UNIFIED ENGINEERING

Spring Semester 2002

Fluid Dynamics

Previous Quiz Problems I
(With School Solutions)

NAME: SCHOOL SOLUTIONS

UNIFIED ENGINEERING QUIZ Q7F
Spring Semester 2000
05/04/00

Problem I

(a) Describe the type of **fluid flows** to which the thin airfoil theory may be applied. Your answer should stress the physical properties of the fluid flow.

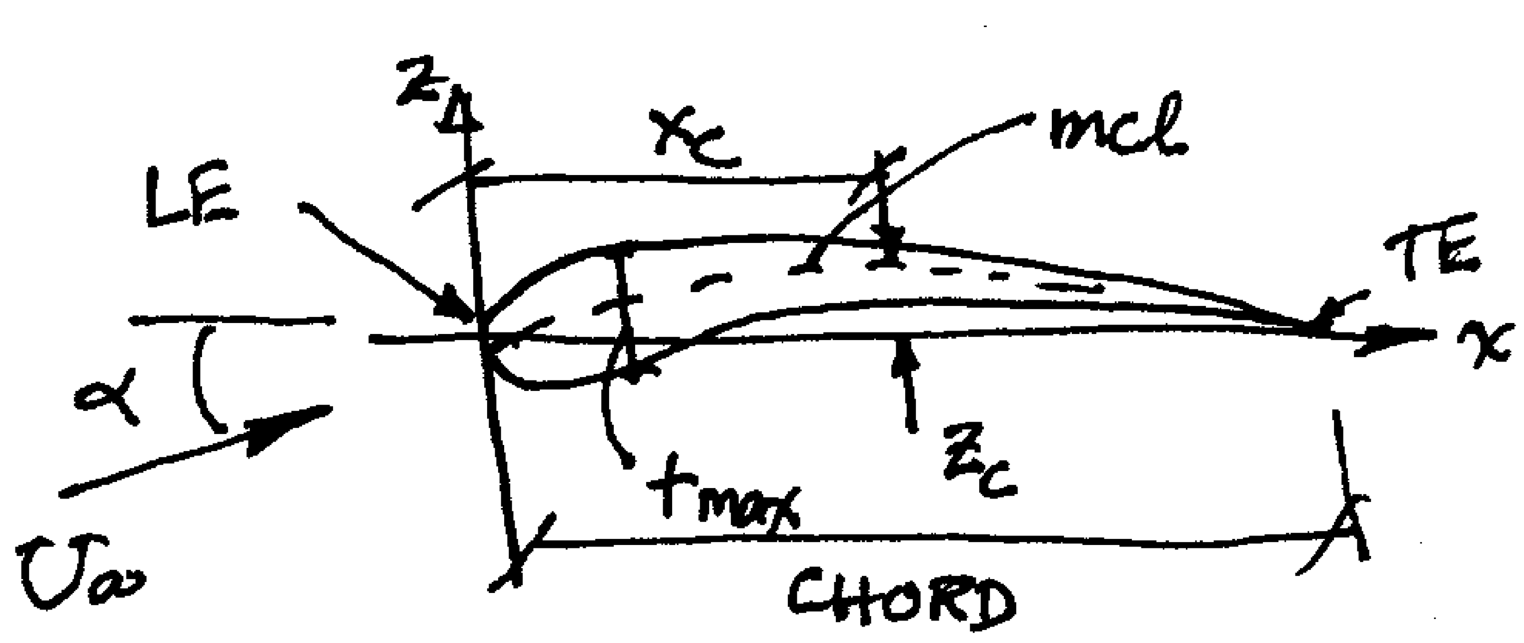
(b) Describe the type of **airfoils** to which the thin airfoil theory may be applied. Your answer should stress the geometrical properties of the airfoils. Use diagrams to illustrate your answers.

(c) Describe how **lift** is generated in the thin airfoil theory model. Use diagrams to illustrate your answers.

10 (a)
- INVISCID FLUID – COEFFICIENT OF VISCOSITY IS ZERO
- INCOMPRESSIBLE FLUID – DENSITY IS CONSTANT
- STEADY FLOW – FLOW PATTERN IS INDEPENDENT OF TIME
- IRROTATIONAL FLOW – FLOW HAS ZERO VORTICITY
- TWO-DIMENSIONAL FLOW – FLOW IS INDEPENDENT OF THE LATERAL DIRECTION: $\frac{\partial}{\partial y} \equiv 0$.

10 (b) THIN AIRFOILS – AIRFOIL IS REPLACED BY MEAN CAMBER LINE

VERY SMALL MAXIMUM MEAN CAMBER
VERY SMALL ANGLE OF ATTACK
} – VORTICITY IS DISTRIBUTED ALONG THE CHORD



t_{max} : MAXIMUM THICKNESS OF AIRFOIL

z_c : MAXIMUM MEAN CAMBER REFERENCE TO THE CHORD LINE

X_c : LOCATION OF z_c AFT THE LEADING EDGE MEASURED ALONG THE CHORD LINE

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Problem I

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(C) IN THIN AIRFOIL THEORY, LIFT IS GENERATED BY A DISTRIBUTION OF VORTICITY ALONG THE mcl OR EQUIVALENTLY BY A DISTRIBUTION OF VORTICITY ALONG THE CHORD

$$l = \rho_\infty U_\infty \Gamma = \rho_\infty U_\infty \int_0^c \gamma(x)\,dx$$

ρ_∞ = FREE STREAM DENSITY OF FLUID

U_∞ = FREE STREAM VELOCITY

Γ = CIRCULATION

$\gamma(x)$ = VORTICITY DISTRIBUTION .

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Problem II

The mean camber line of a thin airfoil is given by

$$z = \frac{0.21x^2(x-c)}{c^3}$$

where c is the airfoil chord.

(a) Calculate the angle of zero lift α_{l_0} when the airfoil is at small angle of attack.

(b) Write the correct expression (integrals) for the lift coefficient c_l and the moment coefficient about the airfoil leading edge $c_{m_{le}}$ when this airfoil is at small angles of attack. Make **ALL** appropriate substitutions but do **NOT** evaluate the integrals.

Include a **sketch** of this thin airfoil at small angle of attack.

Show all details of your analysis and logic.

