UNIFIED ENGINEERING

Spring Semester 2002

Fluid Dynamics

Previous Quiz Problems II
(With School Solutions)

STAEDTLER®
No. 937 811E
Engineer's Computation Pad

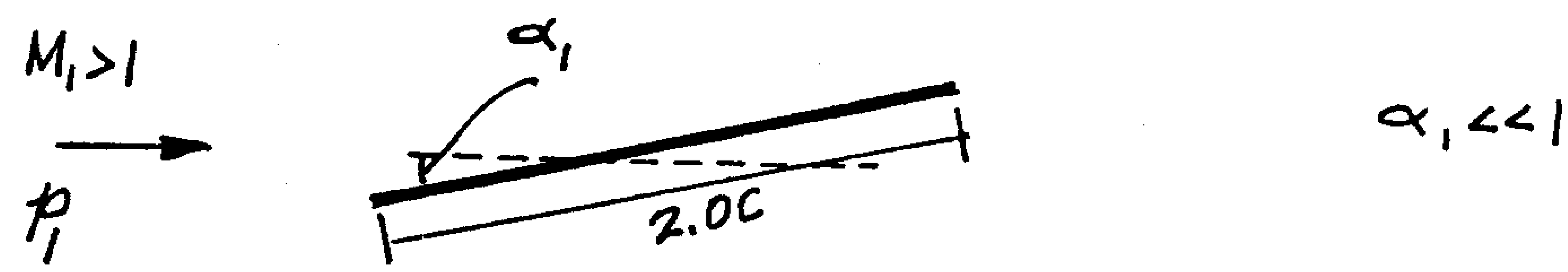
Q4F

APRIL 11, 1996

Student Name:__________________

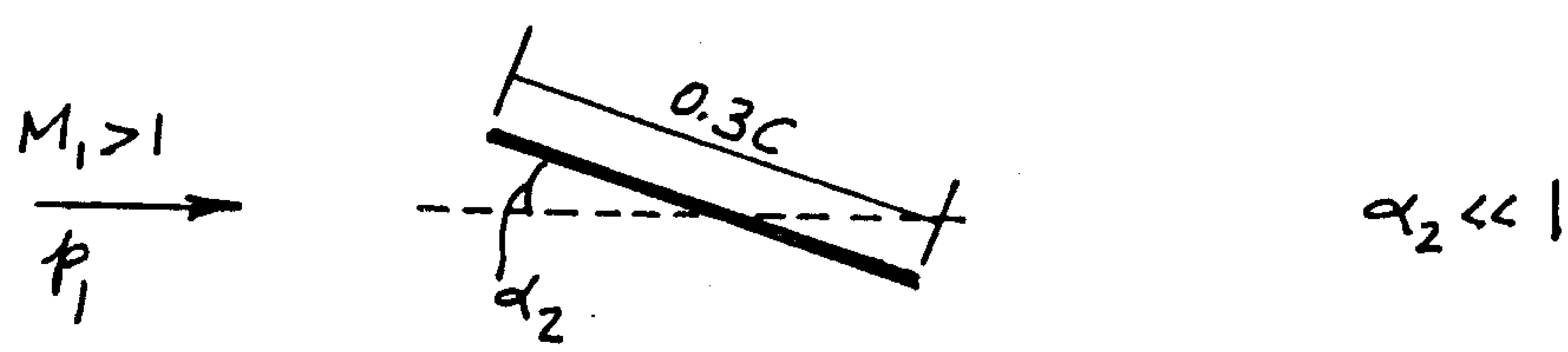
This problem has three(3) parts.

Q4F.1 (a) A flat plate airfoil has a chord 2.0c and is inclined in a supersonic flow as shown.



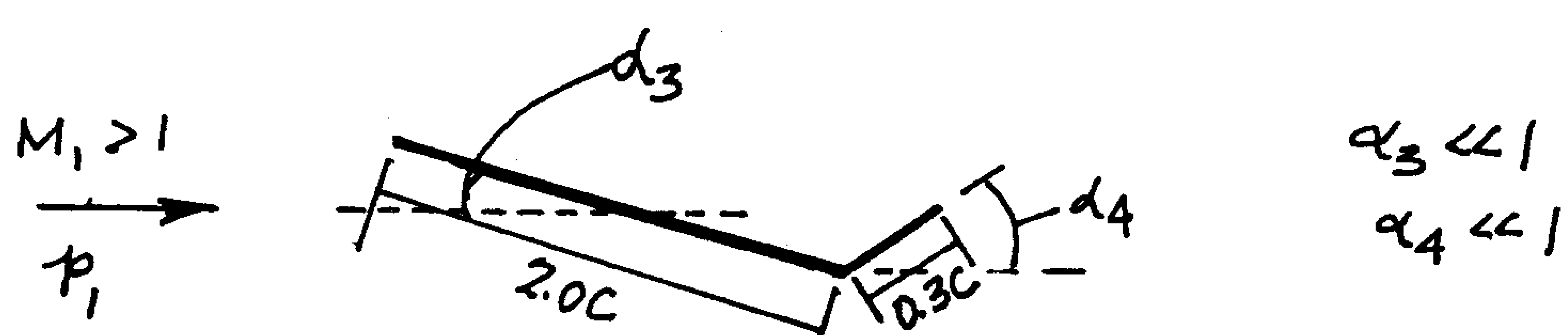
Show all shock waves and expansion waves(Prandtl-Meyer expansion). Compare the pressure on the upper surface with p_1. Compare the pressure on the lower surface with p_1. Does this flat plate airfoil produce lift? Drag? Why?

(b) A flat plate airfoil has a chord 0.3c and is inclined in a supersonic flow as shown.



Show all shock waves and expansion waves(Prandtl-Meyer expansion). Compare the pressure on the upper surface with p_1. Compare the pressure on the lower surface with p_1. Does this flat plate airfoil produce lift? Drag? Why?

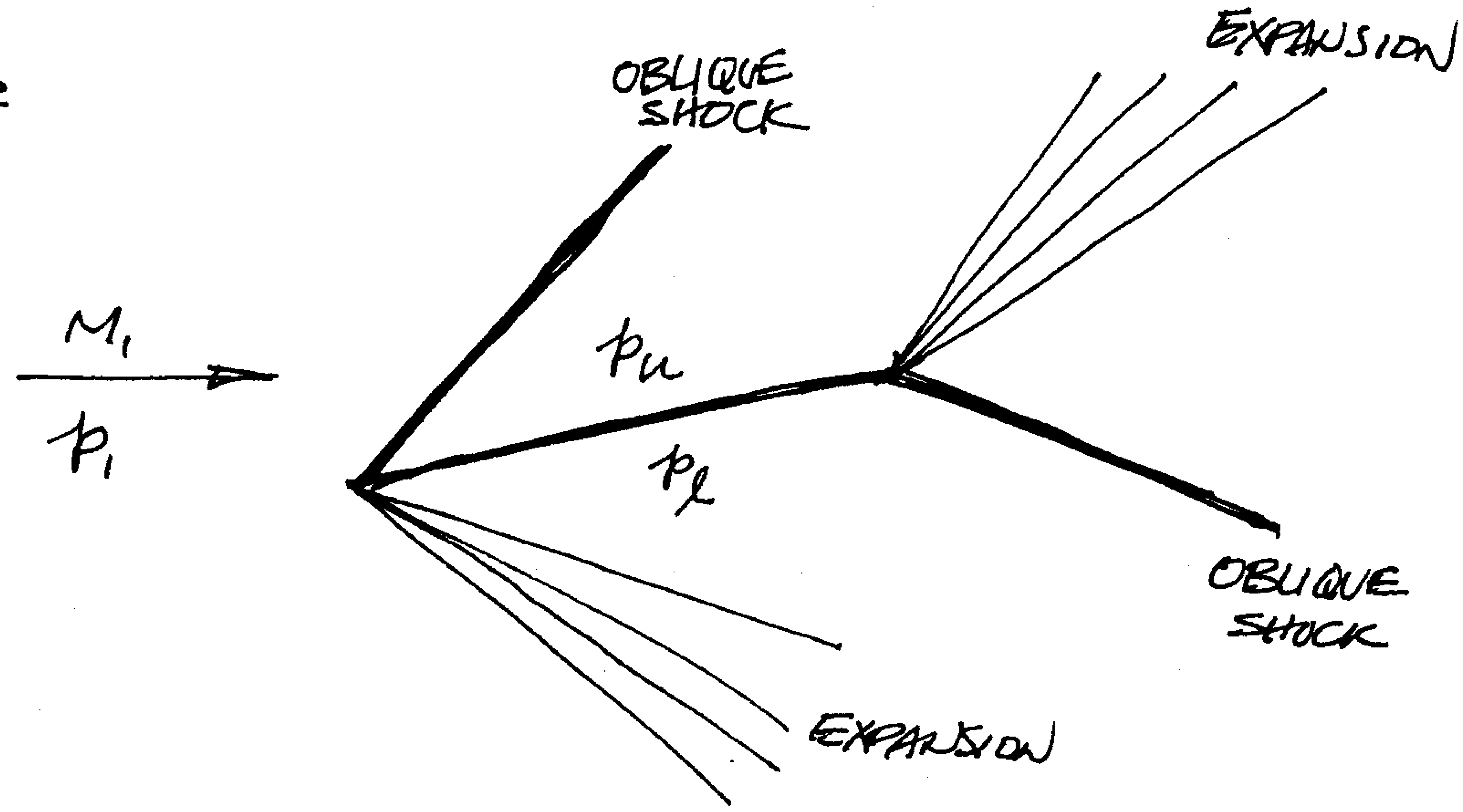
(c) A compound flat plate airfoil has a chord as shown and is inclined in a supersonic flow as shown.



Show all shock waves and expansion waves(Prandtl-Meyer expansion). Compare the pressure on each segment of the upper surface with p_1. Compare the pressure on each segment of the lower surface with p_1. Does this compound flat plate airfoil produce lift? Drag? Why?

