UNIFIED ENGINEERING

Spring Semester 2003

Fluid Dynamics

Previous Quiz Problems Part 1
(Spring '02 quiz with solutions)

UNIFIED ENGINEERING
FLUID DYNAMICS
QUIZ 2F
March 7, 2002
Problem I

Consider a flapped airfoil as shown. The slope of the mean camber line is :

$$\left(\frac{dy}{dx}\right)^{mcl} = 0 \quad \text{for} \quad 0 < x < kc$$

$$\left(\frac{dy}{dx}\right)^{mcl} = -\delta \quad \text{for} \quad kc < x < c$$

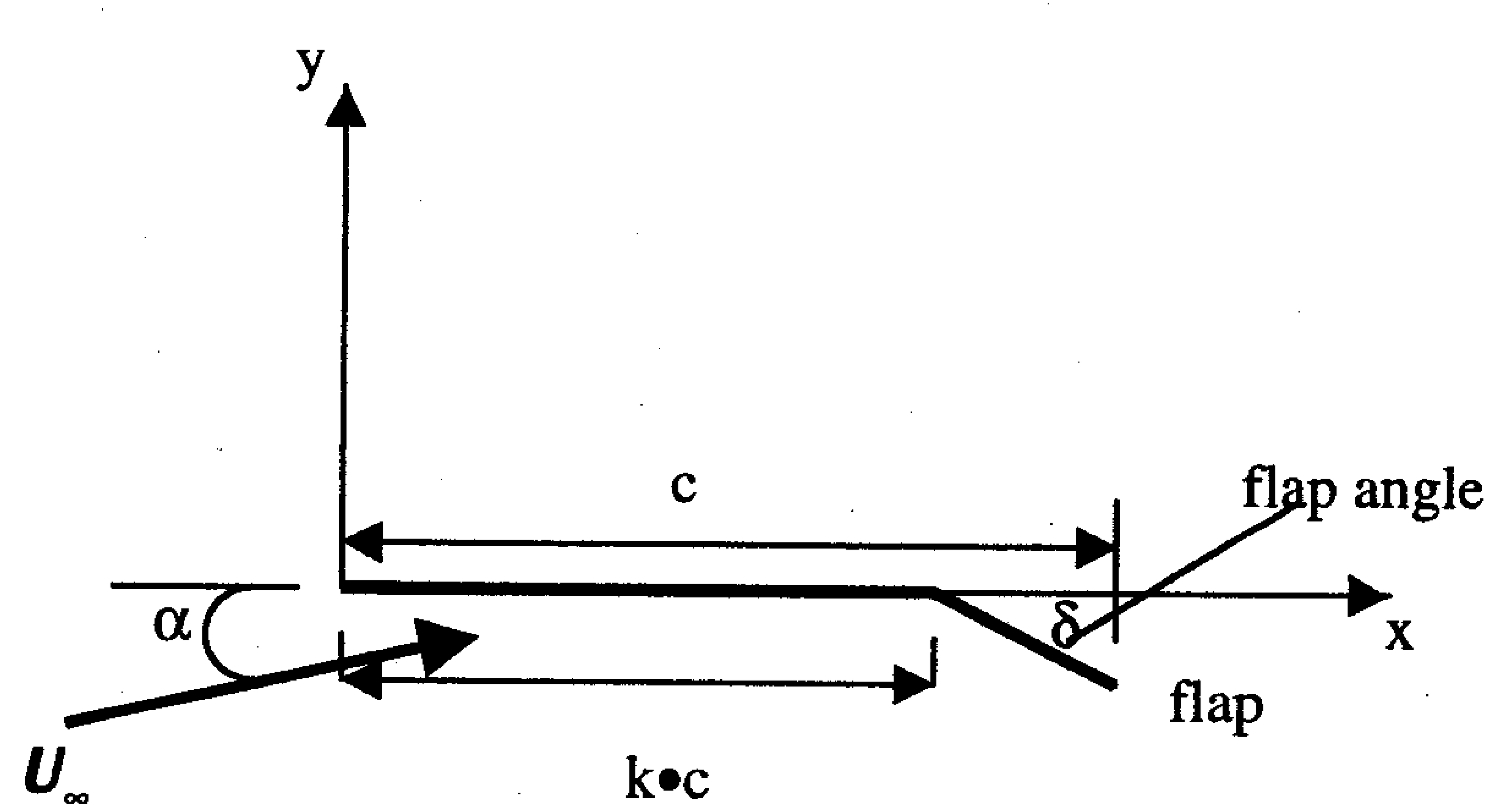
Assume angle of attack α and flap angle δ to be small and therefore consistent with the approximations of the Thin Airfoil Theory (TAT).

