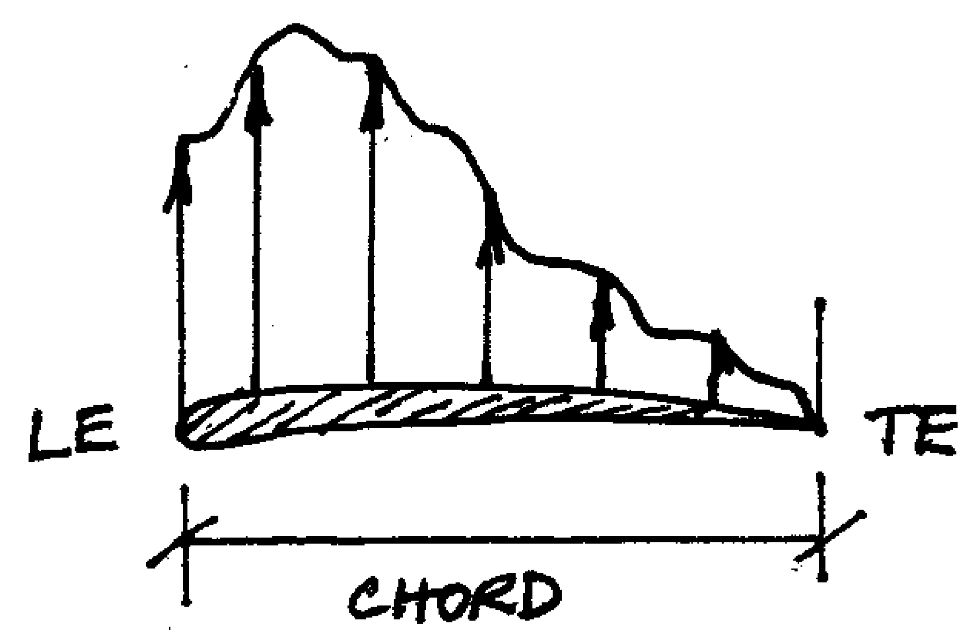
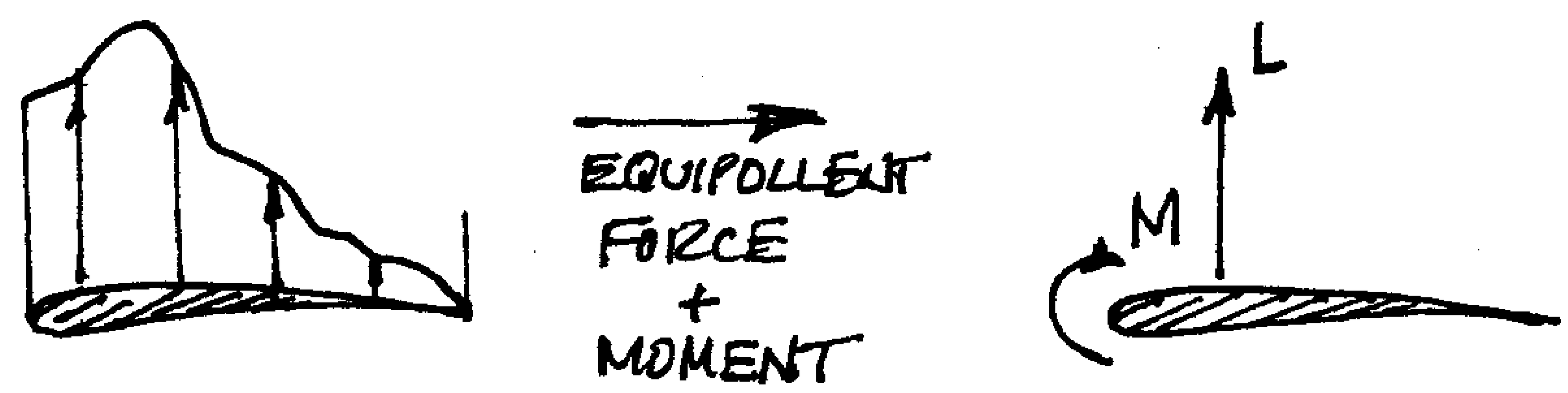
FORCES AND MOMENTS ON AIRFOILS