SCHOOL SOLUTIONS

Q4F.1 (a)

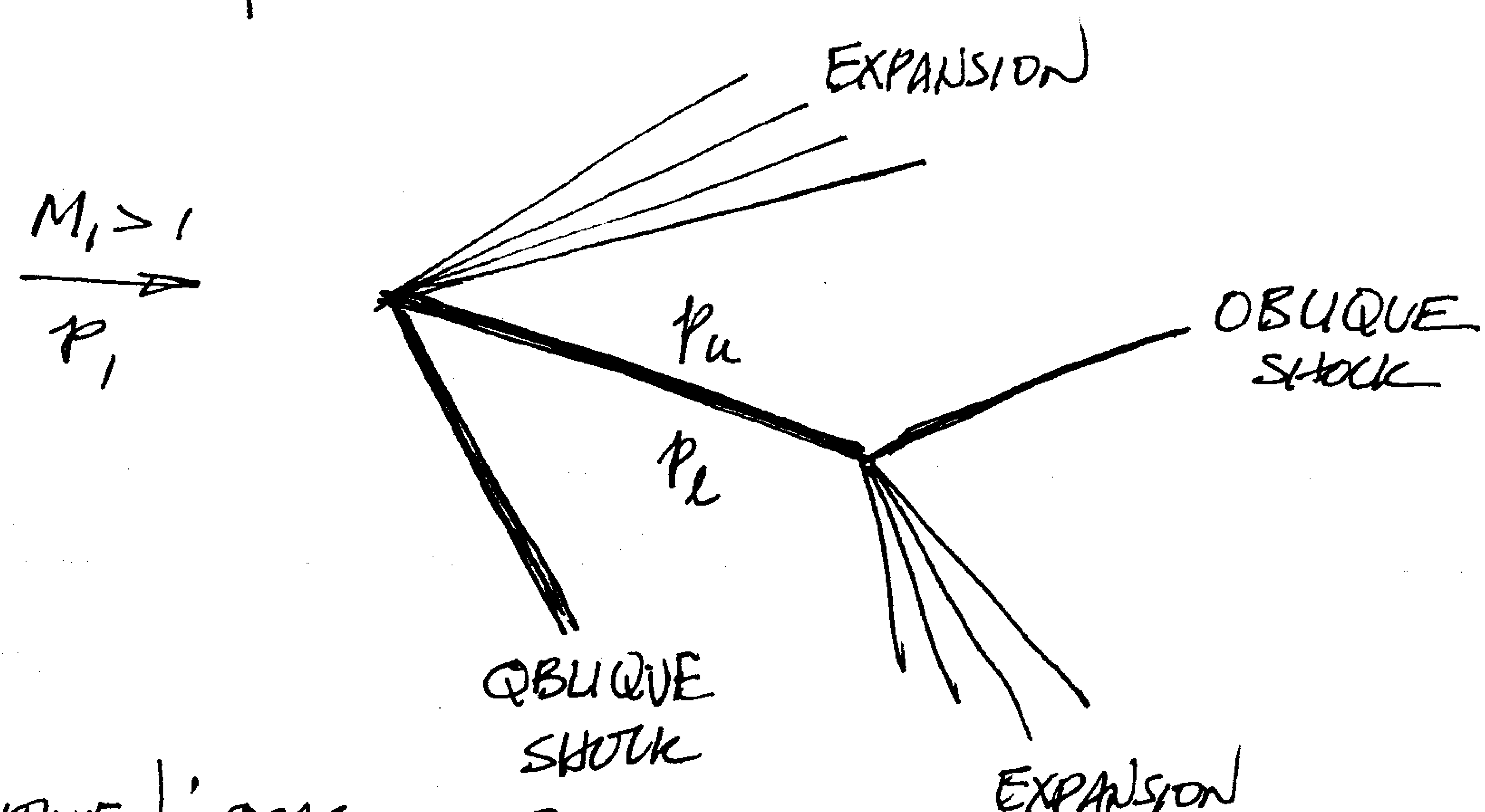


$p_u > p_1$

$p_\ell < p_1$

LIFT IS NEGATIVE	DRAG IS POSITIVE
$p_\ell < p_u$	$\Delta S > 0$

Q 4F. (b)



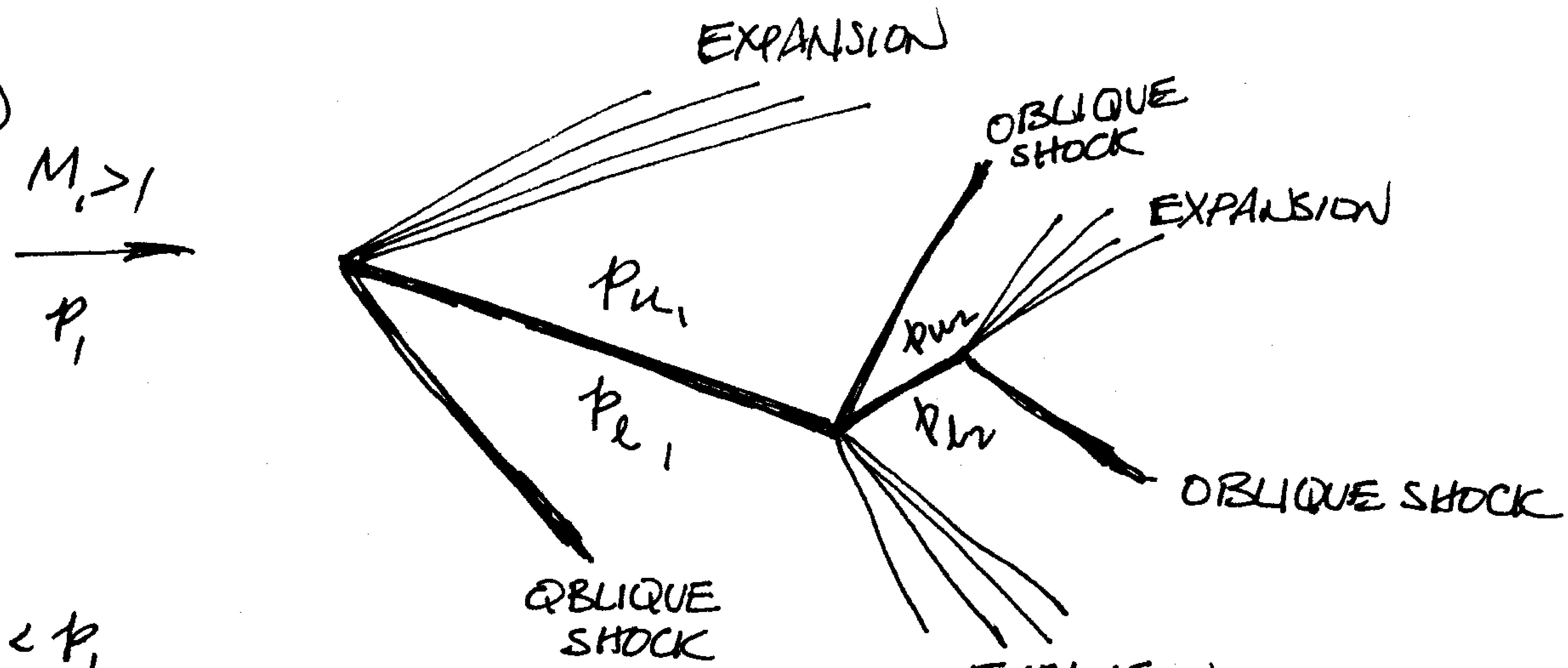
$p_u < p_1$

$p_\ell > p_1$

LIFT IS POSITIVE	DRAG IS POSITIVE
$p_\ell > p_u$	$\Delta S > 0$

SCHOOL SOLUTIONS

Q4F.1 (c)