(a) 5

$$\alpha_{l_0} = -\frac{1}{\pi}\int_0^{\pi}\left(\frac{dz}{dx}\right)_{\theta}^{mcl}(\cos\theta - 1)\,d\theta$$

$$z = \frac{0.21}{c^3}x^2(x-c) = \frac{0.21}{c^3}(x^3 - cx^2)$$

$$\frac{dz}{dx} = \frac{0.21}{c^3}(3x^2 - 2cx)$$

$$x = \frac{c}{2}(1-\cos\theta) = c\sin^2\frac{\theta}{2}$$

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$$\left(\frac{dz}{dx}\right)_\theta^{mcl} = \frac{0.21}{c^3}\left(3\left(\frac{c}{2}(1-\cos\theta)\right)^2 - 2c\left(\frac{c}{2}(1-\cos\theta)\right)\right)$$

$$= \frac{0.21}{c^3}\left[3\frac{c^2}{4}(1-2\cos\theta+\cos^2\theta) - c^2(1-\cos\theta)\right]$$

$$= \frac{0.21}{c}\left[0.75(1-2\cos\theta+\cos^2\theta) - 1 + \cos\theta\right]$$

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$$\left(\frac{dz}{dx}\right)_\theta^{mcl} = \frac{0.21}{c}\left[-0.25 - 0.50\cos\theta + 0.75\cos^2\theta\right]$$

LET: $a = -0.25$, $b = -0.50$, $d = 0.75$

$$\left(\frac{dz}{dx}\right)_\theta^{mcl} = \frac{0.21}{c}\left[a + b\cos\theta + d\cos^2\theta\right]$$

$$\therefore \alpha_{l_0} = -\frac{1}{\pi}\int_0^\pi \frac{0.21}{c}\left[a + b\cos\theta + d\cos^2\theta\right](\cos\theta - 1)\,d\theta$$

$$\alpha_{l_0} = -\frac{0.21}{\pi c}\int_0^\pi \left(-a + (a-b)\cos\theta + (b-d)\cos^2\theta + d\cos^3\theta\right)d\theta$$

WHERE:

$$\int_0^\pi -a\,d\theta = -a\pi$$

$$\int_0^\pi (a-b)\cos\theta\,d\theta = (a-b)\sin\theta\Big|_0^\pi = 0$$

10

$$\int_0^\pi (b-d)\cos^2\theta\,d\theta = (b-d)\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]\Bigg|_0^\pi = (b-d)\frac{\pi}{2}$$

$$\int_0^\pi d\cos^3\theta\,d\theta = d\left[\sin\theta - \frac{1}{3}\sin^3\theta\right]_0^\pi = 0$$

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$$\alpha_{l_0} = -\frac{0.21}{\pi c}\left[-a\pi + (b-d)\frac{\pi}{2}\right]$$

$$= -\frac{0.21}{c}\left(-a + \tfrac{1}{2}b - \tfrac{1}{2}d\right)$$

$$= \frac{0.21}{c}\left(a - 0.5b + 0.5d\right)$$

$$= \frac{0.21}{c}\left(-0.25 - 0.5(-0.50) + 0.5(0.75)\right)$$

$$= \frac{0.21}{c}\left(-0.25 + 0.25 + 0.375\right)$$

$$= (0.21)(0.375)/c$$

$$= 0.07875/c$$

5 $$\underline{\alpha_{l_0} = 0.079/c}$$

5 (b) $$c_l = 2\pi\alpha + 2\int_0^\pi \left(\frac{dz}{dx}\right)_\theta^{mcl} (\cos\theta - 1)\,d\theta$$

$$c_l = 2\pi\alpha + 2\int_0^\pi \frac{0.21}{c}\left[a + b\cos\theta + d\cos^2\theta\right](\cos\theta - 1)\,d\theta$$

5 $$c_{m_{le}} = -\frac{\pi}{2}\left(A_0 + A_1 - \tfrac{1}{2}A_2\right)$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \left(\frac{dz}{dx}\right)_\theta^{mcl} d\theta$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \left[\frac{0.21}{c}\left(a + b\cos\theta + d\cos^2\theta\right)\right] d\theta$$

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$$A_1 = \frac{2}{\pi}\int_0^{\pi}\left(\frac{dz}{dx}\right)_{\theta}^{mcl}\cos\theta\, d\theta$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi}\frac{0.21}{c}\left[a + b\cos\theta + d\cos^2\theta\right]\cos\theta\, d\theta$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi}\left(\frac{dz}{dx}\right)_{\theta}^{mcl}\cos(2\theta)\, d\theta$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi}\frac{0.21}{c}\left[a + b\cos\theta + d\cos^2\theta\right]\cos(2\theta)\, d\theta$$