(a) Calculate the lift coefficient, c_l.

(b) Calculate the moment coefficient about the leading edge of the airfoil, c_{m_0}.

(c) Calculate the incremental effect of the flap, i.e., calculate the change in the lift coefficient and the change in the moment coefficient of this airfoil due to the flap.

SHOW DETAILS OF YOUR WORK. STATE ALL ASSUMPTIONS CLEARLY.



ASSUME $\alpha \ll 1$ AND $\delta \ll 1$
DISTRIBUTE $\delta(x)$ ALONG THE CHORD (X-AXIS)
USE THE TRANSFORMATION

$$x = \frac{c}{2}(1-\cos\theta)$$

ASSUME

$$\gamma(\theta) = 2U_\infty\left[A_0 \cot\frac{\theta}{2} + \sum_1^\infty A_n \sin(n\theta)\right]$$

SUBSTITUTE INTO MASTER EQUATION:

$$C_l = 2\pi\alpha + 2\int_0^\pi \left(\frac{dy}{dx}\right)_\theta^{mcl}(\cos\theta - 1)d\theta$$

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$$C_l = 2\pi(\alpha - \alpha_0)$$

$$\alpha_0 \equiv \text{ANGLE OF ZERO LIFT}$$

$$\alpha_0 = -\frac{1}{\pi}\int_0^\pi \left(\frac{dy}{dx}\right)_\theta^{mcl}(\cos\theta - 1)d\theta$$

THE ZERO-LIFT ANGLE IS MOST INFLUENCED BY THE TRAILING-EDGE REGION WHERE $\theta \to \pi$.

LET'S LOCATE THE HINGE POINT, θ_K.

$$x_K = Kc = \frac{c}{2}(1-\cos\theta_K)$$

$$\therefore \quad \cos\theta_K = 1 - 2K.$$

NOTE:

$$\left(\frac{dy}{dx}\right)_\theta^{mcl} = 0; \quad 0 < \theta < \theta_K$$

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$$\therefore A_0 = \alpha - \frac{1}{\pi}\int_{\theta_K}^\pi \left(\frac{dy}{dx}\right)_\theta^{mcl} d\theta$$

$$A_1 = \frac{2}{\pi}\int_{\theta_K}^\pi \left(\frac{dy}{dx}\right)_\theta^{mcl}\cos\theta\, d\theta$$

$$A_n = \frac{2}{\pi}\int_{\theta_K}^\pi \left(\frac{dy}{dx}\right)_\theta^{mcl}\cos(n\theta)d\theta$$

SUBSTITUTING

$$A_0 = \alpha - \frac{1}{\pi}\int_{\theta_K}^{\pi}(-\delta)\,d\theta = \alpha + \frac{\delta}{\pi}\theta\Big|_{\theta_K}^{\pi}$$

$$A_0 = \alpha + \frac{\delta}{\pi}\left[\pi - \theta_K\right]$$

$$A_1 = \frac{2}{\pi}\int_{\theta_K}^{\pi}(-\delta)\cos\theta\,d\theta = -\frac{2}{\pi}\delta\sin\theta\Big|_{\theta_K}^{\pi}$$