$p_{u_1} < p_1$

$p_{u_2} > p_{u_1}$

$p_{u_2} > p_1$

$p_{l_1} > p_1$

$p_{l_2} < p_{l_1}$

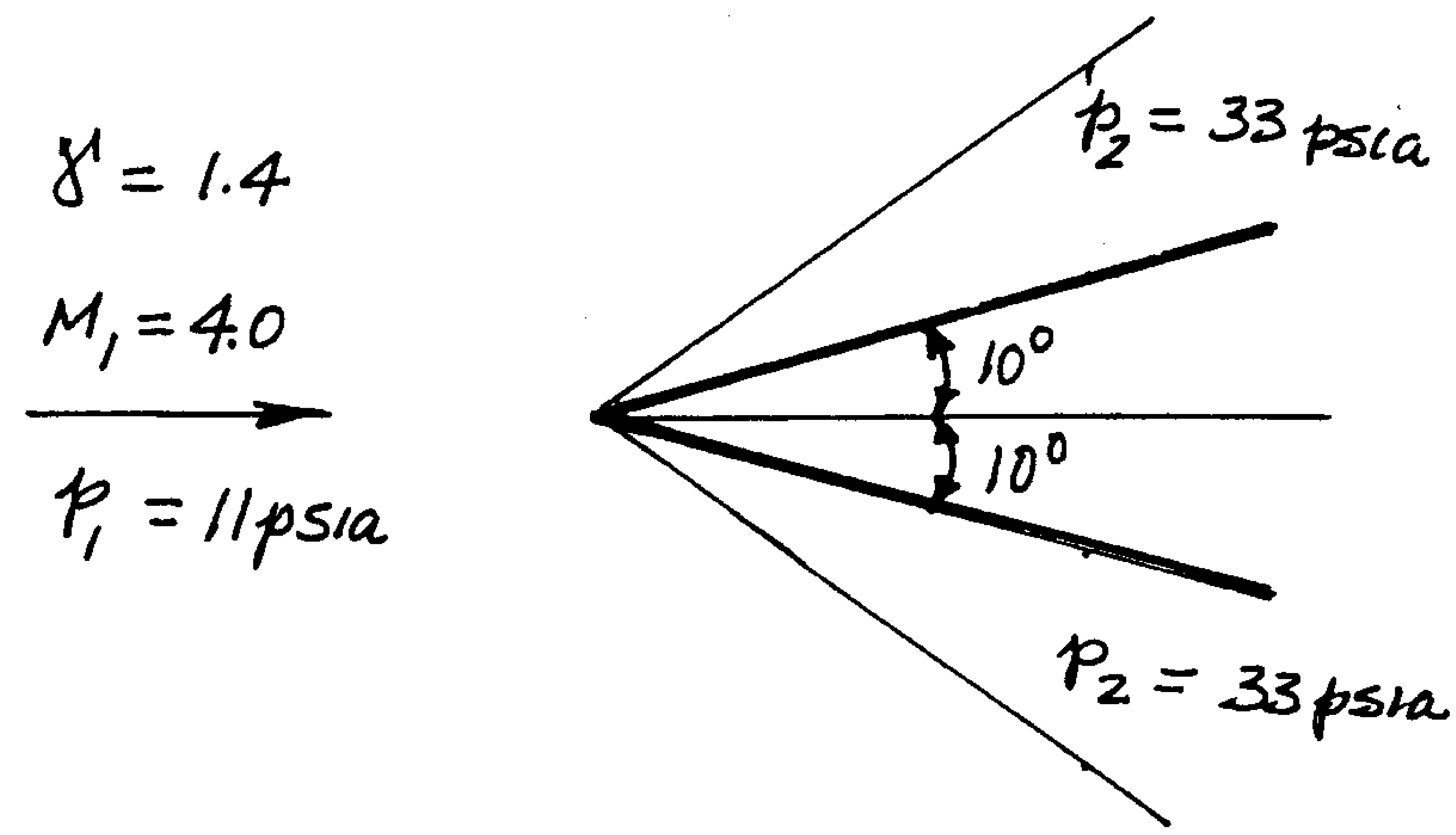
$p_{l_2} < p_1$

$p_{u_2} > p_{l_2}$

LIFT IS POSITIVE	DRAG IS POSITIVE
$p_{u_1} < p_{l_1}$	$\Delta s > 0$
$p_{u_2} > p_{l_2}$	
$\frac{2.0c}{0.3c} = 6.667$	

Student Name:_________________

<u>**Q4F.2**</u> The surface pressure on a 10° half-angle wedge is 33psia. The wedge is a zero angle of attack. The free stream pressure if 11psia, γ = 1.4, and the free stream Mach number is 4.0. <u>Is</u> the above data consistent? <u>Explain</u> your answer fully.



SCHOOL SOLUTIONS

QAF.2

$$\left.\begin{aligned} \theta &= 10^\circ \\ p_2 &= 33 \text{ psia} \\ \alpha &= 0^\circ \\ p_1 &= 11 \text{ psia} \\ \gamma &= 1.4 \\ M_1 &= 4.0 \end{aligned}\right\} \quad \frac{p_2}{p_1} = 3$$

DATA IS INCONSISTENT. OBLIQUE SHOCK CHARTS REQUIRE:

$$\left.\begin{aligned} M_1 &= 4.0, \\ p_2/p_1 &= 3 \end{aligned}\right\} \quad \theta = 12.3^\circ$$

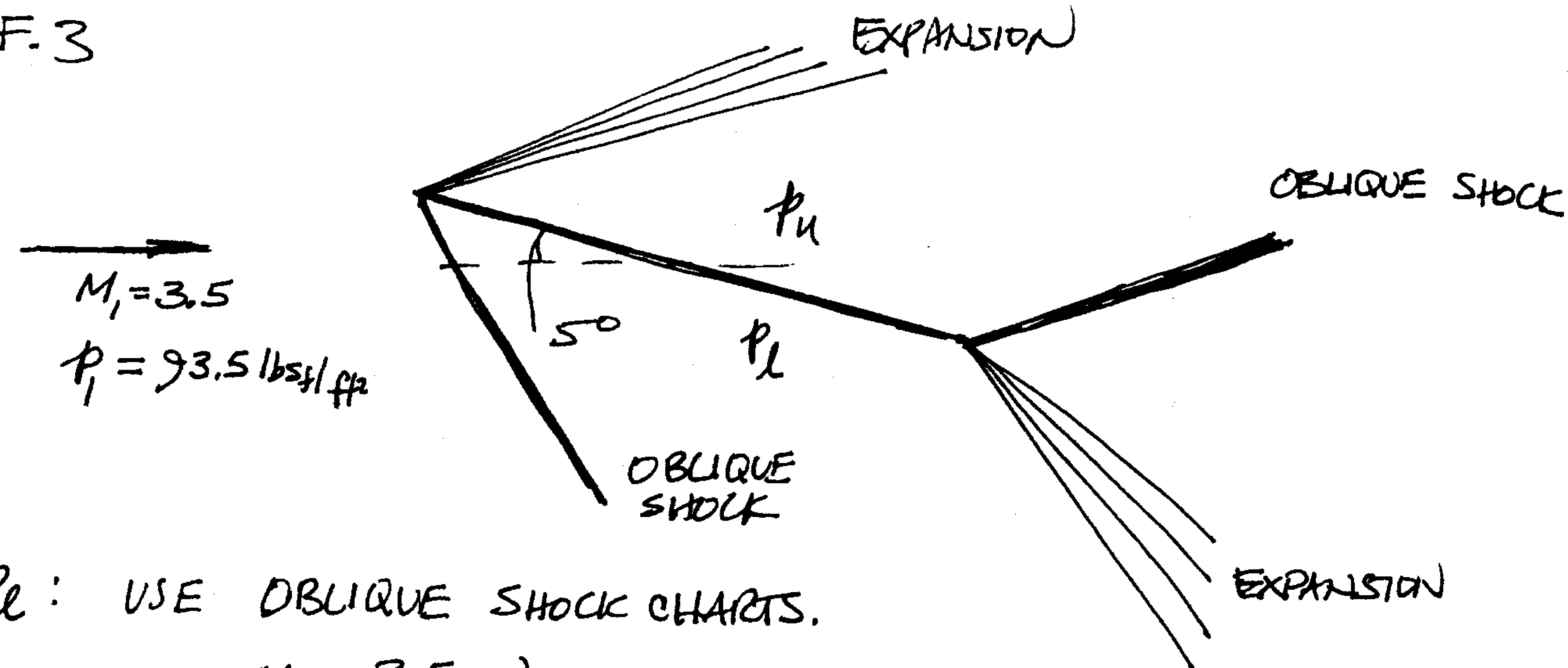
FOR THE GIVEN $M_1 = 4.0$ AND $\theta = 10^\circ$, p_2/p_1 MUST BE LESS THAN 3.0 ($p_2/p_1 = 2.5$). OR, FOR THE GIVEN $\theta = 10^\circ$ AND $p_2/p_1 = 3.0$, M_1 MUST BE CONSIDERABLY GREATER THAN 4.0. OR, FOR THE GIVEN $M_1 = 4.0$ AND $p_2/p_1 = 3$, θ MUST BE GREATER THAN 10°.

Student Name:__________________

Q4F3 A section of a third generation supersonic transport(SST) may be approximated, for wind tunnel testing, by a two-dimensional flat plate. The aircraft will fly at an altitude of 70,000 ft. where the pressure is 93.5 lbs_f / ft^2, density is 0.000159 lbs_f / ft^3, and the speed of sound is 662 mi/hr. The aircraft will fly at a Mach number of 3.50. If the angle of attack is 5°, find the pressure on the upper and lower surfaces using exact methods.

SCHOOL SOLUTIONS

Q4F.3



p_l: USE OBLIQUE SHOCK CHARTS.

$$\left.\begin{array}{l} M_1 = 3.5 \\ \theta = 5^\circ \end{array}\right\} \Rightarrow \frac{p_2}{p_1} \cong 1.42$$

$$p_2 = 1.42\,p_1 = (1.42)(93.5)\ \text{lbsf/ft}^2 = \underline{132.8\ \text{lbsf/ft}^2}$$

p_u: USE PRANDTL-MEYER FUNCTION TABLE.

$$\nu(M_u) = \nu(M_1) + |\theta - \theta_1|$$
$$= \nu(3.5) + |0 - 5^\circ|$$
$$= 58.6^\circ + 5^\circ$$
$$\nu(M_u) = 63.6^\circ$$
$$\therefore \quad M_u \cong 3.84$$

NOW USE SUPERSONIC FLOW (ISENTROPIC) TABLE.

$$\left(\frac{p_1}{p_{0_1}}\right)_{M_1 = 3.5} = 0.01311; \qquad \left(\frac{p_u}{p_{0_1}}\right)_{M_u = 3.84} = 8.19 \times 10^{-3}$$

SCHOOL SOLUTIONS

$$\therefore \left(\frac{p_u}{p_{o_1}}\right)\left(\frac{p_{o_1}}{p_1}\right) = \frac{p_u}{p_1} = (8.19\times10\ )\left(\frac{1}{0.01311}\right)$$

$$= \frac{8.19\times10^{-3}}{1.311\times10^{-2}}$$

$$\frac{p_u}{p_1} = 6.25\times10^{-1}$$

$$\therefore p_u = (6.25\times10^{-1})p_u = (6.25\times10^{-1})(93.5)\ lbs_f/ft^2$$

$$p_u = \underline{58.4\ lbs_f/ft^2}$$

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UNIFIED ENGINEERING
SPRING 1997

QUIZ Q4F

April 3, 1997
10:00 AM - 11:00 AM
37 - 212

RESOURCES ALLOWED: (1) class notes, (2) school solutions, (3) calculator, (4) KUETHE and CHOW.

- Put your name on each page of the quiz.
- Show all your work, especially intermediate work.
- Show the logical path of your work.
- Underscore your answers.