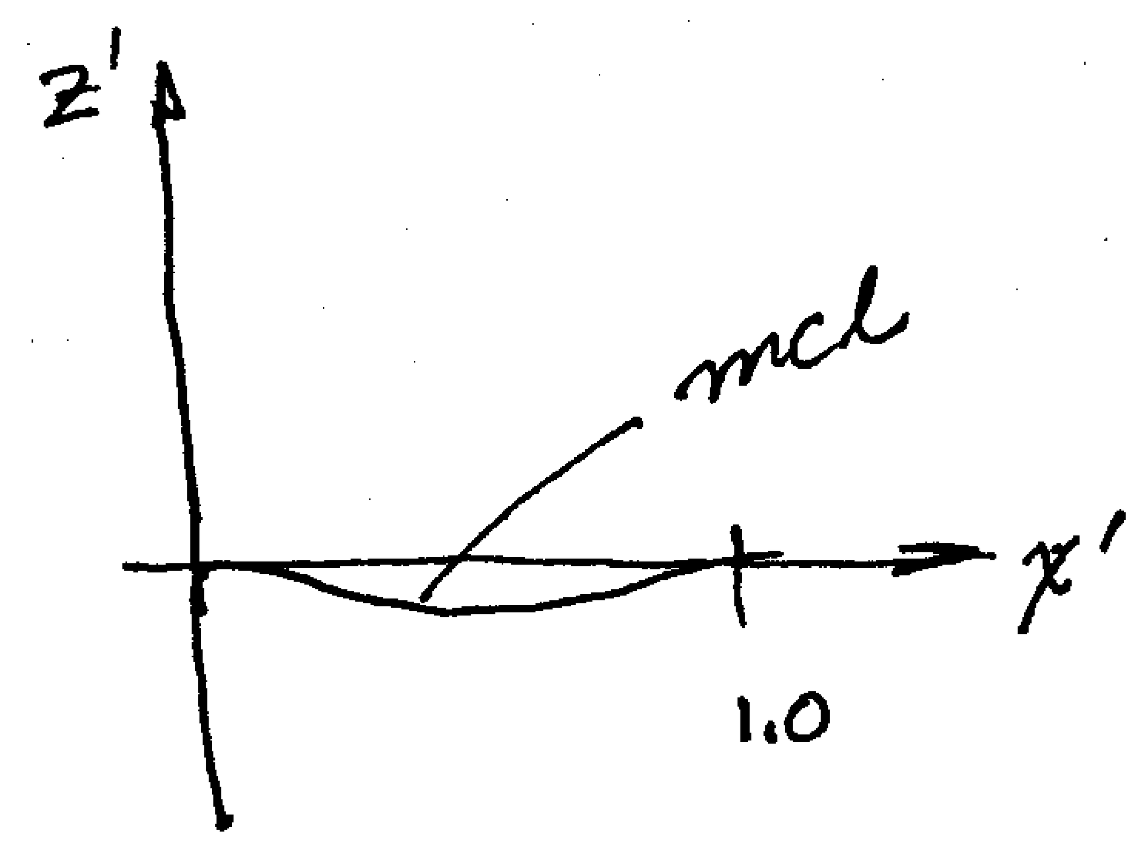
5

$$z = \frac{0.21\, x^2}{c^3}(x - c)$$

LET $x' = \frac{x}{c}$

$z' = z/0.21$

$$z' = x'^2(x' - 1)$$



NAME: SCHOOL SOLUTION

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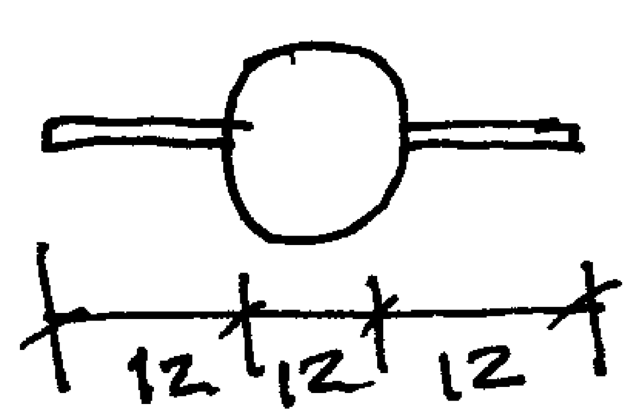
Problem III

(a) An undersea vessel has a hull diameter of 12 meters. Wings extending 12 meters from the hull on each side have a chord of 8 meters. What is the **aspect ratio** of the resulting wing? Support your answer with a valid physical argument.

(b) How does the magnitude of the velocity induced by a vortex vary with distance from the vortex?

10 (a) THE PORTION OF THE HULL LYING BETWEEN THE WINGS IS BY DEFINITION INCLUDED IN THE WING PLANFORM AREA. THIS REGION CONTRIBUTES TO THE OVERALL WING LIFT. [MULTHOPP DEVISED A TRANSFORM SIMILAR TO THE JOUKOWSKI TRANSFORM, WHICH SQUEEZES AN ELLIPTICAL FUSELAGE INTO A LINE ALONG THE MAJOR AXIS AND DISTRIBUTES ITS UPWASH EFFECTS ALONG A TRACE OF THE WING.]



$$AR = \frac{b^2}{S} = \frac{b^2}{b \times c} = \frac{b}{c} = \frac{36}{8}$$

$$AR = 4.5$$

10 (b) THE BIOT-SAVART LAW HOLDS

$$v_{induced} \sim \frac{1}{distance}$$

$$dv = \frac{\gamma(x)\,\delta x}{2\pi (x - x')}$$

NAME: SCHOOL SOLUTION 1

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Problem I

The NACA 23012 airfoil has a theoretical lift coefficient of 0.3, has a maximum camber at 15 percent of the chord, and has a maximum thickness of 0.12c where c is the chord. The equation for the mean camber line is:

$$z/c = 2.6595\ [\ (x/c)^3 - 0.6075\ (x/c)^2 + 0.11471\ (x/c)\ ]$$

for the region $0.0c \le x \le 0.2025c$ and

$$z/c = 0.022083\ [\ 1 - (x/c)\ ]$$

for the region $0.2025c \le x \le 1.0000c$.

For this airfoil, **calculate** the following:

(a) c_ℓ , (b) α_0 (angle of zero lift), and (c) $c_{m_{ac}}$

FIRST, FIND $\frac{dz}{dx}$:

$0.0c \le x \le 0.2025c$: $\frac{dz}{dx} = a\left[3\left(\frac{x}{c}\right)^2 - 2b\left(\frac{x}{c}\right) + d\right]$

$0.2025c \le x \le 1.000c$: $\frac{dz}{dx} = -e$

WHERE: $a = 2.6595$, $b = 0.6075$
$d = 0.11471$, $e = 0.022083$

COORDINATE TRANSFORMATION:

$$x = \frac{c}{2}(1 - \cos\theta)$$

HENCE:

x/c	θ
0	0
0.2025	53.5°
1.0	π

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Problem I

NAME: SHOOL SOLUTION 2

NOTE: $(x/c)^2 = \frac{1}{4}(1-2\cos\theta+\cos^2\theta)$

$x/c = \frac{1}{2}(1-\cos\theta)$

FROM K&C AND/OR CLASS LECTURE NOTES:

(a) $C_\ell = \pi(2A_0 + A_1)$

(b) $\alpha_0 = -\frac{1}{\pi}\int_0^\pi \left(\frac{dz}{dx}\right)_\theta (\cos\theta - 1)\, d\theta$

(c) $C_{mac} = \frac{\pi}{4}(A_2 - A_1)$

WHERE:

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \left(\frac{dz}{dx}\right)_\theta d\theta$$

$$A_1 = \frac{2}{\pi}\int_0^\pi \left(\frac{dz}{dx}\right)_\theta \cos\theta\, d\theta$$

$$A_2 = \frac{2}{\pi}\int_0^\pi \left(\frac{dz}{dx}\right)_\theta \cos(2\theta)\, d\theta$$

SUBSTITUTING TO OBTAIN:

$$A_0 = \alpha - \frac{1}{\pi}\left\{\int_0^{53.5^\circ} a\left[3\left(\tfrac{1}{4}\right)(1-2\cos\theta+\cos^2\theta) - 2b\left(\tfrac{1}{2}\right)(1-\cos\theta) + d\right]d\theta + \int_{53.5^\circ}^{180^\circ}(-e)\, d\theta\right\}$$