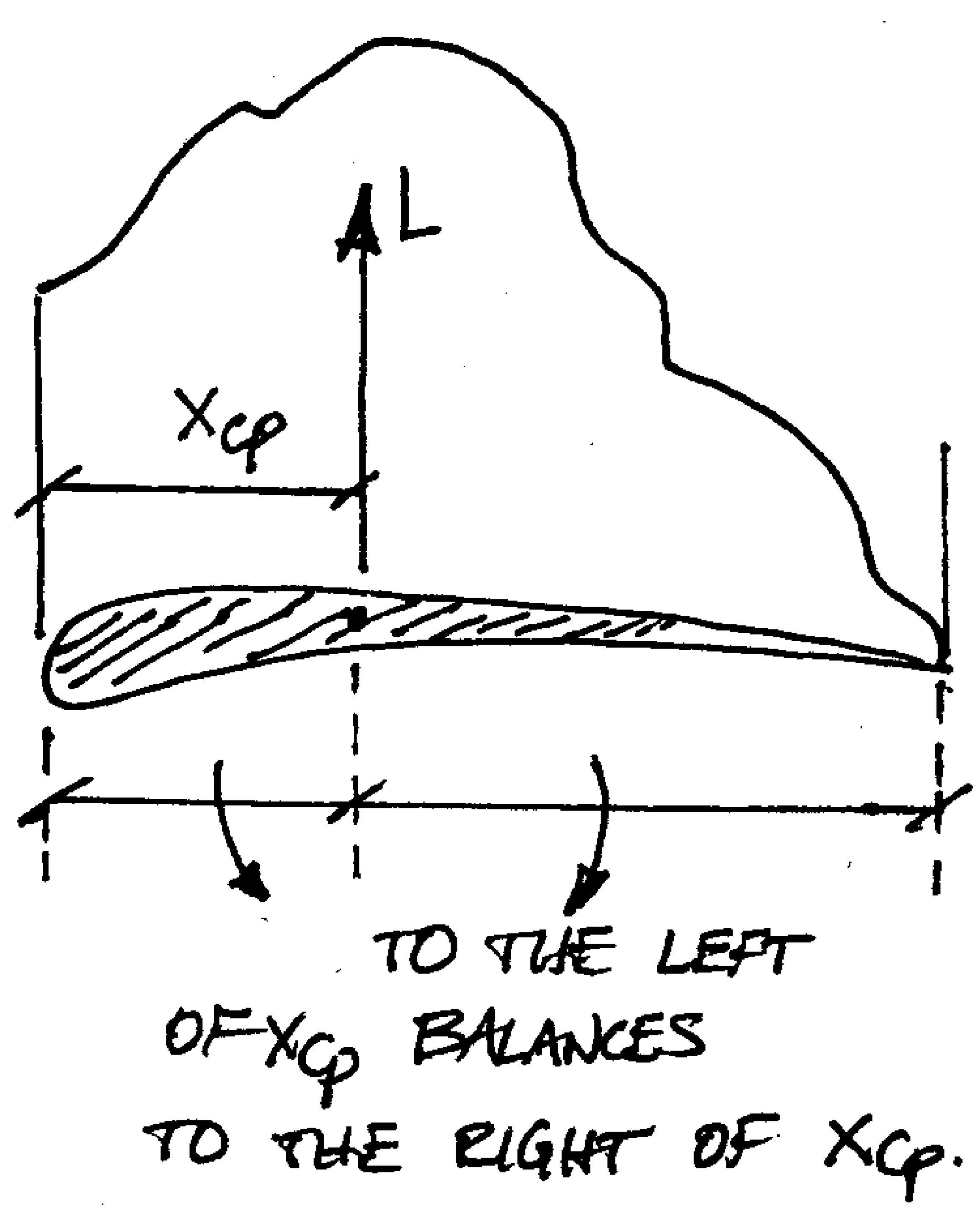
CONSIDER AN AIRFOIL WITH A PRESSURE DISTRIBUTION AS SHOWN.