QUIZ SCORING

PROBLEM I (33.3%)
PROBLEM II (33.3%)
PROBLEM III (33.3%)
TOTAL SCORE:

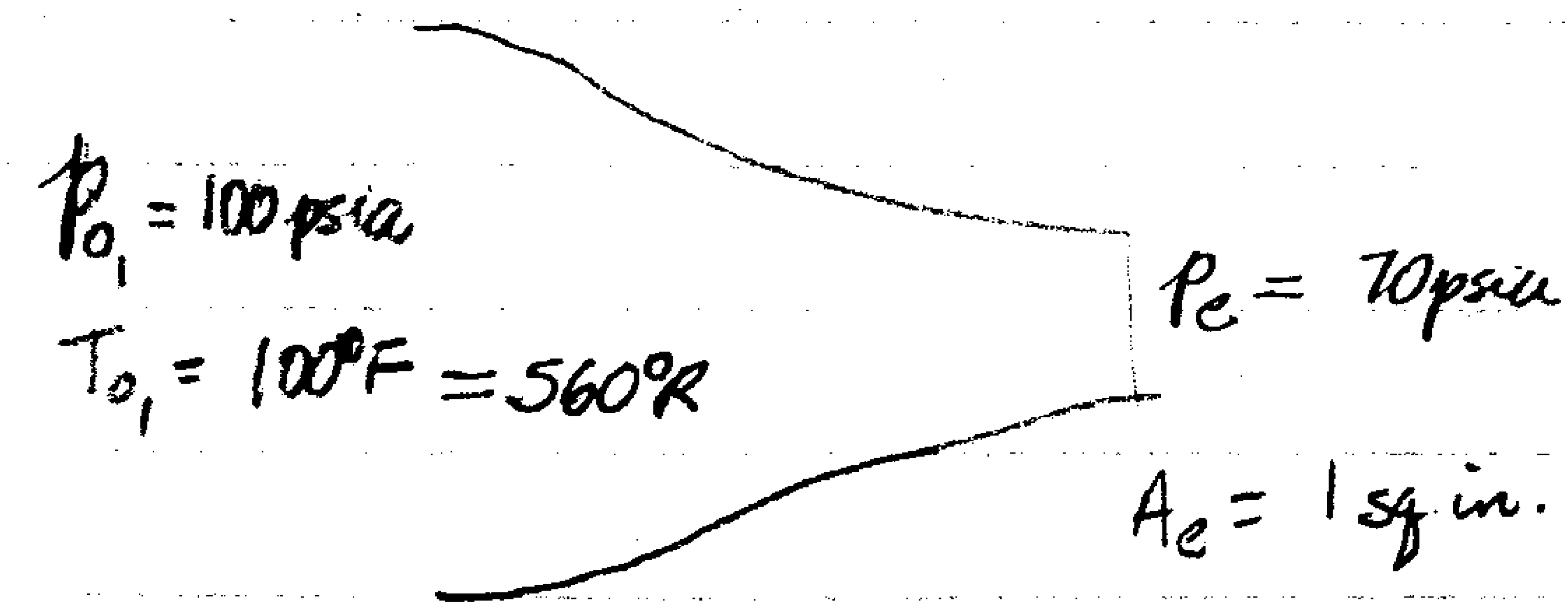
NAME:

UNIFIED ENGR QUIZ Q4F
Spring 1997
04/03/97

Problem I

A convergent nozzle with an exit area of 1.0 sq. in. has been properly designed to pass air isentropically from a stagnation state of 100 psia, 100°F, to an exit pressure of 70 psia.
(a) **Find** the mass flow rate.
(b) If the back pressure of the region into which the nozzle exhausts is raised to 90 psia, **what** is the mass flow rate?
(c) **Show** a diagram of the flow in the nozzle in each of the two cases.

PROBLEM #1



(a) $\dot{m} = ?$

(5)
$$\frac{p_e}{p_{o_1}} = \frac{70}{100} = 0.7 \rightarrow M_e = 0.73 \; ; \; \frac{A^*}{A_e} = 0.931$$

$$\therefore A^* = A_e \times 0.931 = (1) \times (0.931) = 0.931 \text{ sq. in.}$$

(10)
$$\frac{m}{A^*} = \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} p_o \rho_o}$$

$$\gamma = 1.4 \; , \quad p_o = \rho_o R T_o$$

$$m = A^* \sqrt{(1.4)\left(\frac{2}{2.4}\right)^{\frac{2.4}{0.4}} p_o \frac{p_o}{RT_o}}$$

(5)
$$m = 0.931 \sqrt{(1.4)(0.8333)^6 \frac{p_o^2}{RT_o}}$$

$$= (0.931)(1.183)(0.8333)^3 p_o / \sqrt{RT_o}$$

$$= (0.931)(1.4)(0.8333)^3 p_o / \sqrt{\gamma R T_o}$$

$$= (0.931)(1.4)(0.8333)^3 \frac{p_o}{a_o} \; ; \quad [0.931] = \text{sq. in.}$$

$$[p_{o_1}] = \text{lbf/sq in}^2$$

$$= 2.09 \approx 2.1 \text{ lbm/sec.}$$

$a = 49.1\sqrt{T}$

$[T] = °R$

$R_{air} = 1716 \frac{\text{ft-lb}}{\text{lbm-°R}}$

$[a] = \text{ft/sec}$

$g_c = 32.2 \frac{\text{lbm-ft/sec}^2}{\text{lbf}}$

(b) $\dot{m} = ?$

4

$$p_e = 90 \text{ psia}$$

$$\frac{p_e}{p_{0_1}} = \frac{90}{100} = 0.9 \rightarrow M_e = 0.39 \;;\; \frac{A^*}{A_e} = 0.616$$

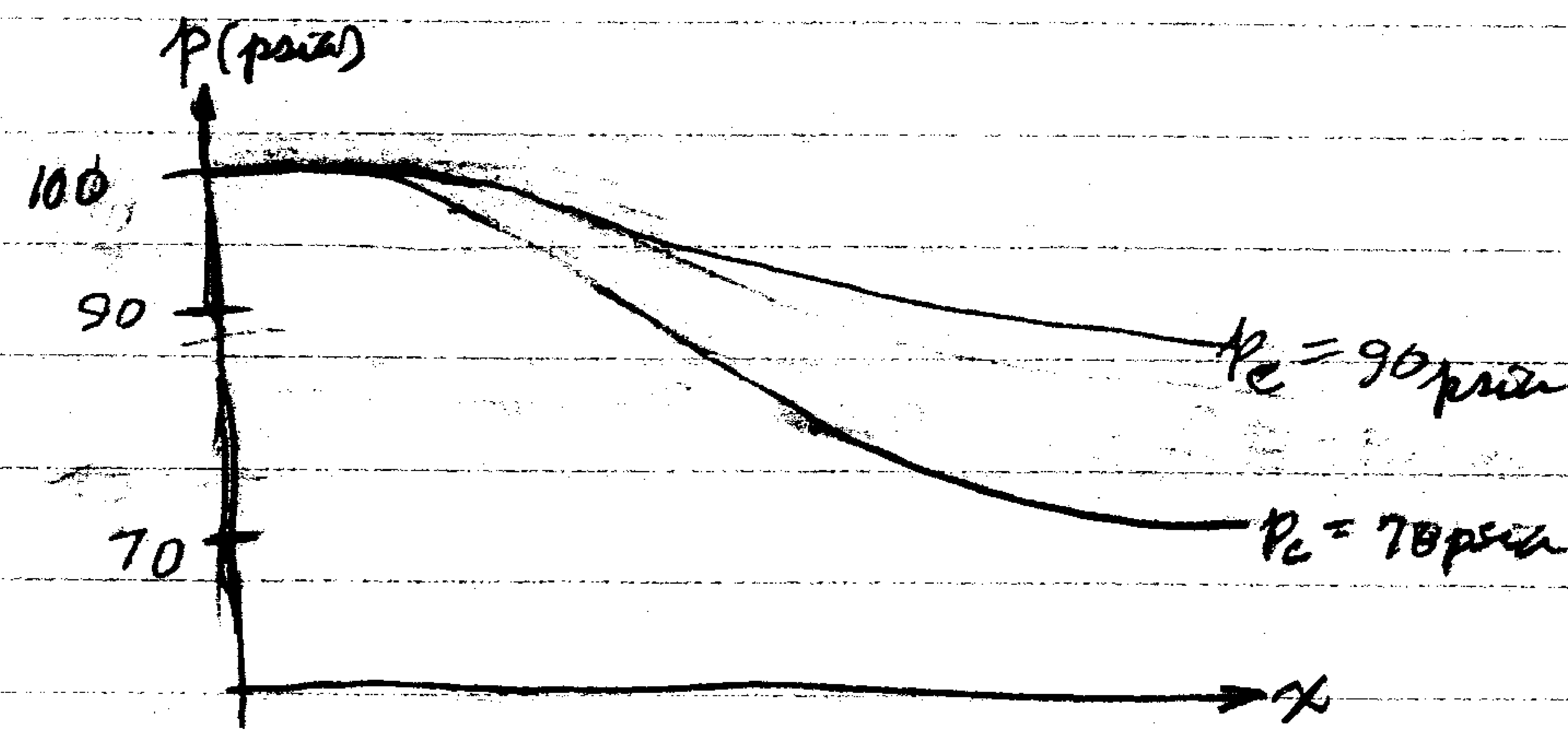
$$\therefore A^* = A_e \times 0.616 = (1)(0.616) = 0.616 \text{ sq. in.}$$