(10)

$$A_1 = +\frac{2}{\pi}\delta\sin\theta_K$$

$$A_n = \frac{2}{\pi}\int_{\theta_K}^{\pi}(-\delta)\cos(n\theta)\,d\theta$$

$$A_n = -\frac{2}{\pi}\delta\frac{1}{n}\sin(n\theta)\Big|_{\theta_K}^{\pi}$$

$$A_n = +\frac{2}{\pi}\frac{\delta}{n}\sin(n\theta_K)$$

FOR A GENERAL $(dy/dx)_{\theta}^{mcl}$ WE HAVE CALCULATED

$$C_l = \pi(2A_0 + A_1)$$

$$C_{m_0} = \frac{\pi}{4}(A_2 - A_1) \; ; \; C_{m_{LE}} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)$$

SUBSTITUTING

(8)

$$C_l = 2\pi\left\{\left[\alpha + \frac{\delta}{\pi}(\pi - \theta_K)\right] + \frac{1}{\pi}\delta\sin\theta_K\right\}$$

(a) $$\boxed{C_l = 2\pi\left\{\alpha + \frac{\delta}{\pi}\left[\pi - \theta_K + \sin\theta_K\right]\right\}}$$

$$C_{m_0} = \frac{\pi}{4}(A_2 - A_1)$$

SUBSTITUTING:

$$C_{m_0} = \frac{\pi}{4}\left\{ \frac{2}{\pi}\frac{\delta}{2}\sin(2\theta_k) - \frac{2}{\pi}\delta\sin(\theta_k)\right\}$$

$$C_{m_0} = \frac{1}{2}\left[\frac{\delta}{2}\sin(2\theta_k) - \delta\sin\theta_k\right]$$

(b) $$\boxed{C_{m_0} = \frac{\delta}{2}\left[\frac{1}{2}\sin(2\theta_k) - \sin\theta_k\right]}$$

$$C_{m_{LE}} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)$$

SUBSTITUTING:

$$C_{m_{LE}} = -\frac{\pi}{2}\left[\alpha + \frac{\delta}{\pi}(\pi - \theta_k) + \frac{2}{\pi}\delta\sin\theta_k - \frac{1}{2}\frac{2}{\pi}\frac{\delta}{2}\sin(2\theta_k)\right]$$

(8)

$$C_{m_{LE}} = -\frac{1}{2}\left[\pi\alpha + \delta\left(\pi - \theta_k + 2\sin\theta_k - \frac{1}{2}\sin(2\theta_k)\right)\right]$$

(b') $$\boxed{C_{m_{LE}} = -\frac{1}{2}\left\{\pi\alpha + \delta\left[\pi - \theta_k + 2\sin\theta_k - \frac{1}{2}\sin(2\theta_k)\right]\right\}}$$

BY SETTING $\alpha = 0$, THE INCREMENTAL EFFECT OF THE FLAP IS OBTAINED:

(9) (c)

$$\Delta C_\ell = 2\delta(\pi - \theta_k + \sin\theta_k)$$

$$\Delta C_{m_{LE}} = -\frac{\delta}{2}\left[\pi - \theta_k + 2\sin\theta_k - \frac{1}{2}\sin(2\theta_k)\right]$$

C_{m_0} IS INDEPENDENT OF α

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Problem II

Consider the case where the spanwise circulation distribution for a wing is parabolic,

