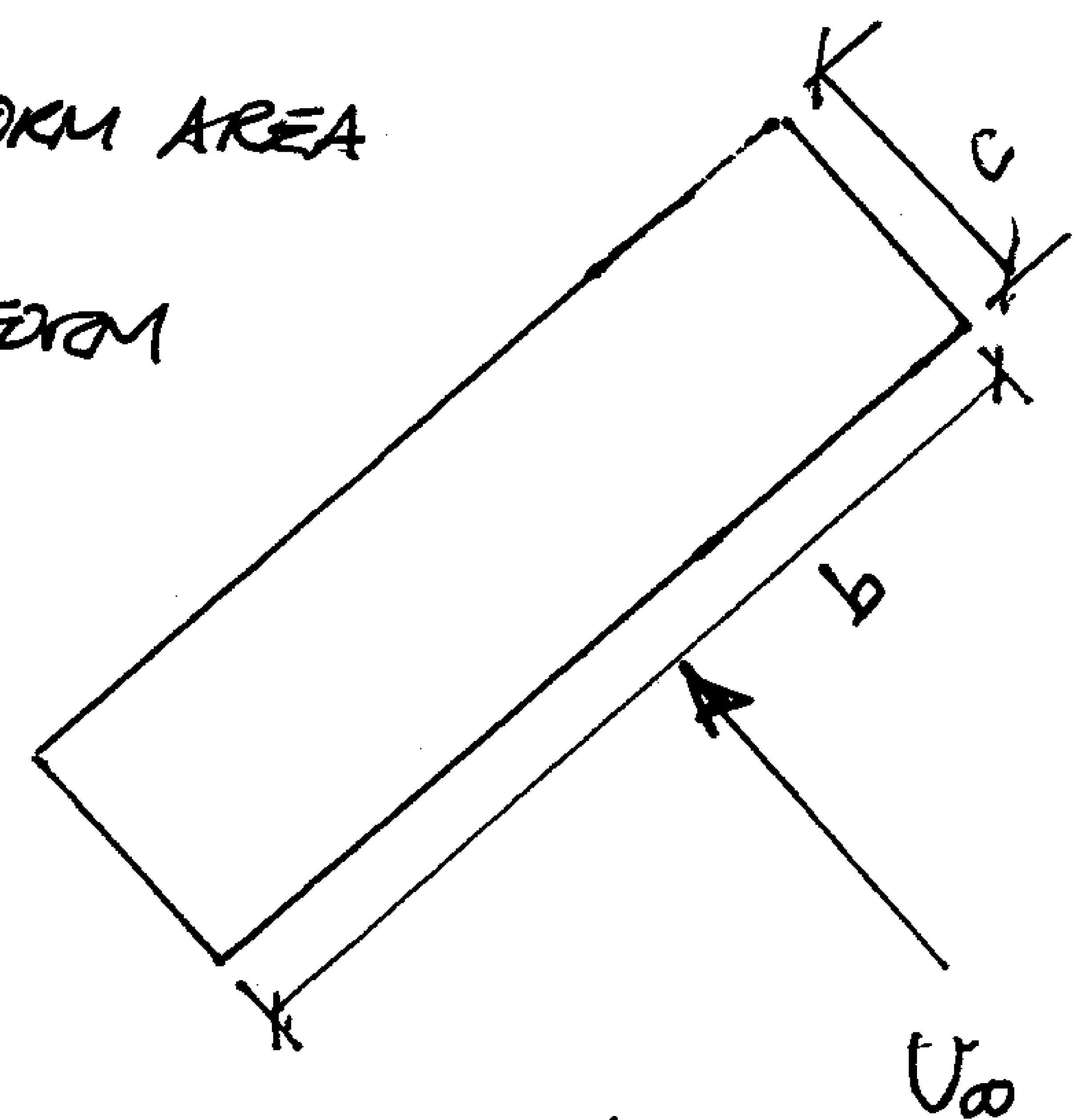
WINGS OF LARGE ASPECT RATIO

TO THIS POINT IN OUR DISCUSSION WE HAVE LIMITED OUR CONSIDERATION TO TWO-DIMENSIONAL SECTIONS OF A WING, E.G. AIRFOILS. WE WILL NOW EXTEND OUR DISCUSSION TO THREE-DIMENSIONAL WINGS. AN IMPORTANT NON-DIMENSIONAL PARAMETER TO DESCRIBING FLOW OVER WINGS IS ITS ASPECT RATIO, AR. BY DEFINITION,