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Problem I

NAME: SCHOOL SOLUTION 3

$$A_1 = \frac{2}{\pi}\left\{ \int_0^{53.5^\circ} a\left[3(\tfrac{1}{4})(1-2\cos\theta+\cos^2\theta) - 2b(\tfrac{1}{2})(1-\cos\theta) + d\right]\cos\theta\, d\theta + \int_{53.5^\circ}^{180^\circ} (-e)\cos\theta\, d\theta \right\}$$

$$A_2 = \frac{2}{\pi}\left\{ \int_0^{53.5^\circ} a\left[3(\tfrac{1}{4})(1-2\cos\theta+\cos^2\theta) - 2b(\tfrac{1}{2})(1-\cos\theta) + d\right]\cos 2\theta\, d\theta + \int_{53.5^\circ}^{180^\circ} (-e)\cos 2\theta\, d\theta \right\}$$

OR:

$$A_0 = \alpha - \frac{1}{\pi}\left\{ \int_0^{53.5^\circ} (A + B\cos\theta + C\cos^2\theta)\, d\theta - \int_{53.5}^{180^\circ} e\, d\theta \right\}$$

$$A_1 = \frac{2}{\pi}\left\{ \int_0^{53.5^\circ} (A + B\cos\theta + C\cos^2\theta)\cos\theta\, d\theta - \int_{53.5}^{180^\circ} e\cos\theta\, d\theta \right\}$$

$$A_2 = \frac{2}{\pi}\left\{ \int_0^{53.5^\circ} (A + B\cos\theta + C\cos^2\theta)\cos 2\theta\, d\theta - \int_{53.5^\circ}^{180^\circ} e\cos 2\theta\, d\theta \right\}$$

ALSO NOTE:

$$\alpha_0 = \alpha - (A_0 + \tfrac{1}{2}A_1)$$

$$A = a\left(\tfrac{3}{4} - b + d\right) = 2.6595\left(\tfrac{3}{4} - 0.6075 + 0.11471\right) = 0.68$$

$$B = a\left(b - \tfrac{3}{2}\right) = 2.6595(0.6075 - 1.500) = -2.37$$

$$C = \tfrac{3}{4}a = \tfrac{3}{4}(2.6595) = 2.0$$

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Problem I

NAME: SCHOOL SOLUTION 4

WHERE:

$$\int_{\theta_1}^{\theta_2} d\theta = \theta_2 - \theta_1$$

$$\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta = \sin\theta \Big|_{\theta_1}^{\theta_2} = \sin\theta_2 - \sin\theta_1$$

$$\int_{\theta_1}^{\theta_2} \cos^2\theta \, d\theta = \frac{\theta}{2} + \frac{\sin(2\theta)}{4} \Big|_{\theta_1}^{\theta_2} = \frac{\theta_2}{2} + \frac{\sin(2\theta_2)}{4} - \frac{\theta_1}{2} - \frac{\sin(2\theta_1)}{4}$$

$$\int_{\theta_1}^{\theta_2} \cos^3\theta \, d\theta = \sin\theta - \frac{\sin^3\theta}{3} \Big|_{\theta_1}^{\theta_2} = \sin\theta_2 - \frac{\sin^3\theta_2}{3} - \sin\theta_1 + \frac{\sin^3\theta_1}{3}$$

$$\int_{\theta_1}^{\theta_2} \cos(2\theta) \, d\theta = \frac{1}{2}\sin(2\theta) \Big|_{\theta_1}^{\theta_2} = \frac{1}{2}\sin(2\theta_2) - \frac{1}{2}\sin(2\theta_1)$$

$$\int_{\theta_1}^{\theta_2} \cos\theta\cos(2\theta) \, d\theta = \int_{\theta_1}^{\theta_2} \cos\theta\,(2\cos^2\theta - 1)\, d\theta = \int_{\theta_1}^{\theta_2} [2\cos^3\theta - \cos\theta]\, d\theta$$

$$= 2\left\{\sin\theta_2 - \frac{\sin^3\theta_2}{3} - \sin\theta_1 + \frac{\sin^3\theta_1}{3}\right\} - \left\{\sin\theta_2 - \sin\theta_1\right\}$$

$$\int_{\theta_1}^{\theta_2} \cos^2\theta\cos(2\theta)\, d\theta = \int_{\theta_1}^{\theta_2} \cos^2\theta\,(2\cos^2\theta - 1)\, d\theta = \int_{\theta_1}^{\theta_2} [2\cos^4\theta - \cos^2\theta]\, d\theta$$

$$= 2\left\{\frac{3\theta}{8} + \frac{3\sin(2\theta)}{16} + \frac{\cos^3\theta\sin\theta}{4} \Big|_{\theta_1}^{\theta_2}\right\} - \left\{\frac{\theta}{2} + \frac{\sin(2\theta)}{4} \Big|_{\theta_1}^{\theta_2}\right\}$$

$$= 2\left\{\frac{3\theta_2}{8} + \frac{3\sin(2\theta_2)}{16} + \frac{\cos^3\theta_2\sin\theta_2}{4} - \frac{3\theta_1}{8} - \frac{3\sin(2\theta_1)}{16} - \frac{\cos^3\theta_2\sin\theta_1}{4}\right\}$$

$$- \left\{\frac{\theta_2}{2} + \frac{\sin(2\theta_2)}{4} - \frac{\theta_1}{2} - \frac{\sin(2\theta_1)}{4}\right\}$$