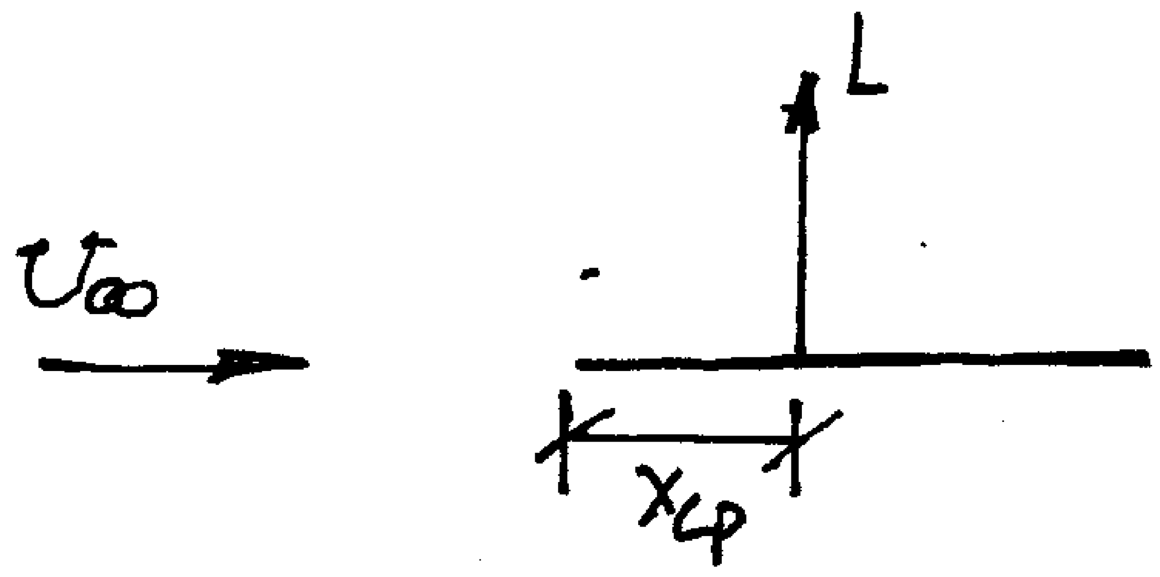
IT IS CONVENIENT, USEFUL TO RESOLVE THE PRESSURE DISTRIBUTION INTO ITS EQUIPOLLENT FORCE AND MOMENT. THAT IS,



WE DEFINE THE CENTER OF PRESSURE TO BE THE LOCATION ON THE AIRFOIL CHORD ABOUT WHICH THE MOMENT IS ZERO. DENOTE THE CENTER OF PRESSURE X_{cp}. HENCE, WE HAVE



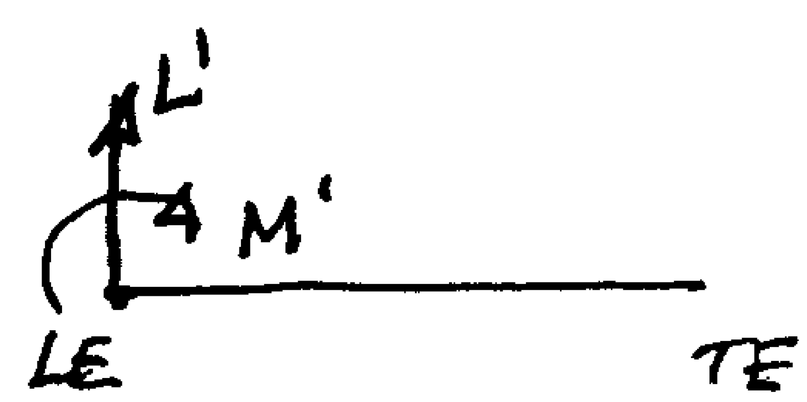
USING THE CENTER OF PRESSURE CONCEPT, WE MODEL AN AIRFOIL AS FOLLOWS:



L "ACTS" THROUGH x_{cp}.