$$AR \equiv \frac{b^2}{S}$$

b = WING SPAN
S = WING PLANFORM AREA



IN THE CASE OF A RECTANGULAR PLANFORM OF SPAN b AND CHORD c,

$$AR = \frac{b^2}{S} = \frac{b^2}{(b)(c)} = \frac{b}{c}$$

FOR RECTANGULAR PLANFORMS, WINGS OF LARGE ASPECT RATIO ARE WINGS WITH LARGE SPAN RELATIVE TO CHORD. THE TABLE BELOW GIVES VALUE OF ASPECT RATIO FOR SEVERAL AIRCRAFT TYPES.

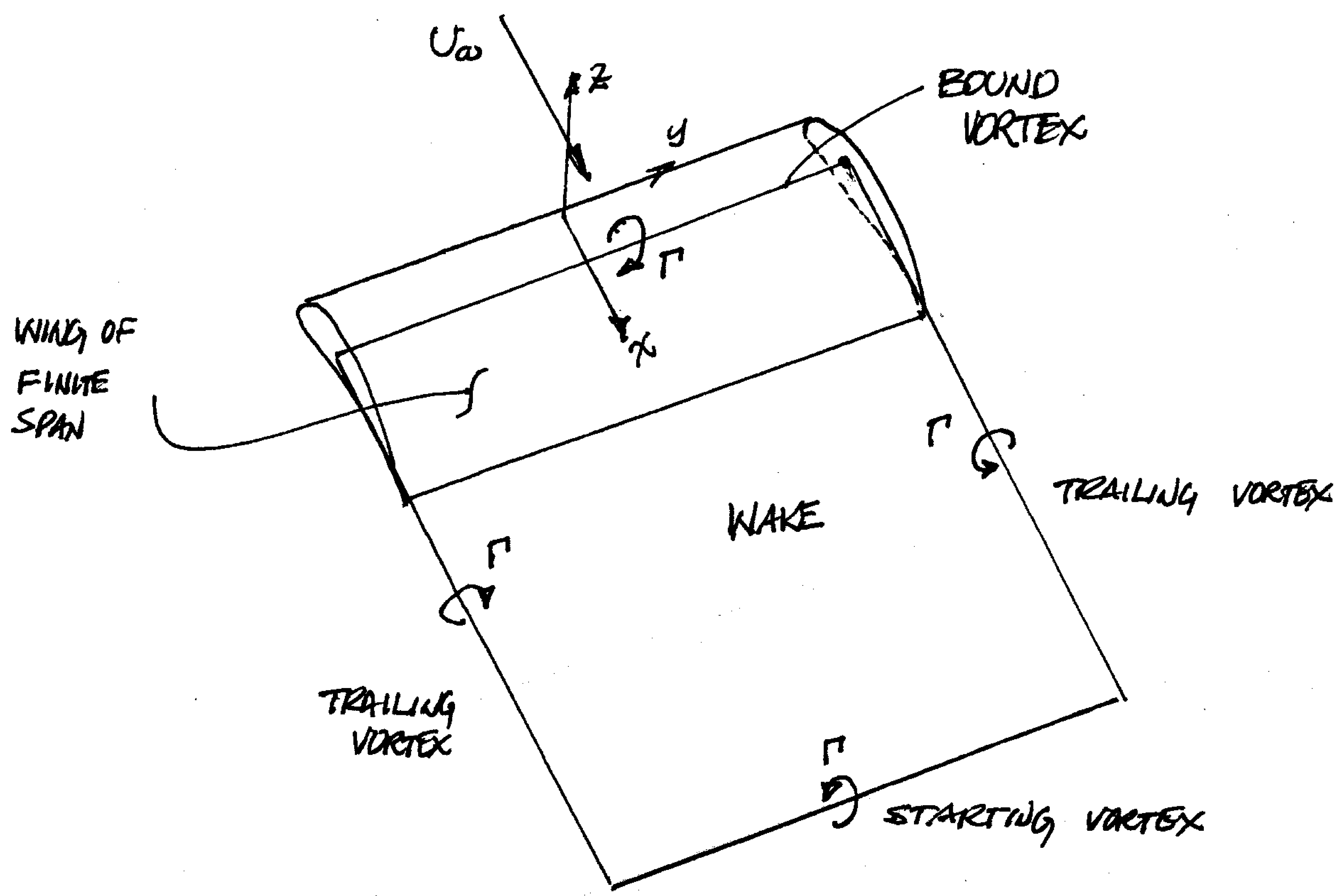
TABLE OF ASPECT RATIOS

AIRCRAFT	ASPECT RATIO
CESSNA 172	7.32
TUPOLEV 154	7.03
McDONNELL-DOUGLAS DC9	8.25
BOEING 737	8.83
BOEING 747	6.95
LOCKHEED 5A	7.75
DASSAULT MIRAGE III	1.94
McDONNELL-DOUGLAS F-4	2.78

WINGS IN STEADY, INVISCID FLOW

THE FIVE MOST IMPORTANT CONCEPTS IN UNDERSTANDING AND MODELING FLOW OVER WINGS ARE:

1. BOUND VORTICITY
2. WAKE VORTICITY
3. DOWNWASH, w_i
4. INDUCED FLOW FIELD (BIOT-SAVART LAW)
5. EFFECTIVE ANGLE OF ATTACK, α_{eff}

