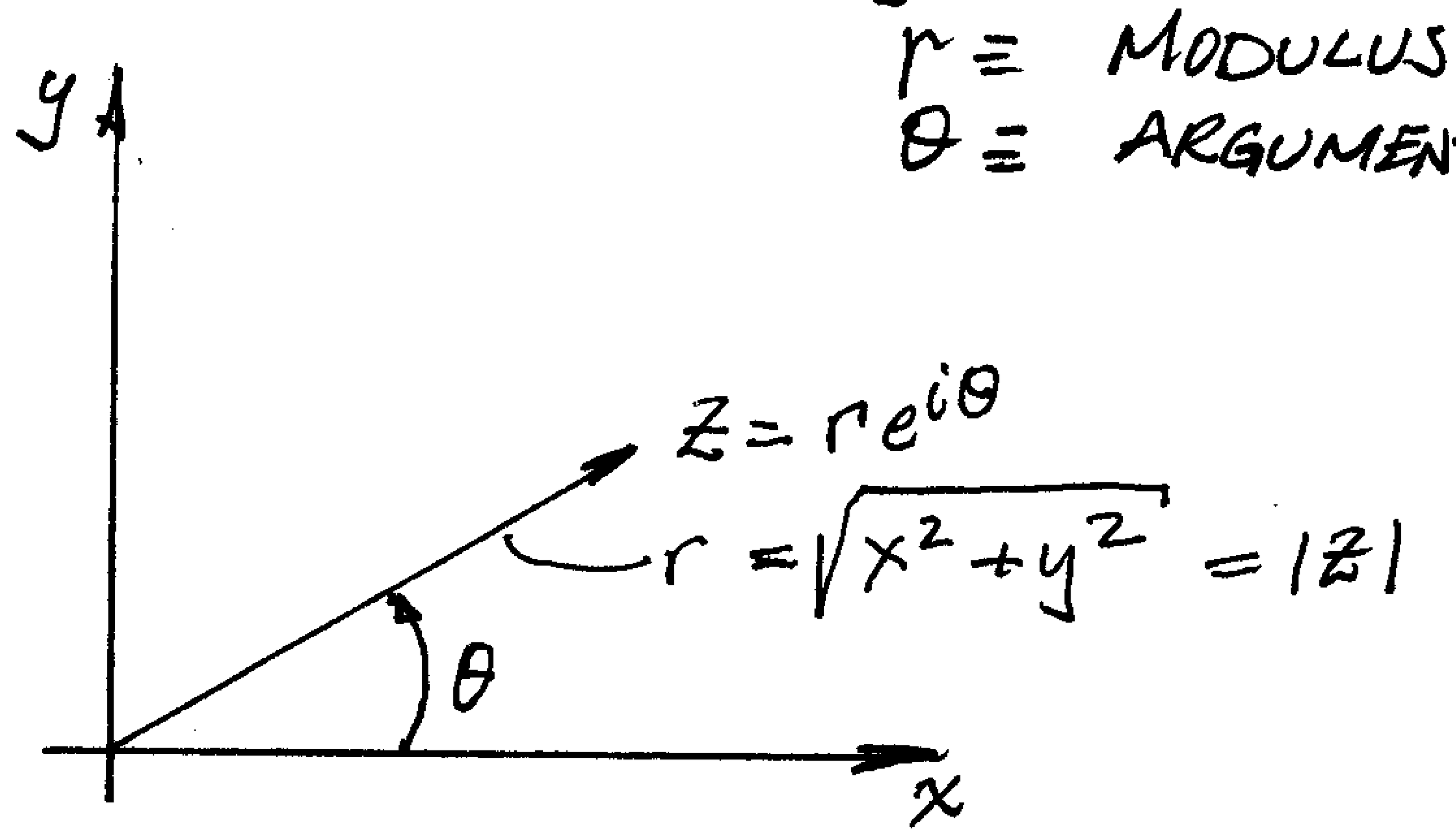
OUR FIRST OBJECTIVE IS TO EXAMINE FLOW OVER AN AIRFOIL BASED ON WHAT WE NOW KNOW ABOUT FLOW OVER A CYLINDER. WE WILL TRANSFORM A CYLINDER INTO AN AIRFOIL. THE JOUKOWSKI TRANSFORMATION IN COMPLEX VARIABLES ENABLES THE MAPPING OF A CYLINDER INTO AN AIRFOIL. THE JOUKOWSKI TRANSFORMATION IS BASED ON CONFORMAL MAPPING PRINCIPLES.

COMPLEX VARIABLES

A COMPLEX VARIABLE, z, MAY BE WRITTEN AS FOLLOWS:

$$z = x + iy = re^{i\theta}$$

$x \equiv$ REAL PART
$y \equiv$ IMAGINARY PART
$r \equiv$ MODULUS OF z
$\theta \equiv$ ARGUMENT OF z



$z = x + iy$
$z = r\cos\theta + ir\sin\theta$
$z = re^{i\theta}$

WE STATE, WITHOUT PROOF, THE FOLLOWING:

$$z^n = r^n e^{in\theta}$$

$$z = z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$$

$$z = z_1 \times z_2 = re^{i\theta} = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$z = z_1 / z_2 = (r_1 / r_2) e^{i(\theta_1 - \theta_2)}$$

$$\bar{z} = x - iy \equiv \text{COMPLEX CONJUGATE OF } z$$

$$\ln z = \ln r + i\theta$$

$$w(z) = \phi + i\psi = f(x+iy) \equiv \text{COMPLEX POTENTIAL}$$

ϕ = VELOCITY POTENTIAL
ψ = STREAM FUNCTION

$\left\{ \begin{array}{l} \text{INCOMPRESSIBLE} \\ \text{IRROTATIONAL} \\ \text{INVISCID} \end{array} \right.$

$$\left. \begin{array}{l} \phi_x = \psi_y \\ \phi_y = -\psi_x \end{array} \right\} \text{ CAUCHY-RIEMANN EQUATIONS}$$

$$u = \phi_x = \psi_y$$

$$v = -\psi_x = \phi_y$$

TABLE OF COMPLEX POTENTIALS, z-PLANE

UNIFORM STREAM, PARALLEL TO X-AXIS	$w = U_\infty z$
SOURCE AT ORIGIN	$w = (K/2\pi) \ln z$
DOUBLET AT ORIGIN, AXIS ALONG X-AXIS	$w = (\mu/2\pi)/z$
CIRCULAR CYLINDER, RADIUS a, UNIFORM STREAM	$w = U_\infty [z + a^2/z]$
VORTEX AT ORIGIN	$w = (i\Gamma/2\pi) \ln z$
CIRCULAR CYLINDER, WITH CIRCULATION	$w = U_\infty [z + a^2/z] + (i\Gamma/2\pi) \ln(z/a)$