$M_{x_{cp}} = 0$

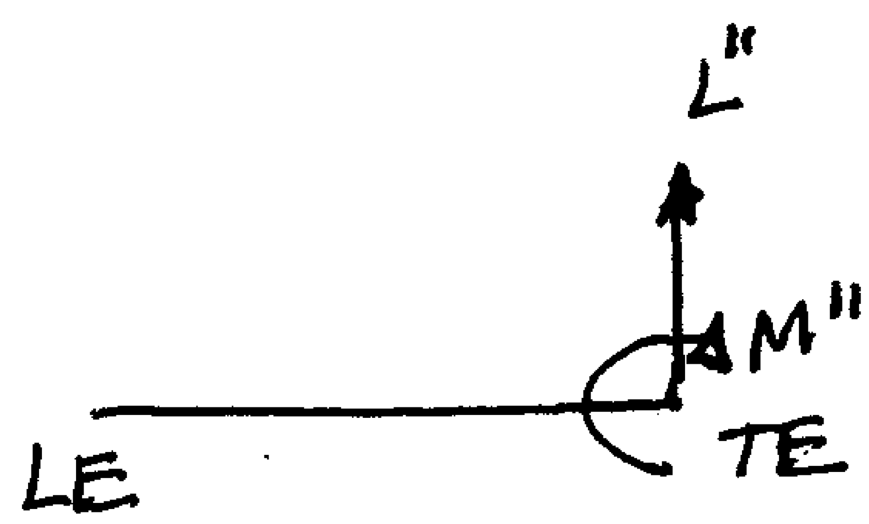
WE MAY EXTEND THE EQUIPOLLENT FORCES PLUS MOMENTS CONCEPT TO THE LEADING EDGE OF THE AIRFOIL AND WRITE