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Problem I

NAME: SCHOOL SOLUTION 5

HENCE:

$$\int_0^{53.5^\circ} d\theta = 53.5^\circ = \underline{\underline{0.93^r}}; \quad \int_{53.5^\circ}^{180^\circ} d\theta = 3.14 - 0.93 = \underline{\underline{2.21}};$$

$$\int_{\theta_1=0}^{\theta_2=53.5^\circ} \cos d\theta = \sin 53.5^\circ - \sin 0^\circ = \underline{\underline{0.80}}; \quad \int_{53.5^\circ}^{180^\circ} \cos d\theta = \sin 180^\circ - \sin 53.5 = \underline{-0.80};$$

$$\int_0^{53.5^\circ} \cos^2\theta d\theta = \frac{0.93}{2} + \frac{\sin(107^\circ)}{4} = 0.465 + 0.24 = \underline{0.705}; \quad \int_{53.5^\circ}^{180^\circ} \cos^2\theta d\theta = \frac{3.14}{2} - \frac{0.93}{2} - 0.24 = \underline{\underline{0.865}};$$

$$\int_0^{53.5^\circ} \cos^3\theta d\theta = 0.80 - \frac{(0.80)^3}{3} = \underline{\underline{0.63}}; \quad \int_{53.5^\circ}^{180^\circ} \cos^3\theta d\theta = -0.80 + \frac{(0.80)^3}{3} = \underline{\underline{-0.63}};$$

$$\int_0^{53.5^\circ} \cos(2\theta) d\theta = \frac{1}{2}(0.956) = \underline{\underline{0.48}}; \quad \int_{53.5^\circ}^{180^\circ} \cos(2\theta) d\theta = -\frac{1}{2}(0.956) = \underline{\underline{-0.48}};$$

$$\int_0^{53.5^\circ} \cos\theta \cos(2\theta) d\theta = 2\left\{0.80 - \frac{(0.80)^3}{3}\right\} - \left\{0.80\right\}$$
$$= 1.26 - 0.80 = \underline{\underline{0.40}};$$

$$\int_{53.5^\circ}^{180^\circ} \cos\theta \cos(2\theta) d\theta = -0.80 + \frac{(0.80)^3}{3} + 0.80 = \underline{\underline{0.17}};$$

$$\int_0^{53.5^\circ} \cos^2\theta \cos(2\theta) d\theta = 2\left\{\frac{0.93}{8} \cdot 3 + \frac{3}{16}(0.956) + \frac{(0.211)(0.80)}{4}\right\}$$
$$- \left\{\frac{0.93}{2} + \frac{1}{4}(0.956)\right\} = 0.\underline{\underline{43}};$$

$$\int_{53.5^\circ}^{180^\circ} \cos^2\theta \cos(2\theta) d\theta = 2\left\{\frac{3\times 3.14}{8} - 3\left(\frac{0.93}{8}\right) - \frac{3}{16}(0.956) - \frac{(0.211)(0.80)}{4}\right\} - \left\{\frac{3.14}{2} - \frac{0.93}{2} - \frac{0.956}{4}\right\}$$

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$$\int_{53.5^\circ}^{180^\circ} \cos^2\theta \cos(2\theta)\,d\theta = \underline{0.35}$$

THEREFORE

$$A_0 = \alpha - \frac{1}{\pi}\left\{ (0.68)(0.93) - (2.37)(0.80) + (2.0)(0.705) - (0.022)(2.21) \right\} = \underline{\alpha - 0.031}$$

$$A_1 = \frac{2}{\pi}\left\{ (0.68)(0.80) - (2.37)(0.705) + (2.0)(0.63) + (0.022)(0.80) \right\} = \underline{0.095}$$

$$A_2 = \frac{2}{\pi}\left\{ (0.68)(0.48) - (2.37)(0.40) + (2.0)(0.43) - (0.022)(-0.48) \right\} \quad \underline{0.158}$$

SUBSTITUTING

(a) $C_\ell = \pi\left(2(\alpha - 0.031) + 0.095\right) = \underline{\pi(2\alpha + 0.033)}$

(b) $\alpha_0 = \alpha - \left(\alpha - 0.031 + \frac{1}{2}(0.095)\right) = \underline{-0.017}$

(c) $C_{m_{ac}} = \frac{\pi}{4}(0.158 - 0.095) = \underline{0.049}$

NAME: SCHOOL SOLUTION

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Problem II

Consider an airplane that weighs 14,700 N and cruises in level flight at 300 km/hr at an altitude of 3000m. The wing has a surface area of 17.0 sq. meters and an aspect ratio of 6.2. Assume that the lift coefficient is a linear function of the angle of attack and that $\alpha_o = -1.2°$. If the load distribution is elliptic, **calculate (a)** the value of the circulation in the plane of symmetry (Γ_o), (b) the downwash velocity (w_i), (c) the induced drag coefficient (C_{D_i}), (d) the geometric angle of attack (α_{geo}), and (e) the effective angle of attack (α_a). **Draw** a diagram of the wing in the above situation. **Draw** a diagram of a section of the wing in the above situation.

KUETHE & CHOW, TABLE 3:

AT $h = 3km = 3000m$

$T = -4.5°C$; $a = 329 \frac{m}{sec}$; $p = 7.01 \times 10^4 \frac{N}{m^2}$

$\rho = 0.909 \frac{kg}{m^3}$, $\mu = 1.684 \frac{kg}{m\text{-}sec}$

FIRST, CALCULATE THE SPAN, b:

$$b = \sqrt{AR\, S} = \sqrt{(6.2)(17)} = \underline{\underline{10.27\ m}}$$

CALCULATE C_L:

$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 S} = \frac{14{,}700 \frac{kg\text{-}m}{sec^2}}{\frac{1}{2}(0.909)\frac{kg}{m^3}\left(\frac{300 \times 10^3}{3600}\right)^2 \frac{m^2}{sec^2}(17)\,m^2}$$

$$\underline{\underline{C_L = 0.274}}$$

(a) CIRCULATION Γ_0 (ELLIPTIC LOAD):

$$\Gamma_o = \frac{2U_\infty S}{\pi b} C_L = \frac{2\left(\frac{300 \times 10^3}{3600}\right)\frac{m}{sec} \cdot (17)\,m^2}{(3.14)(10.27)\,m}(0.274)$$

$$\Gamma_o = 24.1 \frac{m^2}{sec}$$

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Problem II

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(b) DOWNWASH VELOCITY w_i:

$$w_i = \frac{\Gamma_0}{2b} = \frac{24.1\ \frac{m^2}{sec}}{(2)(10.27)\ m} = \underline{\underline{1.17\ \frac{m}{sec}}}$$

(c) INDUCED DRAG COEFFICIENT, C_{D_i}:

$$C_{D_i} = \frac{C_L^2}{\pi\, AR} = \frac{(0.274)^2}{(3.14)(6.2)} = \underline{\underline{3.85 \times 10^{-3}}}$$

(d) GEOMETRIC ANGLE OF ATTACK, α_{geo}:

$$\alpha_{geo} = \left(1 + \frac{2}{AR}\right)\frac{1}{2\pi}C_L = \left(1 + \frac{2}{6.2}\right)\left(\frac{1}{(2)(3.14)}\right)(0.274)$$