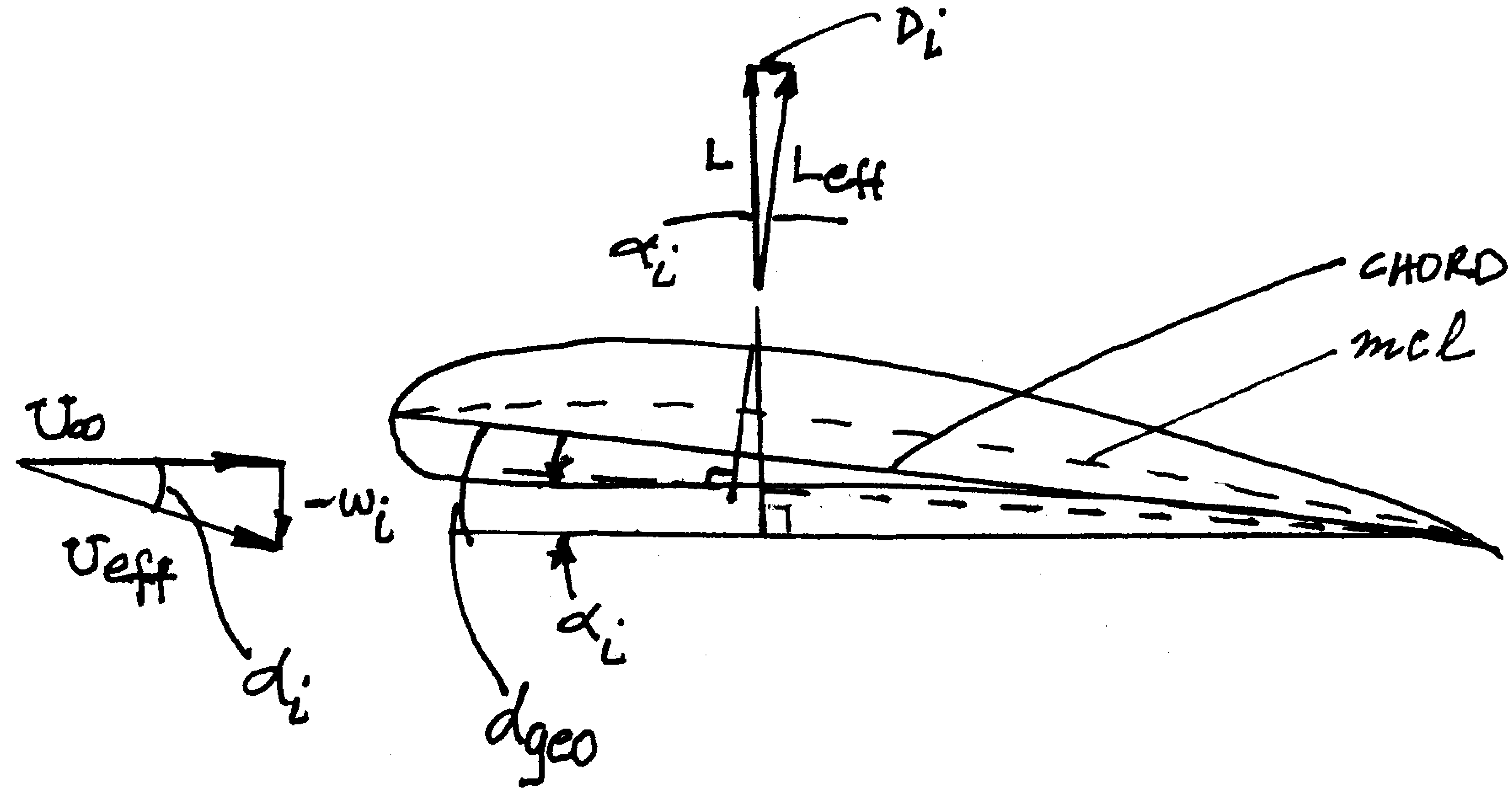


HOW WOULD THIS PICTURE CHANGE IF THE CIRCULATION OVER THE WING VARIES WITH SPAN? THAT IS

$$LIFT = \rho U_\infty \int_{SPAN} \Gamma(y)\,dy$$

THE WAKE WOULD CONSIST OF TRAILING VORTICES SHEDDED OFF THE TRAILING EDGE OF THE WING.

KELVIN'S THEOREM ENSURES THE PRESENCE OF A TRAILING WAKE. THIS TRAILING WAKE CONSISTS OF THE VORTICITY SHEDDED BY THE WING OF FINITE SPAN. BIOT-SAVART LAW, IN TURN, REQUIRES AN INDUCED VELOCITY FIELD THAT IMPACTS THE ENTIRE FLOW PATTERN. HENCE:



$$\alpha_{eff} = \alpha_{geo} - \alpha_i$$

$$\tan \alpha_i = -\frac{w_i}{U_\infty}$$

FOR SMALL α_i :

$$\tan\alpha_i = \frac{\sin\alpha_i}{\cos\alpha_i} \cong \alpha_i$$

$$\alpha_i = -\frac{w_i}{U_\infty}$$

$$L = L_{eff} \cos\alpha_i$$

$$D_i = L_{eff} \sin\alpha_i$$

WHERE D_i IS THE INDUCED DRAG OR IS THE DRAG DUE TO LIFT.

LET'S CALCULATE THE DOWNWASH VELOCITY, w_i. USING THE BIOT-SAVART LAW AND INTEGRATING ALONG THE SPAN,

$$w_i(y) = \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{\Gamma(y')\,dy'}{y-y'}$$

AN APPROXIMATION TO THE ABOVE INTEGRAL IS