$L' = L$

$M' = -L x_{cp}$

AND AT THE TRAILING EDGE

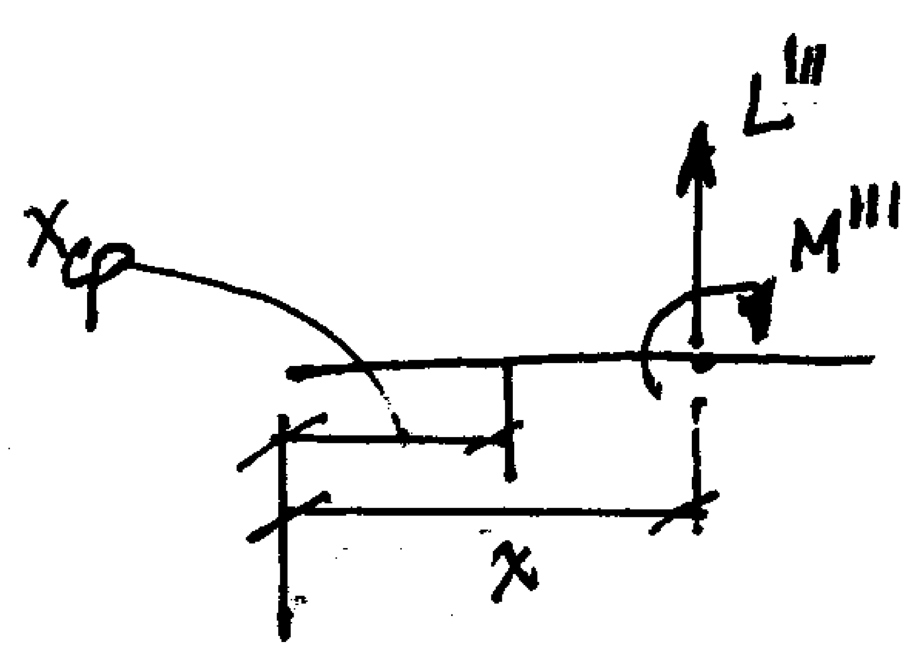


$L'' = L$

$M'' = L(c - x_{cp})$

c = CHORD

LIKEWISE, FOR AN ARBITRARY POINT



$L''' = L$

$M''' = L(x - x_{cp})$

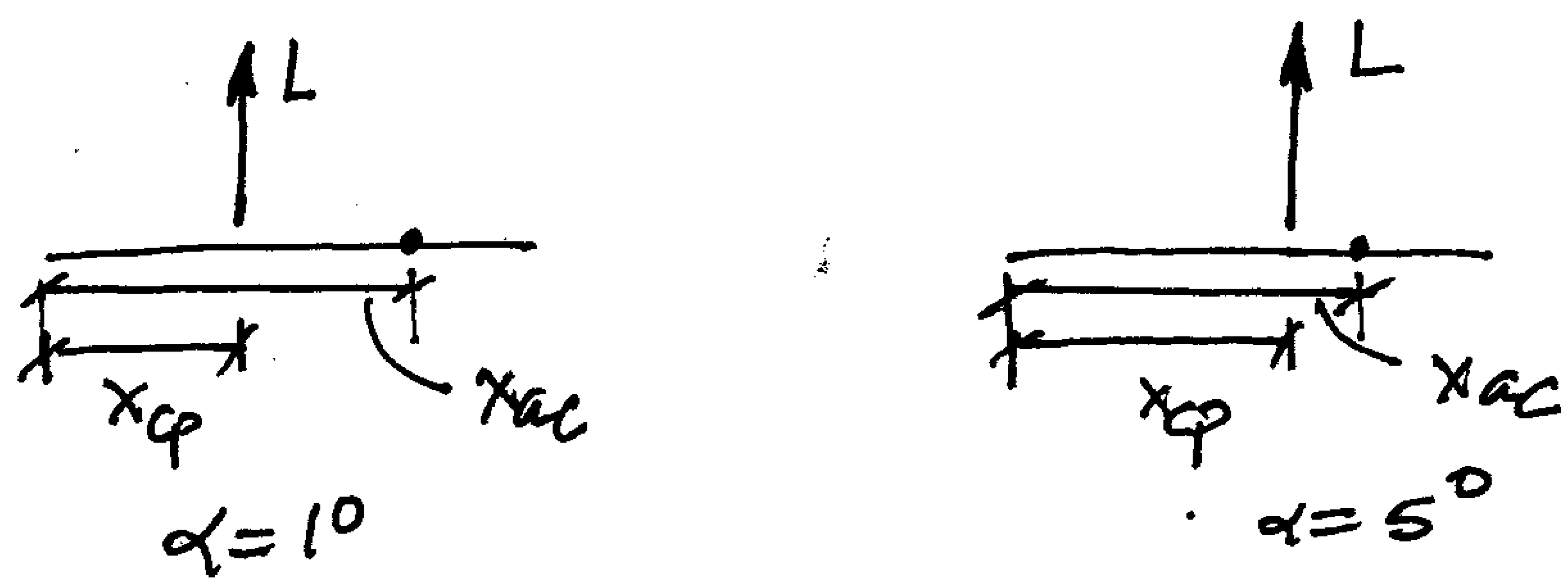
ASSUME WE HAVE EXPERIMENTAL DATA OR THEORETICAL CALCULATIONS FOR C_ℓ AND C_m AT $x = c/4$. FROM THIS DATA/RESULTS, WE MAY SOLVE FO x_{cp}.

$$x_{cp} = \frac{c}{4} - \frac{M_{x=c/4}}{L}$$

OR

$$x_{cp} = x - \frac{M_x}{L}$$

AS ANGLE OF ATTACK α CHANGES, THE LIFT L ALSO CHANGES AND SO DOES x_{cp}.



THERE IS SOME POINT FOR WHICH THE MOMENT IS CONSTANT AS α CHANGES. FOR SUCH A POINT L GETS LARGER BUT x_{cp} MOVES SUCH THAT CHANGE IN MOMENT ARM BALANCES THE CHANGE IN L. THIS POINT IS THE AERODYNAMIC CENTER x_{ac}. THUS WE WANT $C_{m_{ac}}$ TO BE CONSTANT WITH RESPECT TO α, ANGLE OF ATTACK.

$$\frac{\partial C_{m_{ac}}}{\partial \alpha} = 0$$

CONSIDER AN ARBITRARY POINT, X

$$M''' = L(x - x_{cp})$$

NOW SET $x = x_{ac}$

$$M''' = M_{ac} = L(x_{ac} - x_{cp})$$

$$\frac{1}{2}\rho U_\infty^2 c^2 C_{M_{ac}} = \frac{1}{2}\rho U_\infty^2 c\, C_\ell\,(x_{ac} - x_{cp})$$

$$c\, C_{M_{ac}} = C_\ell\,(x_{ac} - x_{cp})$$

TAKE $\frac{\partial}{\partial \alpha}$ AND SET RESULT EQUAL TO ZERO.