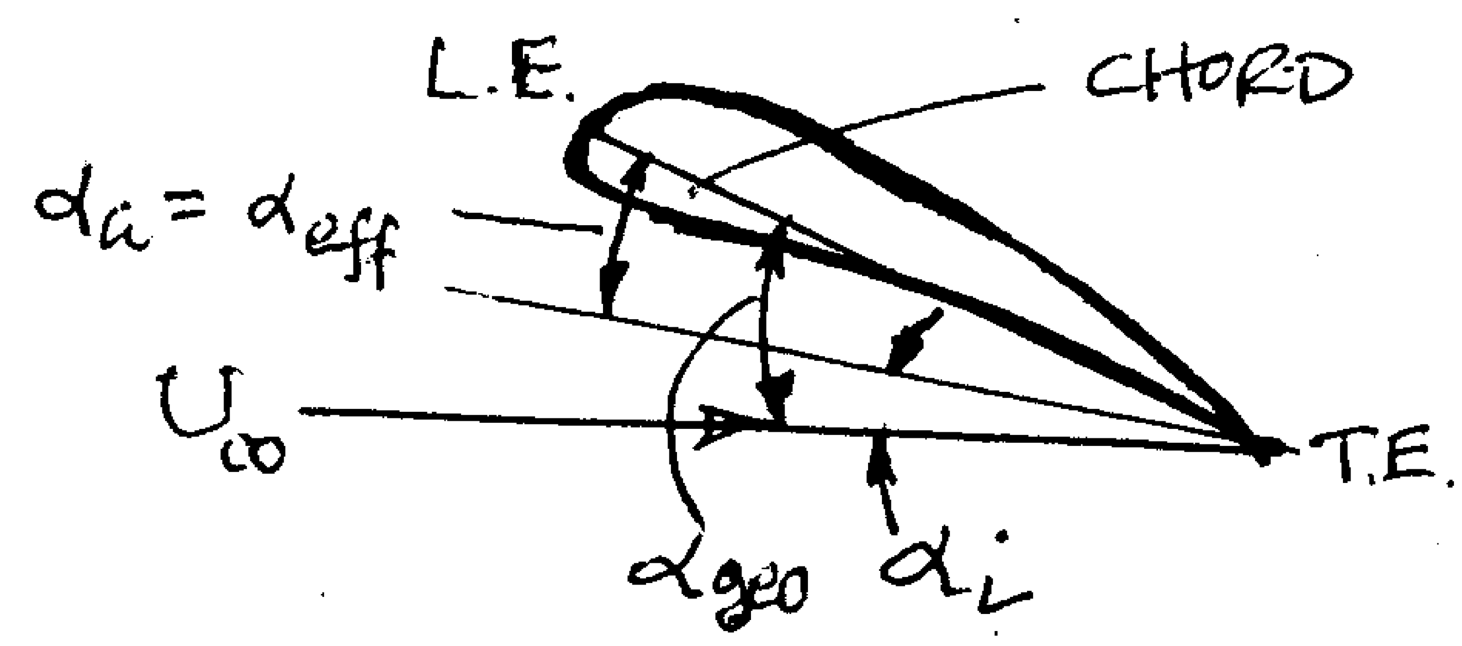
$$\alpha_{geo} = 0.058^r = \underline{\underline{3.32^\circ}}$$

(e) EFFECTIVE ANGLE OF ATTACK, α_a;

$$\alpha_a \equiv \alpha_{eff} = \alpha_{geo} - \alpha_i = \alpha_{geo} - \frac{w_i}{U_\infty}$$

$$= 0.058 - \frac{1.17\ \frac{m}{sec.}}{\frac{300 \times 10^3}{3600}\ \frac{m}{sec.}} = 0.044^r = \underline{\underline{2.52^\circ}}$$

WING SECTION:



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Problem III

Using complex variables, **find** (a) the stream function and (b) the velocity potential when the <u>*complex potential*</u> is

$w(z) = A (z + b^2/z)$

What must A be? **What** is b ?

$$w(z) = A\left(z + \frac{b^2}{z}\right) = A\left(re^{i\theta} + \frac{b^2}{r}e^{-i\theta}\right)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\therefore w(z) = A\left(r\cos\theta + \frac{b^2}{r}\cos\theta\right) + iA\left(r\sin\theta - \frac{b^2}{r}\sin\theta\right)$$

$$= \phi + i\psi$$

(a) $\therefore \underline{\phi = A\left(r\cos\theta + \frac{b^2}{r}\cos\theta\right) = A\cos\theta\left(r + \frac{b^2}{r}\right)}$

(b) $\underline{\psi = A\left(r\sin\theta - \frac{b^2}{r}\sin\theta\right) = A\sin\theta\left(r - \frac{b^2}{r}\right)}$

NOTE:

$$\phi = \phi_1 + \phi_2$$

$\phi_1 = Ar\cos\theta \longrightarrow$ UNIFORM STREAM

$\phi_2 = A\frac{b^2}{r}\cos\theta \longrightarrow$ DOUBLET FLOW

WHERE:

$$u_r = \frac{\partial\phi}{\partial r} = A\cos\theta\left(1 - \frac{b^2}{r^2}\right)$$

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -A\sin\theta\left(1 + \frac{b^2}{r^2}\right)$$

THIS RESULT MATCHES FLOW OVER A CIRCULAR CYLINDER OF RADIUS $\underline{r = b}$ AND UNIFORM STREAM $\underline{V_\infty = A}$.

NAME: SCHOOL SOLUTION

UNIFIED ENGR QUIZ Q2F
Spring 1997
03/06/97

Problem I

A Cessna 172 is cruising at 10,00 ft. on a standard day (ρ = 0.001756 slugs/ ft^3.) at 130 mi/hr. If the airplane weighs 2300 lb., **what C_L** is required to maintain level flight? A Cessna 172 has a wing span of 36 ft and an aspect ratio of 7.3.