$$\Gamma(y) = \Gamma_0\left(1 - \left(\frac{y}{b/2}\right)^2\right)$$

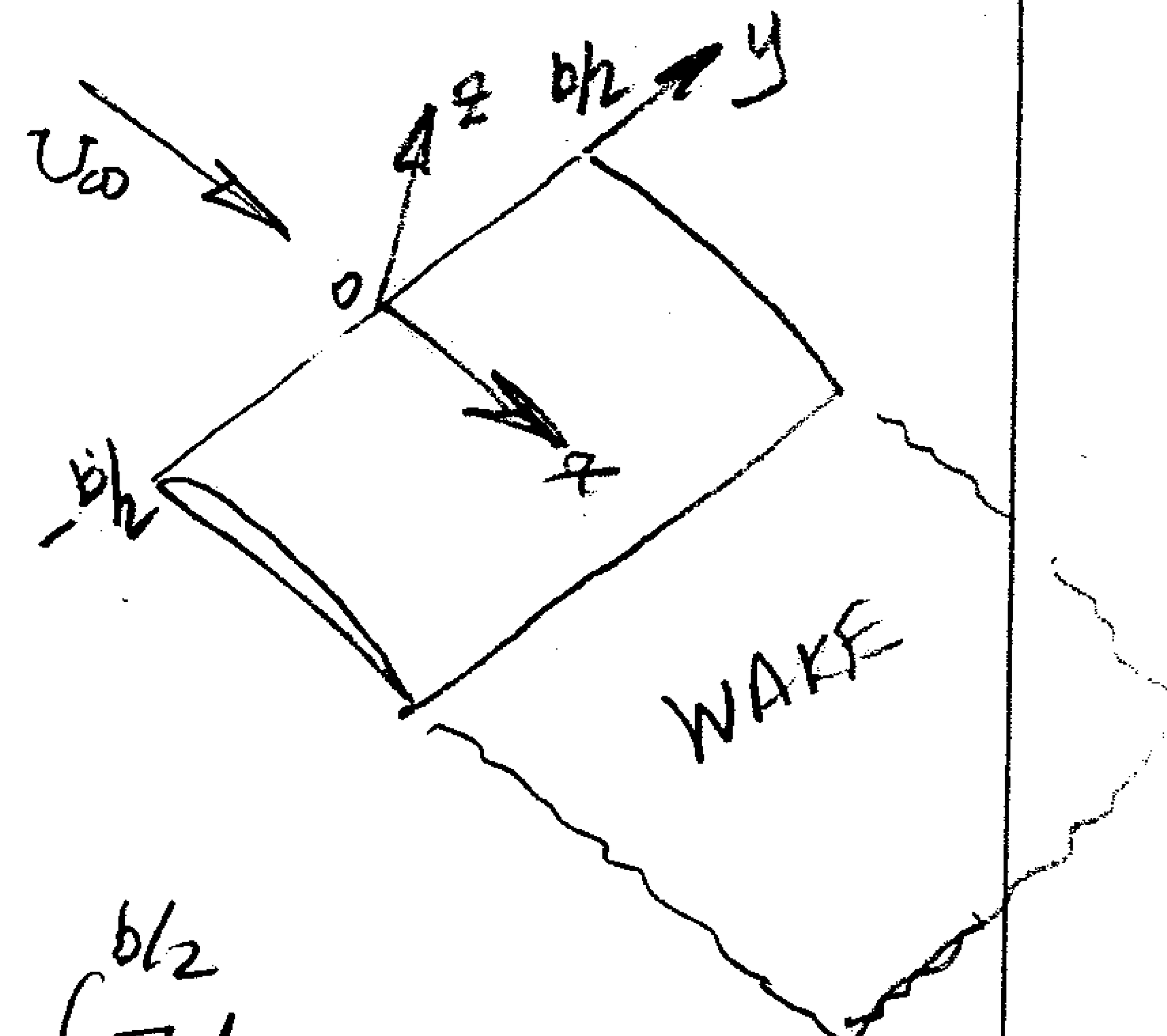
If the total lift generated by wing with the parabolic circulation distribution is to be equal to the total lift generated by a wing with an elliptic circulation distribution, **(a)** what is the relation between the Γ_0 values for the two distributions? **(b)** What is the relation between the induced downwash velocities at the plane of symmetry for the two configurations?

The elliptic spanwise circulation distribution is:

$$\Gamma(y) = \Gamma_0\left(1 - \left(\frac{y}{b/2}\right)^2\right)^{1/2}$$

SHOW DETAILS OF YOUR WORK. STATE ALL ASSUMPTIONS CLEARLY.