$$c\,\frac{\partial C_{M_{ac}}}{\partial \alpha} = \frac{\partial C_\ell}{\partial \alpha}(x_{ac} - x_{cp}) - C_\ell \frac{\partial x_{cp}}{\partial \alpha} = 0 \quad (*)$$

$$\frac{\partial x_{ac}}{\partial \alpha} \equiv 0$$

RECALL, AGAIN

$$M''' = L(x - x_{cp})$$

NOW SET $x = x_{c/4} = c/4$

$$M_{c/4} = L(c/4 - x_{cp})$$

$$\frac{1}{2}\rho U_\infty^2 c^2 C_{m_{c/4}} = \frac{1}{2}\rho U_\infty^2 c\, C_\ell\,(c/4 - x_{cp})$$

$$c\, C_{m_{c/4}} = C_\ell\,(c/4 - x_{cp})$$

TAKE $\frac{\partial}{\partial \alpha}$:

$$c \frac{\partial C_{M c/4}}{\partial \alpha} = \frac{\partial C_\ell}{\partial \alpha}\left(\frac{c}{4} - x_{cp}\right) - C_\ell \frac{\partial x_{cp}}{\partial \alpha}$$

$$c \frac{\partial C_{M c/4}}{\partial \alpha} = \frac{\partial C_\ell}{\partial \alpha} \frac{c\, C_{M c/4}}{C_\ell} - C_\ell \frac{\partial x_{cp}}{\partial \alpha}$$

$$\therefore \frac{\partial x_{cp}}{\partial \alpha} = \frac{c}{C_\ell^2}\left(C_{M c/4} \frac{\partial C_\ell}{\partial \alpha} - C_\ell \frac{\partial C_{M c/4}}{\partial \alpha} \right) \quad (**)$$

COMBINING EQUATION (*) AND (**):

$$\frac{\partial C_\ell}{\partial \alpha}(x_{ac} - x_{cp}) - C_\ell \frac{c}{C_\ell^2}\left(C_{M c/4} \frac{\partial C_\ell}{\partial \alpha} - C_\ell \frac{\partial C_{M c/4}}{\partial \alpha} \right) = 0$$

SOLVE FOR x_{ac} :

$$x_{ac} = -\frac{c}{C_\ell} \frac{1}{\frac{\partial C_\ell}{\partial \alpha}} \left(C_\ell \frac{\partial C_{M c/4}}{\partial \alpha} - C_{M c/4} \frac{\partial C_\ell}{\partial \alpha} \right) + x_{cp}$$

$$x_{ac} = -c \frac{\partial C_{M c/4}/\partial \alpha}{\partial C_\ell/\partial \alpha} + \frac{C_{M c/4}}{C_\ell} + x_{cp}$$

WE HAVE SHOWN ABOVE THAT

$$x_{cp} = \frac{c}{4} - \frac{M_{x=c/4}}{L} = \frac{c}{4} - \frac{\frac{1}{2}\rho U_\infty^2 c^2 C_{M c/4}}{\frac{1}{2}\rho U_\infty^2 c\, C_\ell}$$

$$x_{cp} = \frac{c}{4} - c\frac{C_{M c/4}}{C_\ell}$$

THEREFORE, BY SUBSTITUTING WE MAY EXPRESS

$$X_{AC} = -C \frac{\partial C_{M_{c/4}} / \partial \alpha}{\partial C_{\ell} / \partial \alpha} + \frac{C}{4}$$