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 A}$$, L = LIFT = 2,300 lbs. , A = WING PLANFORM AREA

$$A = \frac{b^2}{AR}$$; b = WING SPAN ; AR = ASPECT RATIO

$$\therefore \; C_L = \frac{2 AR\, L}{\rho U_\infty^2 b^2}$$

$$C_L = \frac{(2)(7.3)(2300)}{(0.0018)(130)^2\left(\frac{1}{0.6818}\right)^2(36)^2}$$

$$\checkmark \quad \underline{C_L = 0.396}$$

NAME: SCHOOL SOLUTION

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Problem II

The mean camber line of a thin airfoil is given by:

$$y/c = -k\{a + b(x/c) + d(x/c)^2\}$$

where

c = chord and k, a, b, and d are assumed known real constants. **Find** the following:

(a) α_{L_0}, (b) C_{m_0}, and (c) C_L.

$$\alpha_{L_0} = -\frac{1}{\pi}\int_0^{\pi}\left(\frac{dy}{dx}\right)_\theta (1-\cos\theta)\,d\theta$$

$$C_{m_0} = \frac{\pi}{4}(A_2 - A_1) \quad ; \quad C_L = \pi(2A_0 + A_1)$$

$$A_0 = \alpha + \frac{1}{\pi}\int_0^{\pi}\left(\frac{dy}{dx}\right)_\theta d\theta \;;\; A_1 = -\frac{2}{\pi}\int_0^{\pi}\left(\frac{dy}{dx}\right)_\theta \cos\theta\,d\theta \;;\; A_2 = -\frac{2}{\pi}\int_0^{\pi}\left(\frac{dy}{dx}\right)_\theta \cos(2\theta)\,d\theta$$

$$y/c = -k\left[a + b\left(\frac{x}{c}\right) + d\left(\frac{x}{c}\right)^2\right] \;;\; x/c = -\sin^2\frac{\theta}{2}$$

$$\frac{d(y/c)}{d(x/c)} = -k\left[b + 2d\left(\frac{x}{c}\right)\right] \;;\; \left(\frac{d(y/c)}{d(x/c)}\right)_\theta = -k\left[b - 2d\sin^2\frac{\theta}{2}\right]$$

$$\therefore \int_0^{\pi}\left(\frac{dy}{dx}\right)_\theta d\theta = \int_0^{\pi}\left(\frac{d(y/c)}{d(x/c)}\right)_\theta d\theta = \int_0^{\pi} -k\left(b - 2d\sin^2\frac{\theta}{2}\right)d\theta = \pi k(d-b)$$

$$\int_0^{\pi}\left(\frac{dy}{dx}\right)_\theta \cos\theta\,d\theta = \int_0^{\pi} -k\left(b - 2d\sin^2\frac{\theta}{2}\right)\cos\theta\,d\theta = -\pi k d/2$$

$$\int_0^{\pi}\left(\frac{dy}{dx}\right)_\theta \cos 2\theta\,d\theta = \int_0^{\pi} -k\left(b - 2d\sin^2\frac{\theta}{2}\right)\cos 2\theta\,d\theta = 0$$

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NAME: SCHOOL SOLUTION

Probem II

SUBSTITUTING:

$$A_0 = \alpha + \frac{1}{\pi}\pi k(d-b) = \alpha + k(d-b)$$

$$A_1 = -\frac{2}{\pi}\cdot\left(-\pi k d/2\right) = kd$$

$$A_2 = 0$$

(a) $$\alpha_{L_0} = -\frac{1}{\pi}\left[\int_0^\pi \left(\frac{dy}{dx}\right)_\theta d\theta - \int_0^\pi \left(\frac{dy}{dx}\right)_\theta \cos\theta\, d\theta\right]$$

$$\alpha_{L_0} = -\frac{1}{\pi}\left[\pi k(d-b) + \pi k d/2\right]$$

✓ $$\underline{\alpha_{L_0} = -3kd/2 + kb}$$

(b) ✓ $$\underline{C_{m_0} = \frac{\pi}{4}(A_2 - A_1) = -\frac{\pi}{4}A_1 = -\frac{\pi k d}{4}}$$

(c) $$C_L = \pi(2A_0 + A_1) = \pi\left[2(\alpha + k(d-b)) + kd\right]$$

✓ $$\underline{C_L = 2\pi\alpha + (3d-2b)\pi k}$$

NAME: SCHOOL SOLUTION

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Problem III

Using complex variables, **show** that when the complex potential is

w(z) = A(z + 1/z) - iBln(z),

where B is a real constant, the flow consists of a uniform stream past the circle r = 1, combined with circulation about the circle. **What** is A? **What** is B?

$$w(z) = A\left(z + \frac{1}{z}\right) - iB\ln z \quad ; \quad z = re^{i\theta}$$

$$\therefore \; w(z) = A\left(re^{i\theta} + \frac{e^{-i\theta}}{r}\right) - iB\ln(re^{i\theta}) \; ; \quad e^{i\theta} = \cos\theta + i\sin\theta$$

$$w(z) = A\left(r\cos\theta + \frac{1}{r}\cos\theta\right) + B\theta$$
$$+ iA\left(r\sin\theta - \frac{1}{r}\sin\theta\right) - iB\ln r = \phi + i\psi$$

$$\therefore \; \phi = A\left(r\cos\theta + \frac{1}{r}\cos\theta\right) + B\theta$$

$$\psi = A\left(r\sin\theta - \frac{1}{r}\sin\theta\right) - B\ln r$$

OR $$\phi = \phi_1 + \phi_2 + \phi_3$$

✓ $\phi_1 = Ar\cos\theta \rightarrow$ UNIFORM STREAM

✓ $\phi_2 = A\frac{1}{r}\cos\theta \rightarrow$ DOUBLET

✓ $\phi_3 = B\theta \longrightarrow$ PLANE VORTEX ABOUT THE ORIGIN

WHERE $u_r = \frac{\partial\phi}{\partial r} = A\cos\theta\left(1 - \frac{1}{r^2}\right)$

$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -A\sin\theta\left(1 + \frac{1}{r^2}\right) + \frac{B}{r}$

✓ THIS RESULT MATCHES FLOW OVER A CIRCULAR CYLINDER OF RADIUS $r = 1$, UNIFORM STREAM $A = U_\infty$, AND VORTEX OF STRENGTH $\Gamma = 2\pi B$.

UNIFIED ENGINEERING

QUIZ

Prof. Harris, 33-406, ext. 3-0911, Weslhar@mit.edu Spring 1996

Q2E.1

For a particular airfoil section, the pitching moment coefficient about a point 1/3 chord behind the leading edge varies with lift coefficient in the following manner:

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C_L	0.2	0.4	0.6	0.8
C_M	-0.02	0.000	+0.02	+0.04

Find the aerodynamic center. **Show** the location of the aerodynamic center on a diagram of the airfoil section. **Find** C_{M_0}.