$$w_i = \frac{\Gamma_0}{\pi b e}$$

Γ_0 = MID-SPAN CIRCULATION
b = WING SPAN
e = OSWALD'S CONSTANT

LIFT AND DRAG CONSIDERATIONS

BY ANALOGY FROM THIN AIRFOIL THEORY, FOR A WING WE WRITE:

$$L_{eff} = 2\pi \alpha_{eff} \cdot \frac{1}{2}\rho U_{eff}^2 S$$

$$C_{L_{eff}} = 2\pi \alpha_{eff} = 2\pi(\alpha_{geo} - \alpha_i)$$

USING MOMENTUM BALANCE AND THE BIOT-SAVART LAW, WE MAY WRITE

$$L \approx \rho U_\infty \int_{-b/2}^{b/2} \Gamma(y)\,dy$$

$$D \approx \rho \int_{-b/2}^{b/2} w(y)\,\Gamma(y)\,dy$$

SPECIAL CASE: ELLIPTICALLY LOADED WING

$$\Gamma(y) = \Gamma_0 \left[1 - \left(y / b/_2 \right)^2 \right]^{1/2}$$

$$\alpha_i = \frac{\Gamma_0}{2bU_\infty}$$

$$\alpha_i = \frac{C_L}{\pi AR}$$ (MORE LIFT PRODUCES MORE DOWNWASH)

$$C_L = \frac{\pi b \Gamma_0}{2U_\infty S}$$

$$C_L = 2\pi \frac{\alpha_{geo}}{1+\frac{2}{AR}}$$

$$w_i = \frac{\Gamma_0}{2b} = \text{CONSTANT}$$

$$C_{D_i} = \frac{C_L^2}{\pi AR}$$

IN THE LIMIT OF

α_i = CONSTANT
α_{geo} = CONSTANT
ZERO WING TWIST

$$c(y) = c_0 \left[1 - \left(y / b/_2 \right)^2 \right]^{1/2}$$