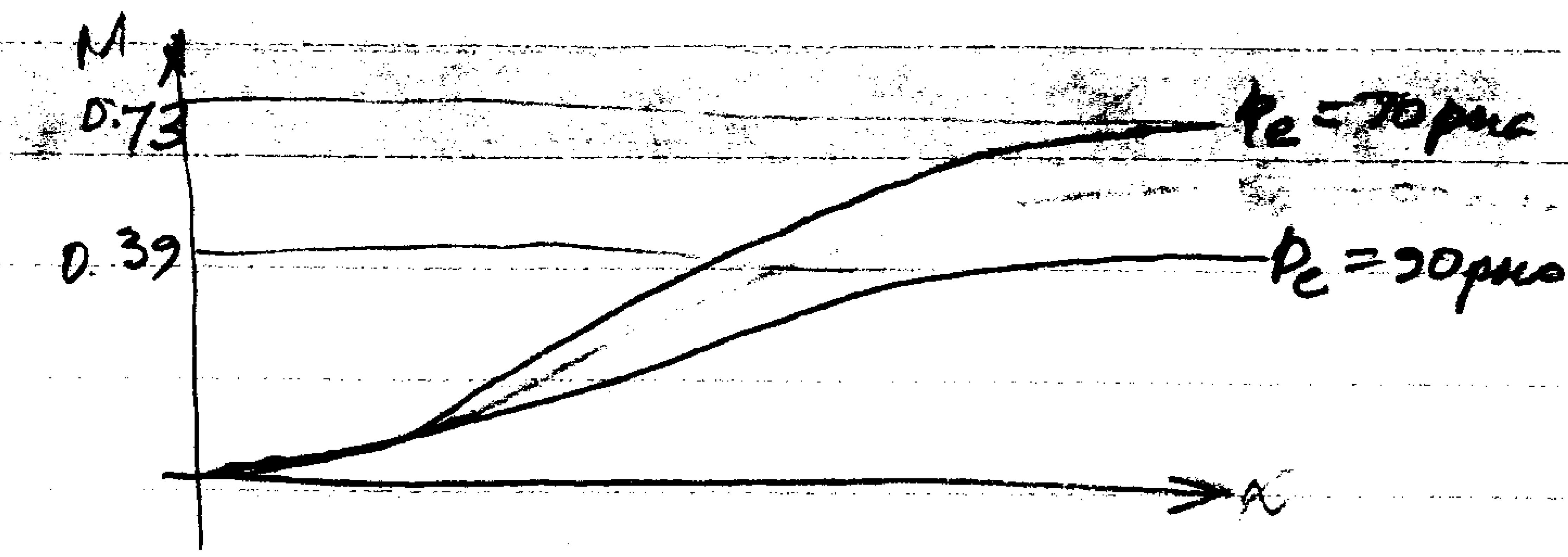
$$\therefore \dot{m} = (0.616)\sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\; p_0 \rho_0$$

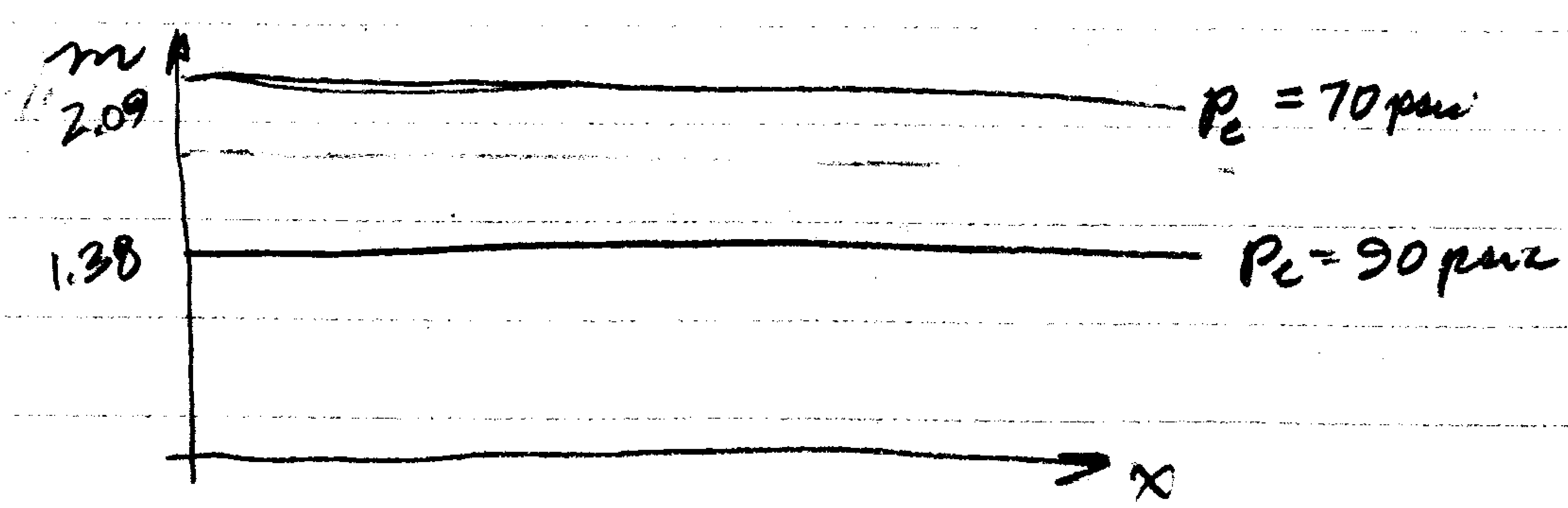
$$= (0.616)(1.4)(0.8333)^3 \frac{p_0}{a_0} = \underline{1.38 \text{ lbm/sec}}$$

(c)

10







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Spring 1997
04/03/97

Problem II

A three-dimensional bow shock wave is generated by the Shuttle Orbiter during entry. However, there is a region where the shock wave is essentially **normal** to the free-stream as shown. The velocity of the Shuttle is 7.62 km/sec. at an altitude of 75.0 km. The free-stream temperature and the static pressure at this altitude are 200.15°K and 2.50 N/sq. meter, respectively. Use the **normal** shock relations to **calculate** the values of the following parameters downstream of the normal shock wave:

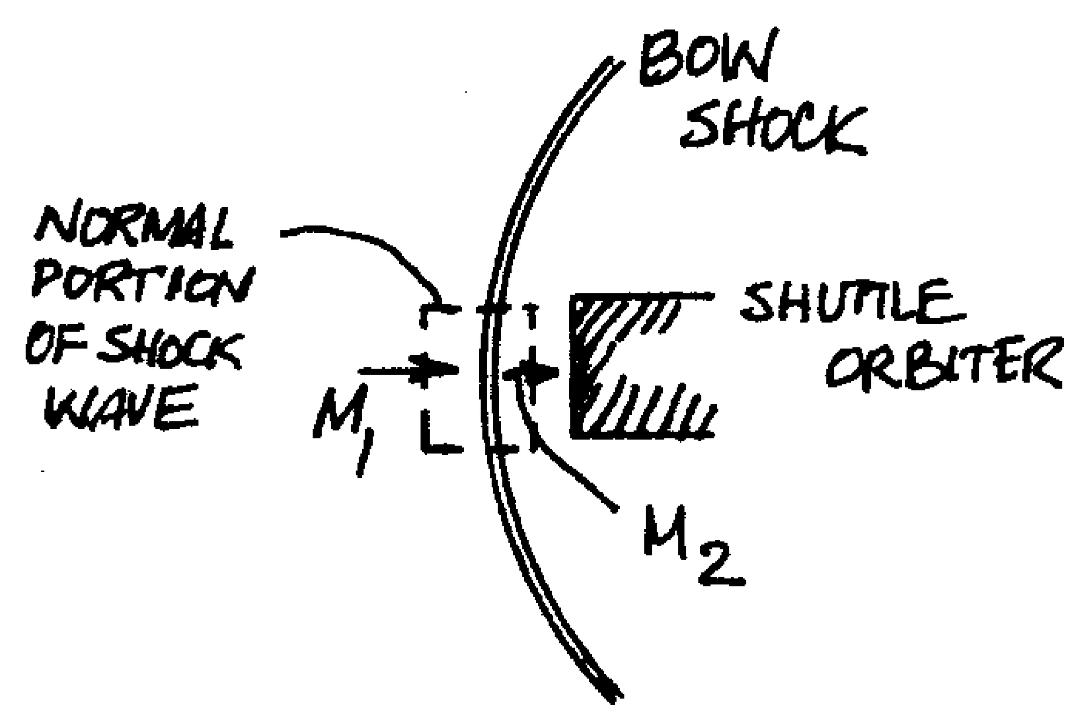
(a) static pressure:

(b) stagnation pressure:

(c) static temperature:

(d) stagnation temperature:

(e) Mach number:



Assume $\gamma = 1.4$.

PROBLEM #2

$$M_1 = \frac{V_1}{a_1} = \frac{7.62 \times 1000 \text{ m/sec.}}{20.04\sqrt{T} \text{ m/sec}} = \frac{7620}{20.04\sqrt{200.15}}$$

(5)

$$M_1 = \frac{7620}{(20.04)(14.15)} = \frac{7620}{283.51} = \boxed{26.88}$$

a) p_2

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1} = \frac{(2)(1.4)}{1.4+1}(26.88)^2 - \frac{1.4-1}{1.4+1}$$

$$= \left(\frac{2.8}{2.4}\right)(26.88)^2 - \frac{0.4}{2.4} = (1.167)(722.53) - 0.167$$

(5)

$$= 843.19 - 0.167$$

$$= 843.03$$

$$\therefore p_2 = (843.03)(2.50) = 2,107.56 \frac{N}{m^2}$$

$$\boxed{p_2 = 2,107.56 \frac{N}{m^2}}$$

b) p_{o2}

$$\frac{p_{o2}}{p_{o1}} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} \left(\frac{\rho_2}{\rho_1}\right) = \left(\frac{T_1}{T_2}\right)^{\frac{1}{1.4-1}} \left(\frac{\rho_2}{\rho_1}\right) = \left(\frac{T_1}{T_2}\right)^{2.5} \left(\frac{\rho_2}{\rho_1}\right)$$

where

$$\frac{\rho_2}{\rho_1} = \frac{\frac{\gamma+1}{\gamma-1} + \frac{p_1}{p_2}}{1 + \frac{\gamma+1}{\gamma-1}\frac{p_1}{p_2}} = \frac{\frac{1.4+1}{1.4-1} + \frac{1}{843.03}}{1 + \frac{1.4+1}{1.4-1}\frac{1}{843.03}}$$

$$\frac{\rho_2}{\rho_1} = \frac{6 + 0.0012}{1 + (6)(0.0012)} = \frac{6.0012}{1.0071} = 5.96$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} = (843.03)\left(\frac{1}{5.96}\right) = 141.45$$

$$T_2 = 141.45 \times T_1 = (141.45)(200.15°K)$$

(6)