$$\Gamma_P(y) = \Gamma_{0_P}\left(1-\left(\frac{y}{b/2}\right)^2\right)$$



$$\Gamma_E(y) = \Gamma_{0_E}\left(1-\left(\frac{y}{b/2}\right)^2\right)^{1/2}$$

LET: $s = b/2$

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$$\text{LIFT} = \rho U_\infty \Gamma = \rho U_\infty \int_{-b/2}^{b/2} \Gamma\, dy$$

$$= \rho U_\infty \int_{-s}^{s} \Gamma\, dy$$

$$L_P = L_E$$

$$\rho U_\infty \int_{-s}^{s} \Gamma_{0_P}\left(1-\left(\frac{y}{s}\right)^2\right) dy = \rho U_\infty \int_{-s}^{s} \Gamma_{0_E}\left(1-\left(\frac{y}{s}\right)^2\right)^{1/2} dy$$

10

$$\frac{\Gamma_{0_P}}{\Gamma_{0_E}} = \frac{\int_{-s}^{s}\left(1-\left(\frac{y}{s}\right)^2\right)^{1/2} dy}{\int_{-s}^{s}\left(1-\left(\frac{y}{s}\right)^2\right) dy}$$

$$\frac{\Gamma_{0_E}}{\Gamma_{0_P}} = \frac{\frac{1}{s^2}\int_{-s}^{s}(s^2-y^2)\,dy}{\int_{-s}^{s}\left(\frac{s^2-y^2}{s^2}\right)^{1/2} dy}$$

$$= \frac{\frac{1}{s^2}\int_{-s}^{s}(s^2-y^2)\,dy}{\frac{1}{s}\int_{-s}^{s}\sqrt{s^2-y^2}\,dy}$$

3

$$\frac{\Gamma_{0_E}}{\Gamma_{0_P}} = \frac{\frac{2}{s^2}\int_0^s (s^2-y^2)\,dy}{\frac{2}{s}\int_0^s \sqrt{s^2-y^2}\,dy}$$

$$= \frac{1}{s}\,\frac{\left(s^2 y - \frac{1}{3}y^3\right)_0^s}{\frac{1}{2}\left[y\sqrt{s^2-y^2} + s^2\left(\sin^{-1}\left(\frac{y}{s}\right)\right)\right]_0^s}$$

$$= \frac{2}{s}\,\frac{\frac{2}{3}s^3}{s^2\frac{\pi}{2}}$$

$$= \frac{2\left(\frac{2}{3}\right)}{\frac{\pi}{2}}$$

(a) $$\boxed{\frac{\Gamma_{0_E}}{\Gamma_{0_P}} = \frac{8}{3}\cdot\frac{1}{\pi}}$$

10

$$w_{i_E} = -\frac{\Gamma_{0_E}}{2b} = \text{CONSTANT} = w_{i_{E_{y=0}}}$$

$$w_{i_P}(y) = \frac{1}{4\pi}\int_{-s}^{s}\frac{\frac{d\Gamma_0(y')}{dy'}dy'}{(y-y')} = -\frac{\Gamma_{0_P}}{4\pi}\int_{-s}^{s}\frac{2y'/s^2\,dy'}{(y-y')}$$

$$w_{ip}(y) = -\frac{2T_{op}}{4\pi s^2}\int_{-s}^{s}\frac{y'dy'}{(y-y')}$$

$$w_{ip}(y) = \frac{T_{op}}{\pi s^2}\int_{0}^{s}\frac{y'dy'}{y-y'}$$

$$\frac{\pi s^2}{T_{op}} w_{ip}(y) = -y' - y\ln(y-y')\Big|_0^s$$

$$\frac{\pi s^2}{T_{op}} w_{ip}(y) = -s - y\ln(y-s) + y\ln(y)$$

$$\therefore \frac{\pi s^2}{T_{op}} w_{ip_{y=0}} = -s$$

$$w_{ip_{y=0}} = -T_{op}\frac{1}{\pi s}$$

2

$$\frac{w_{ip_{y=0}}}{w_{iE_{y=0}}} = \frac{T_{op}\frac{1}{\pi s}}{T_{oE}\frac{1}{4s}} = \frac{4}{\pi}\frac{T_{op}}{T_{oE}}$$

$$\boxed{\frac{w_{ip_{y=0}}}{w_{iE_{y=0}}} = \frac{4}{\pi}\cdot\frac{3\pi}{8} = \frac{3}{2}}$$

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Problem III

The lift/drag ratio of a sailplane is 28.5. The sailplane has a wing area of 10.0 meters squared and weighs 3200 N. **(a)** What is C_D when the aircraft is in steady level flight at 170 km/hr at an altitude of 1.0 km where the air density is 1.059 kg/cubic meter.