EXAMPLE PROBLEM

THE MEAN CAMBER LINE OF A THIN AIRFOIL IS GIVEN BY

$$y = c\,k\left[a - b\,\frac{x}{c} - d\left(\frac{x}{c}\right)^2\right]$$

WHERE c IS THE CHORD AND k, a, b, AND d ARE KNOWN REAL CONSTANTS. CALCULATE THE FOLLOWING

(a) α_0 (b) C_{m_0} (c) C_ℓ

SOLUTION

$$x = \frac{c}{2}(1-\cos\theta) = c\sin^2\frac{\theta}{2}$$

$$\frac{dy}{dx} = ck\left(-\frac{b}{c} - 2\frac{d}{c}\frac{x}{c}\right)$$

$$\left(\frac{dy}{dx}\right)_\theta = -ck\left[\frac{b}{c} + 2\frac{d}{c}\cdot\frac{1}{2}(1-\cos\theta)\right]$$

$$\left(\frac{dy}{dx}\right)_\theta = -ck\left(\frac{b+d}{c} - \frac{d}{c}\cos\theta\right)$$

CALCULATE THE LIFT COEFFICIENT, C_ℓ

$$C_\ell = 2\pi\alpha + 2\int_0^\pi \left(\frac{dy}{dx}\right)_\theta (\cos\theta - 1)\,d\theta$$

$$C_\ell = 2\pi\alpha + 2ck\int_0^\pi \left[\frac{d}{c}\cos\theta - \left(\frac{b+d}{c}\right)\right](\cos\theta - 1)\,d\theta$$

$$C_\ell = 2\pi\alpha + 2ck\int_0^\pi \left[\frac{d}{c}\cos^2\theta - \left(\frac{b+2d}{c}\right)\cos\theta + \frac{b+d}{c}\right]d\theta$$

$$C_\ell = 2\pi\alpha + 2ck\left[\frac{d}{c}\frac{\theta}{2} + \frac{\sin 2\theta}{4}\Big|_0^\pi - \frac{b+2d}{c}\sin\theta\Big|_0^\pi + \frac{b+d}{c}\,\theta\Big|_0^\pi \right]$$

$$C_\ell = 2\pi\alpha + 2\pi k\left(\frac{d}{2} + b + d\right)$$

$$\underline{C_L = 2\pi\left(\alpha + k\left(b + \tfrac{3}{2}d\right)\right)}$$

CALCULATE THE ANGLE OF ZERO LIFT, α_0

$$C_\ell = 2\pi(\alpha - \alpha_0)\ , \qquad \alpha_0 = -\frac{1}{\pi}\int_0^\pi \left(\frac{dy}{dx}\right)_\theta (\cos\theta - 1)\,d\theta$$

$$\therefore \quad \alpha_0 = \alpha - \frac{C_\ell}{2\pi}$$

$$= \alpha - \frac{1}{2\pi}\left[2\pi\alpha + 2\pi k\left(b + \tfrac{3}{2}d\right)\right]$$

$$= \alpha - \alpha - k\left(b + \tfrac{3}{2}d\right)$$

$$\underline{\alpha_0 = -k\left(b + \tfrac{3}{2}d\right)}$$

CALCULATE THE MOMENT FOR ZERO LIFT, VIZ

$$C_{m_0} = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}\left[\frac{2}{\pi}\int_0^\pi \left(\frac{dy}{dx}\right)_\theta \cos(2\theta)\,d\theta - \frac{2}{\pi}\int_0^\pi \left(\frac{dy}{dx}\right)_\theta \cos\theta\,d\theta\right]$$

$$C_{m_0} = \frac{1}{2}\int_0^\pi \left(\frac{d}{c}\cos\theta - \frac{b+d}{c}\right)ck\cos(2\theta)\,d\theta - \frac{1}{2}\int_0^\pi \left(\frac{d}{c}\cos\theta - \frac{b+d}{c}\right)ck\cos\theta\,d\theta$$

$$C_{m_0} = \frac{ck}{2}\left[\frac{d}{c}\left(\frac{\sin\theta}{2} + \frac{\sin 3\theta}{6} \right)\Big|_0^{\pi} \right.$$

$$- \frac{b+d}{c}\,\frac{1}{2}\sin(2\theta)\Big|_0^{\pi}$$

$$- \frac{d}{c}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right)\Big|_0^{\pi}$$

$$\left. + \frac{b+d}{c}\sin\theta\,\Big|_0^{\pi} \right] = - \frac{ck}{2}\cdot\frac{d}{c}\cdot\frac{\pi}{2}$$

$$C_{m_0} = - \frac{dk}{4}\pi$$