(c) T_2 $\boxed{T_2 = 28,310.8\ °K}$ ✓

Now return to p_{o_2}/p_{o_1}:

$$p_{o_2}/p_{o_1} = \left(\frac{1}{141.45}\right)^{2.5}(5.96) = (0.0071)^{2.5}(5.96)$$

$$(5\times10^{-5})(8.4\times10^{-2})(5.96)$$

$$= (42.04\times10^{-7})(5.96) = (4.204\times5.96\times10^{-6})$$

$$= 25.06\times10^{-6} = 2.506\times10^{-5}$$

$$\therefore\ p_{o_2} = (2.506\times10^{-5})p_{o_1} = (2.506\times10^{-5})\left(\frac{p_{o_1}}{p_1}\right)p_1$$

where

$$\frac{p_{o_1}}{p_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}} = \left(1+\frac{1.4-1}{2}(26.88)^2\right)^{\frac{1.4}{1.4-1}}$$

(5)

$$\frac{p_{o_1}}{p_1} = \left(1+\frac{0.4}{2}(722.53)\right)^{\frac{1.4}{0.4}} = (1+(0.2)(722.53))^{3.5}$$

$$= (1+144.51)^{3.5} = (145.51)^{3.5}$$

$$= (3,080,906.5)(12.06)$$

$$\frac{p_{o_1}}{p_1} = 37.17\times10^{6}$$

$$\therefore\ p_{o_2} = (2.506\times10^{-5})(3.717\times10^{7})p_1 = (2.506)(3.717)\times10^{+2}(2.5\ldots$$

$$p_{o_2} = (9.314)(2.50)\times10^{2} = 23.287\times10^{2}\ \frac{N}{m^2}$$

$$\boxed{p_{o_2} = 2{,}328.7 \ \frac{N}{m^2}}$$ ✓

(d)

$$T_{o_2} = T_{o_1} = \text{constant}$$

7

$$\frac{T_{o_1}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 = \left(1 + \frac{1.4-1}{2}(26.88)^2\right)$$

$$\frac{T_{o_1}}{T_1} = \left(1 + \frac{0.4}{2}(722.53)\right) = 1 + 144.51 = 145.51$$

$$\therefore T_o = T_{o_1} = T_{o_2} = (145.51)\,T_1 = (1.4551 \times 10^2)(2.0015 \times 10^2)\,°$$

$$\boxed{\begin{array}{l} T_o = 2.912 \times 10^4 \ °K \\ T_o = 29{,}123.8\ °K \end{array}}$$ ✓

(e)

$$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)} = \frac{(1.4-1)(26.88)^2 + 2}{(2)(1.4)(26.88)^2 - (1.4-1)}$$

$$= \frac{(0.4)(722.53) + 2}{(2)(1.4)(722.53) - 0.4} = \frac{289.012 + 2}{2{,}023.084 - 0.4}$$

5

$$= \frac{291.012}{2{,}022.684} = 0.1439$$

$$\boxed{\therefore M_2 = 0.379}$$ ✓

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Spring 1997
04/03/97

Problem III

Consider a "symmetrical diamond" airfoil in a supersonic flow. The wedge angle is 2θ. **Draw** the complete flow fields including all waves generated by the flow interacting with airfoil corresponding to the following four cases:

(a) $\alpha = 0$

(b) $\theta - \alpha > 0$

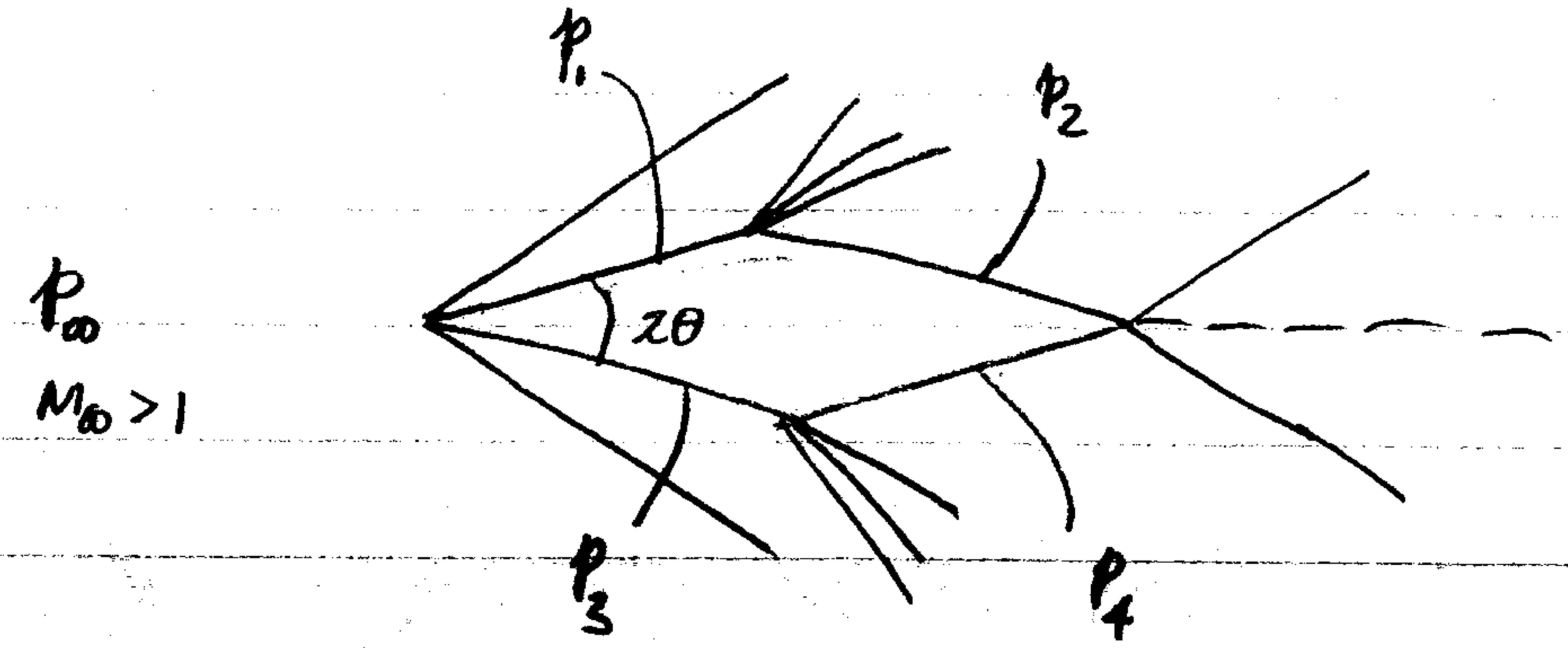
(c) $\theta - \alpha < 0$

(d) $\theta - \alpha = 0$

where α is the angle of attack. Also, In each of the four cases, **compare** the pressure on each of the four surfaces of the airfoil to the free-stream pressure.