SHOW DETAILS OF YOUR WORK. STATE ALL ASSUMPTIONS CLEARLY.

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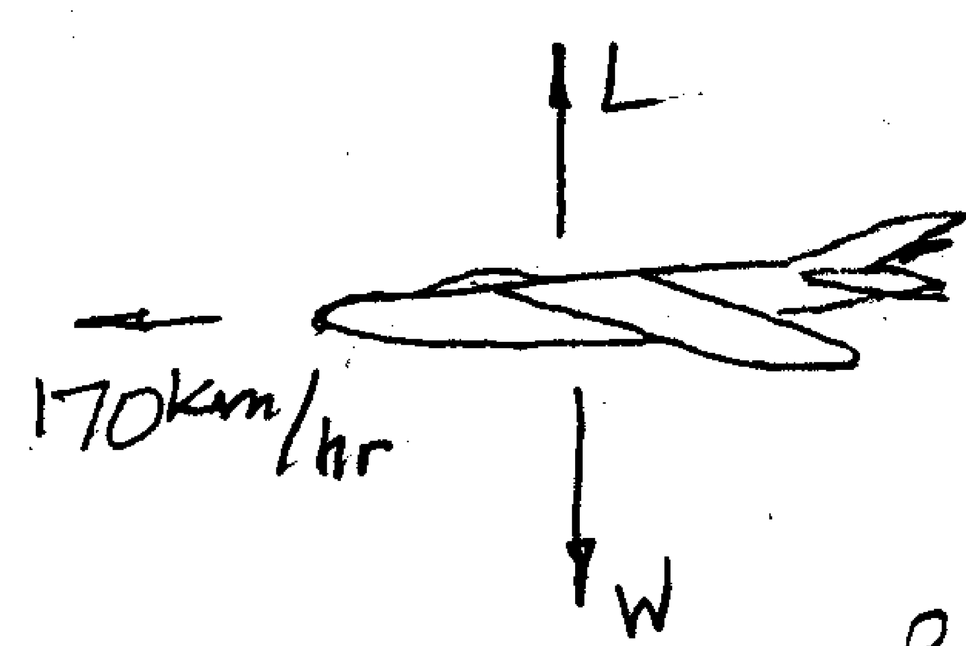
$$\frac{L}{D} = 28.5$$

$$S = 10\, m^3$$

$$W = 3200\, N$$

$$C_D = ?$$



$\rho = 1.059 \frac{kg}{m^3}$

$W = 3200 N$

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} \qquad C_D = \frac{D}{\frac{1}{2}\rho V^2 S}$$

5

$$\frac{C_L}{C_D} = \frac{L}{D} = 28.5$$

$$C_D = \frac{D}{L} C_L$$

$$C_D = \frac{D}{L} \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{D}{L} \frac{W}{\frac{1}{2}\rho V^2 S}$$

3

$$C_D = \frac{1}{28.5} \cdot \frac{3200 \quad N}{\frac{1}{2}(1.059)(10)\left(170 \frac{10^3}{60} \times \frac{1}{60}\right)^2 \quad \frac{kg}{m^3} \cdot m^2 \cdot \frac{m^2}{sec^2}}$$

$$28.5\, C_D = \frac{(2)(3200)}{(1.059)(10)\left(\frac{1.70}{3.6} \times \frac{10^5}{10^3}\right)^2} = \frac{640}{(1.059)(0.223 \times 10^4)}$$

$$28.5\, C_D = \frac{640}{(1.059)(2230)} = \frac{640}{2361.6}$$

$$\boxed{C_D = 0.27/28.5 = 0.0095}$$

$$\boxed{C_D \cong 0.01}$$