AERODYNAMIC CENTER

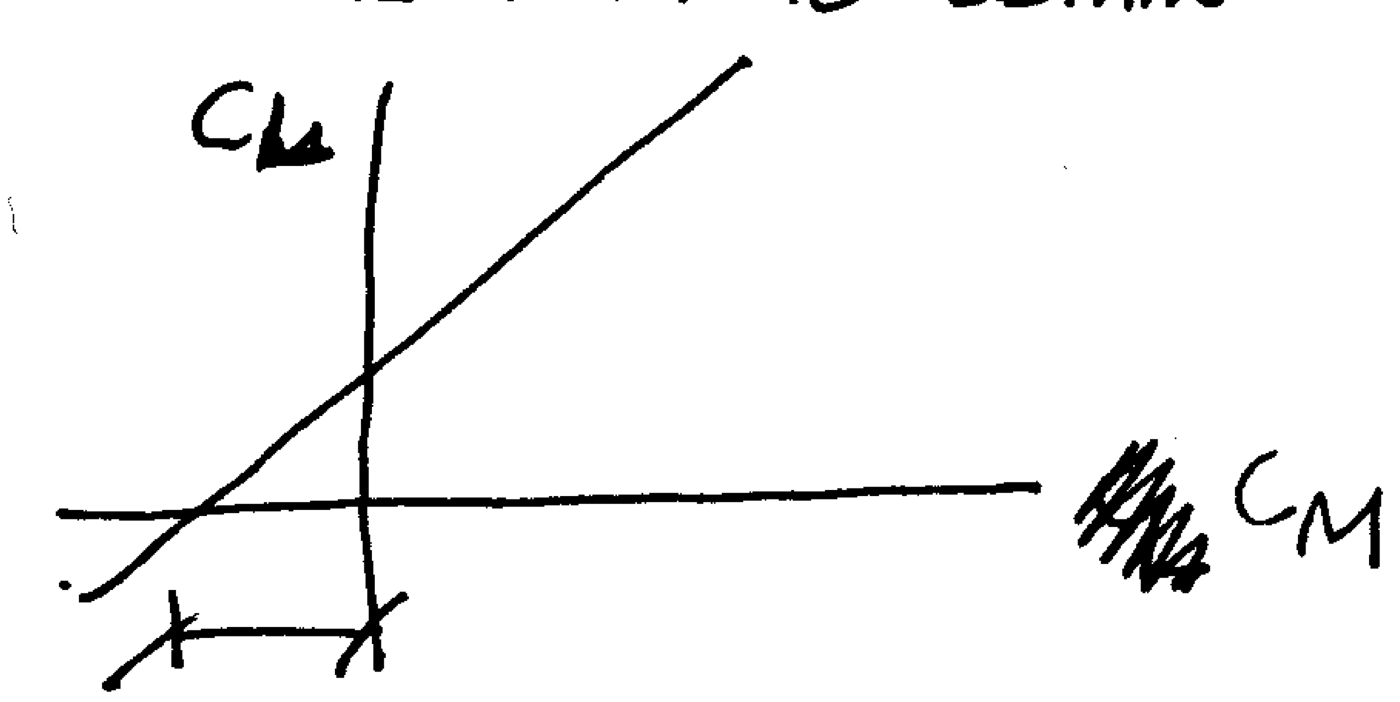
NOTE THAT C_M VARIES LINEARLY WITH C_L. HENCE,

$$\frac{dC_M}{dC_L} = \frac{(0.04)-(-0.02)}{(0.80)-(0.20)} = \frac{0.06}{0.60} = 0.10$$

$$\therefore \frac{x_{ac}}{c} = \frac{1}{3} - \frac{dC_M}{dC_L} = \frac{1}{3} - 0.10 = 0.233$$

C_{M_0}

PLOT THE ABOVE DATA TO OBTAIN



-0.04 $\qquad \therefore C_{M_0} = -0.04$

UNIFIED ENGINEERING

QUIZ

Prof. Harris, 33-406, ext. 3-0911, Weslhar@mit.edu **Spring 1996**

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Q2F.4

An airplane with a wing planform area of 650 ft^2 and a span of 80 ft is flying at an altitude of 35,000 ft at a speed of 500 mph. The ambient temperature is $409° R$. The density is $7.38 \times 10^{-4} \frac{\text{slugs}}{\text{fts}}$. The gross weight is 52,000 lb. **Determine:** C_L, C_{D_i}, α_{geo}. **Assume** an ideal elliptical lift distribution.

ASSUME LEVEL, UN-ACCELERATED FLIGHT

$$L = W = 52{,}000 \text{ lbs.}$$

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{52{,}000}{(\frac{1}{2})(0.000738)(500\times\frac{88}{60})^2(650)} = \frac{104{,}000}{7.38\times10^{-4}\times 5.378\times10^{5}\times 6.50\times10^{2}}$$

$$C_L = \frac{104{,}000}{257.98\times10^{3}} = \frac{10.4\times10^{4}}{25.8\times10^{4}} = \frac{10.4}{25.8} = \underline{0.403}$$

$$C_{D_i} = \frac{C_L^2}{\pi AR\, E} = \frac{C_L^2}{\pi AR} = \frac{(0.403)^2}{(3.14)(80^2/650)} = \frac{(0.403)^2(650)}{(3.14)(80)^2}$$

$$C_{D_i} = \frac{1.62\times10^{-1}\times 6.50\times10^{2}}{3.14\times 6.4\times10^{3}} = \frac{10.53}{20.096}\times10^{-2} = 0.524\times10^{-2} = \underline{0.00524}$$

$$\alpha_{geo} = \frac{1}{2\pi}\left(1+\frac{2}{AR}\right)C_L = \frac{\left[1+\frac{2}{(80)^2/650}\right]}{(2)(3.14)}(0.403)$$

$$\alpha_{geo} = \frac{\left[1+\frac{(2)(650)}{(80)^2}\right](0.403)}{(2)(3.14)} = \frac{(1.203)(0.403)}{(6.28)} = \frac{0.485}{6.28}$$

$$\alpha_{geo} = \underline{0.077}^{\text{ rad}} = 0.077\left(\frac{180}{3.14}\right)^{\circ} = \underline{4.41^{\circ}}$$