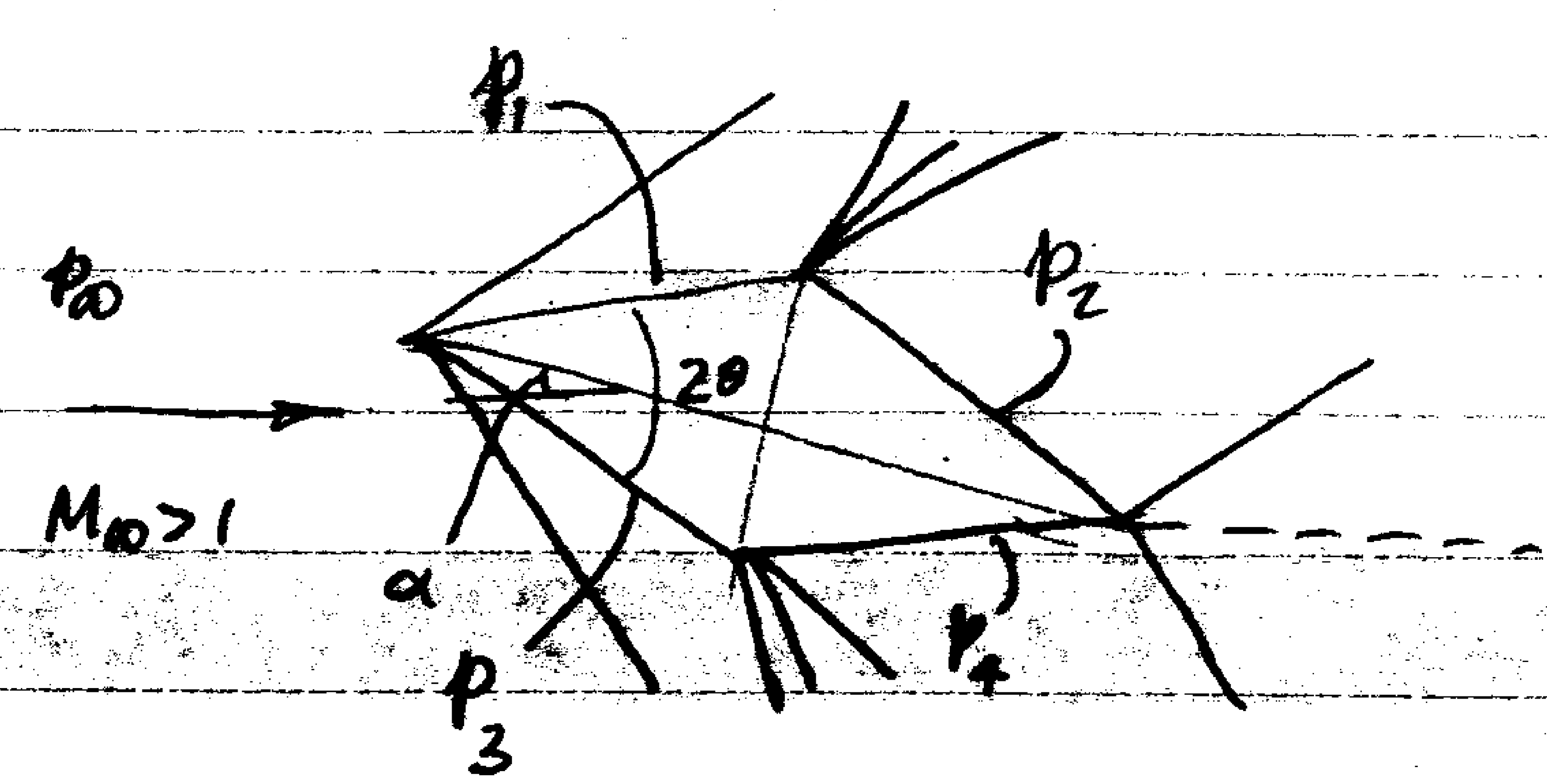
PROBLEM #3

ⓐ $\alpha = 0$



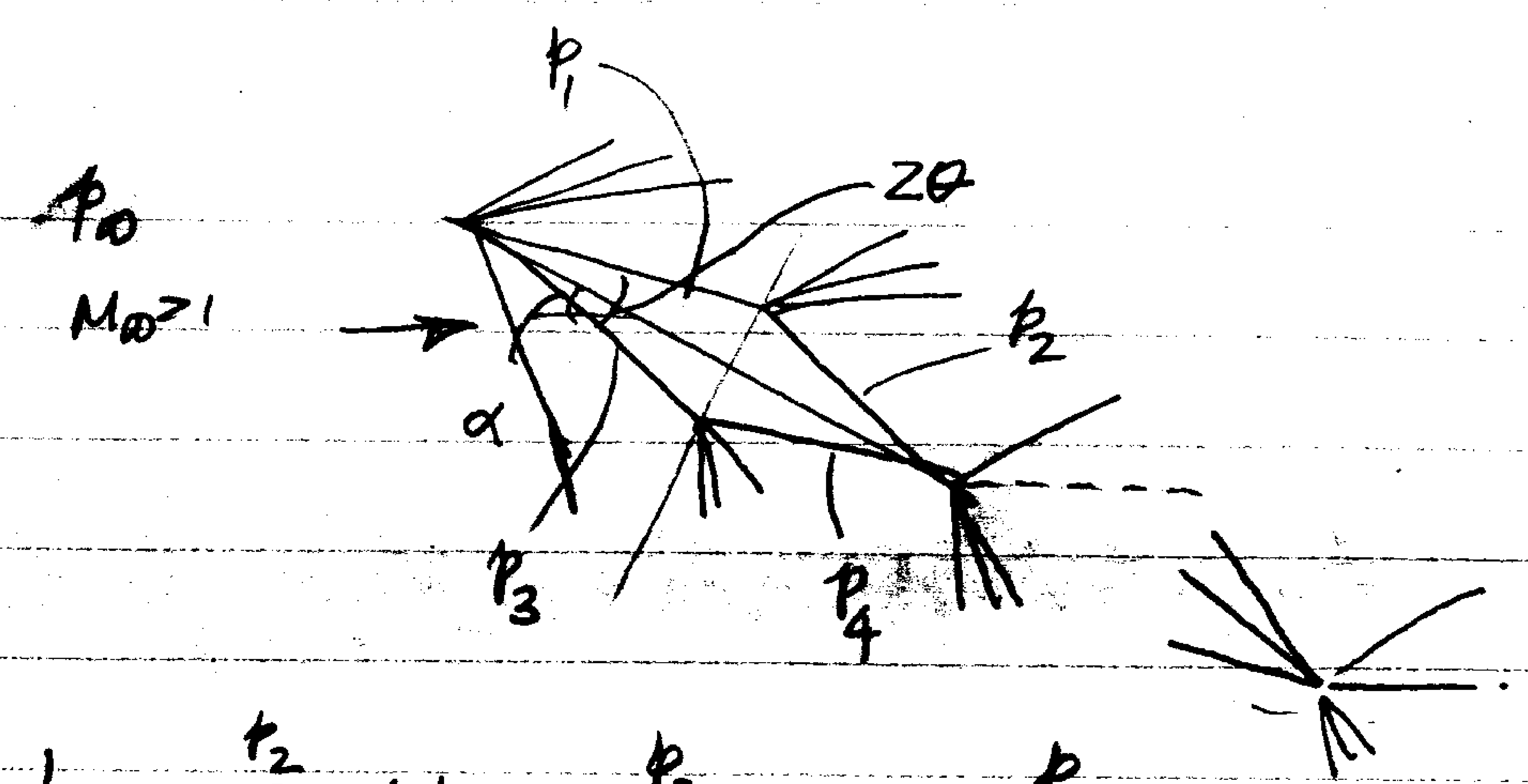
$$\frac{p_1}{p_\infty} > 1 \qquad \frac{p_2}{p_\infty} < 1 \qquad \frac{p_3}{p_\infty} > 1 \qquad \frac{p_4}{p_\infty} < 1$$

ⓑ $(\theta - \alpha) > 0$



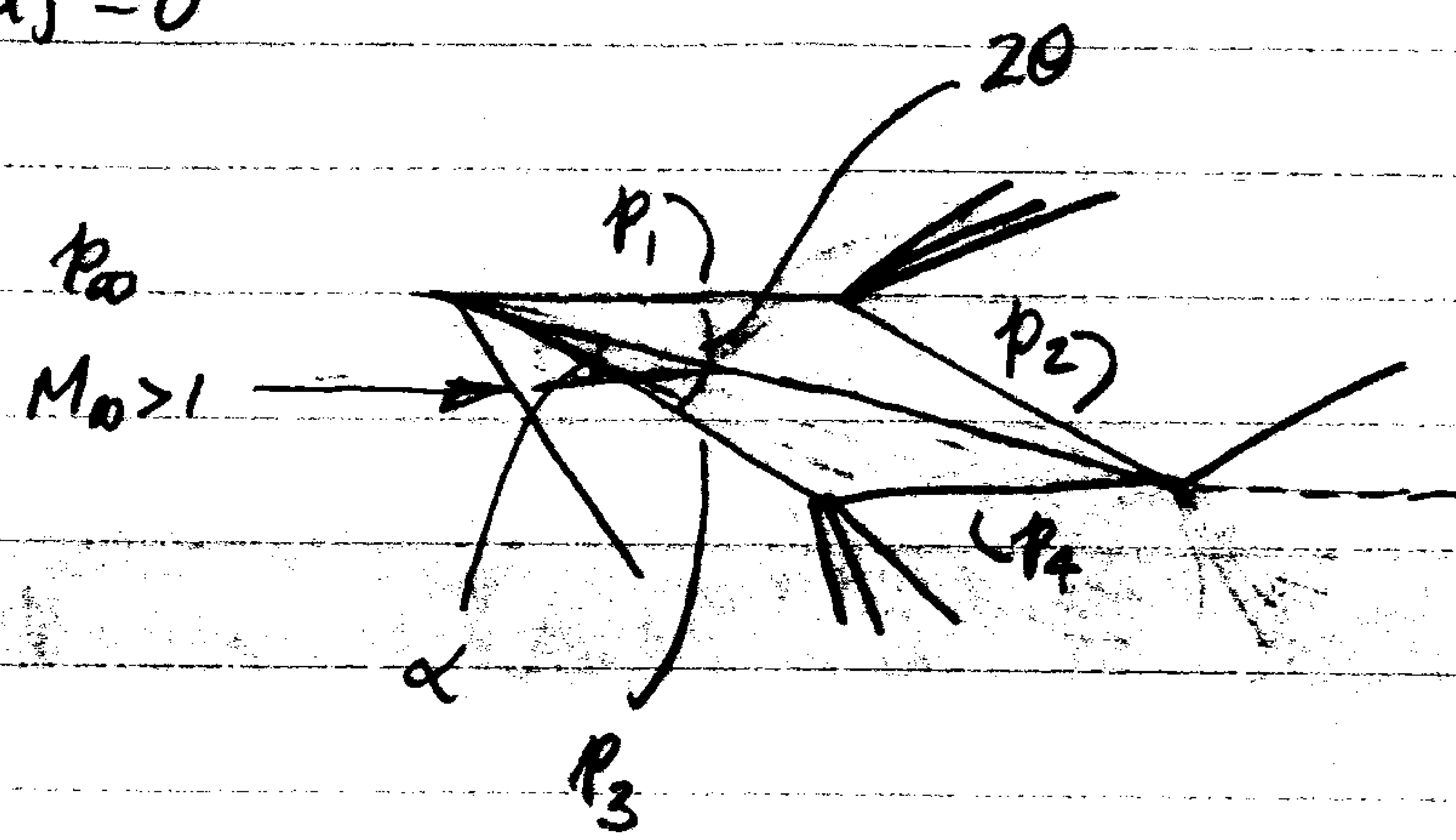
$$\frac{p_1}{p_\infty} > 1 \qquad \frac{p_2}{p_\infty} < 1 \qquad \frac{p_3}{p_\infty} > 1 \qquad \frac{p_4}{p_\infty} < 1$$

(c) $(\theta - \alpha) < 0$



$\frac{p_1}{p_\infty} < 1$ $\frac{p_2}{p_3} < 1$ $\frac{p_3}{p_\infty} > 1$ $\frac{p_4}{p_\infty} > 1$

(d) $(\theta - \alpha) = 0$



$\frac{p_1}{p_\infty} = 1$ $\frac{p_2}{p_\infty} < 1$ $\frac{p_3}{p_\infty} > 1$ $\frac{p_4}{p_\infty} \approx 1$

NAME: SCHOOL SOLUTION

UNIFIED ENGINEERING
SPRING 1998
QUIZ Q4F

APRIL 2, 1998
10:05 AM - 10:55 PM
37 - 212

RESOURCES ALLOWED: (1) class notes, (2) school solutions, (3) calculator, (4) KUETHE and CHOW.

- Put your name on each page of the quiz.
- Show all your work, especially intermediate work.
- Show the logical path of your work.
- Underscore your answers.

QUIZ SCORING

PROBLEM I (20%)	
PROBLEM II (35%)	
PROBLEM III (45%)	
TOTAL SCORE:	

NAME: SCHOOL SOLUTION

UNIFIED ENGINEERING QUIZ Q4F
Spring 1998
04/02/98

Problem I

Air flows in a duct of variable cross section. At one point in the flow one measures:
A_1 = 1.0 sq. ft., T_1 = 65°F, p_1 = 30 psia, M_1 = 2.5.

At a second section, downstream of the first, the area is found to be A_2 = 2.0 sq. ft. If the flow is known to be isentropic, **find:** (a) p_2, (b) T_2. **Draw** a diagram of the flow.

CONVERT T_1°F TO T_1°R: $T_1 = 65°F = (65+460)°R = 525°R$

FOR ISENTROPIC FLOWS:

$$\left(\frac{A^*}{A}\right)_{M=2.5} = 0.3793 \;;\; \therefore \; A^* = A_{M=2.5}(0.3793) = (1)(0.3793)$$

$$A^* = 0.38$$

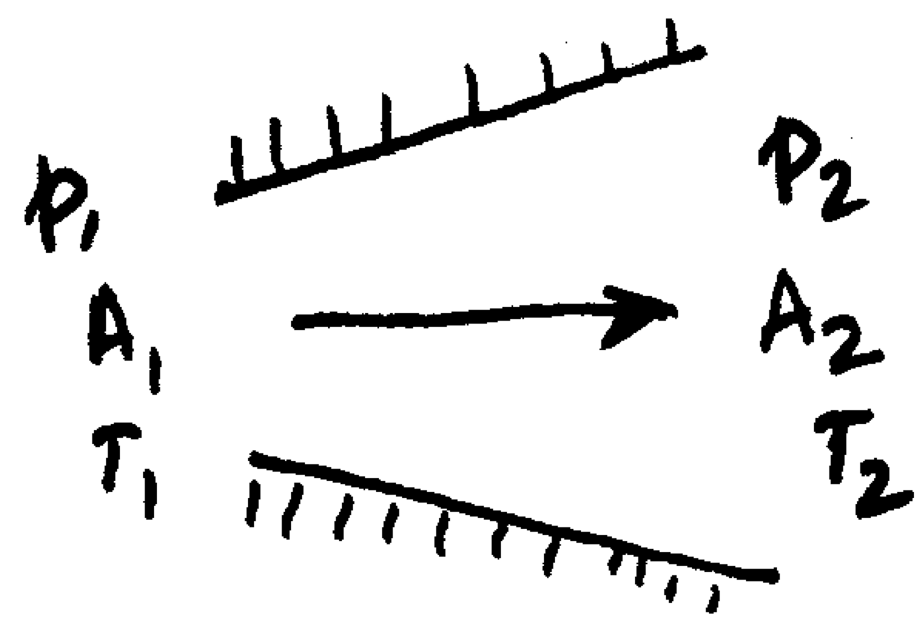
$$\frac{A_2}{A^*} = \frac{2}{0.38} = 5.26 \rightarrow M_2 = 3.23$$

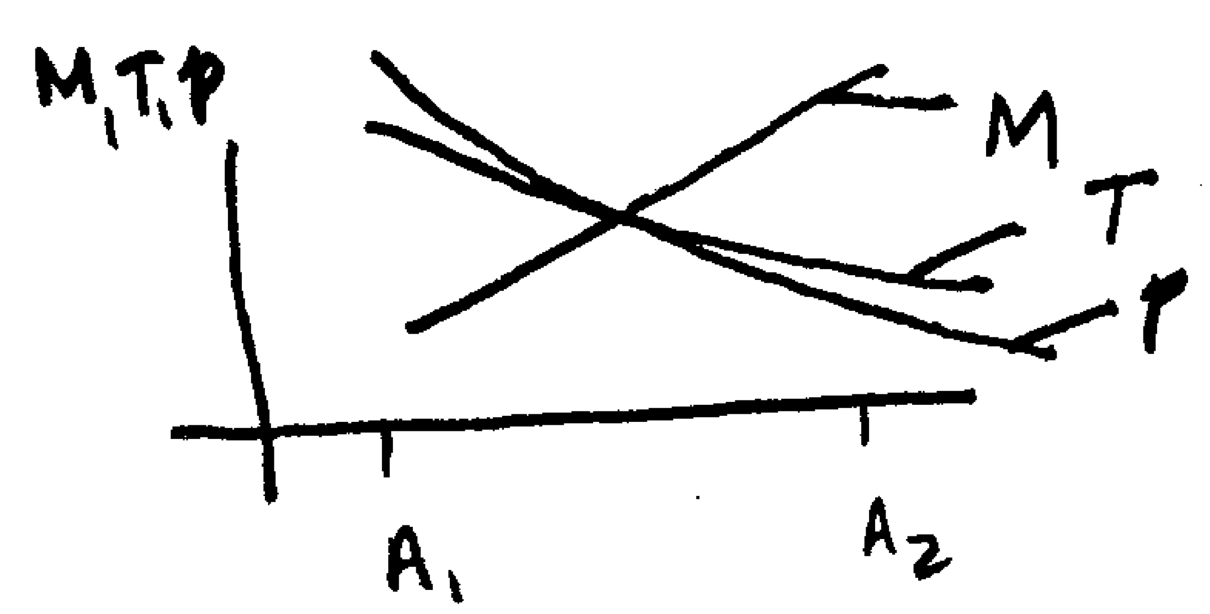
$$p_2 = \left(\frac{p_2}{p_{0_2}}\right)\left(\frac{p_{02}}{p_1}\right)p_1 = \left(\frac{p_2}{p_{0_1}}\right)\left(\frac{p_{01}}{p_1}\right)p_1 = (0.019)\left(\frac{1}{0.056}\right)(30)\,\text{psia}$$

(a) $p_2 = 10.18$ psia

$$T_2 = \left(\frac{T_2}{T_{02}}\right)\left(\frac{T_{02}}{T_1}\right)T_1 = \left(\frac{T_2}{T_{0_1}}\right)\left(\frac{T_{0_1}}{T_1}\right)T_1 = (0.32)\left(\frac{1}{0.44}\right)(525)°R$$

(b) $T_2 = 381.82°R$





NAME: SCHOOL SOLUTION

UNIFIED ENGINEERING QUIZ Q4F
Spring 1998
04/02/98

Problem II

A normal shock wave occurs at an incident Mach number of 1.2 in an ideal gas with γ = 1.3. **Compute** (a) M_2, (b) ρ_2/ρ_1 , (c) T_2/T_1 , (d) p_2/p_1 using the exact shock relations. **Draw** a diagram of the flow. **Repeat** the above calculations for a value of γ = 1.67. **Compare** your results for the two different values of γ and **discuss** the influence of γ on changes in variables across normal shock waves.

NORMAL SHOCK WAVE EXACT SOLUTIONS:

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M_1^2 - 1} \quad ; \quad \frac{\rho_2}{\rho_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{-1}$$

$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2} \quad ; \quad \frac{p_2}{p_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}$$

WE NEED THE FOLLOWING PARAMETERS

γ	$\frac{2}{\gamma-1}$	$\frac{2\gamma}{\gamma-1}$	$\frac{\gamma-1}{2}$
1.3	6.67	8.67	0.15
1.67	2.99	4.99	0.34

HENCE, FOR $\gamma = 1.3$

$$M_2^2 = \frac{M_1^2 + 6.67}{8.67 M_1^2 - 1} = \frac{(1.2)^2 + 6.67}{8.67(1.2)^2 - 1} = \frac{8.11}{11.48} = 0.706$$

$$\underline{M_2 = 0.84}$$

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Problem II

NAME: SCHOOL SOLUTION

$$\frac{\rho_2}{\rho_1} = \frac{1+1.3(1.2)^2}{1+1.3(0.84)^2}\left[\frac{1+0.15(1.2)^2}{1+0.15(0.84)^2}\right]^{-1} = \frac{2.87}{1.92}\left[\frac{1.22}{1.11}\right]^{-1}$$

$$\frac{\rho_2}{\rho_1} = (1.49)(1.1)^{-1} = \underline{1.35}$$

$$\frac{T_2}{T_1} = \frac{1+0.15(1.2)^2}{1+0.15(0.84)^2} = \frac{1.22}{1.11} = \underline{1.1}$$

$$\frac{p_2}{p_1} = \frac{1+1.3(1.2)^2}{1+1.3(0.84)^2} = \frac{2.87}{1.92} = \underline{1.49}$$

NOW, FOR $\gamma = 1.67$

$$M_2^2 = \frac{M_1^2 + 2.99}{4.99 M_1^2 - 1} = \frac{(1.2)^2 + 2.99}{4.99(1.2)^2 - 1} = \frac{4.43}{6.19} = 0.72$$

$$\underline{M_2 = 0.85}$$

$$\frac{\rho_2}{\rho_1} = \frac{1+1.67(1.2)^2}{1+1.67(0.85)^2}\left[\frac{1+0.34(1.2)^2}{1+0.34(0.85)^2}\right]^{-1} = \frac{3.40}{2.20}\left[\frac{1.49}{1.24}\right]^{-1}$$

$$\frac{\rho_2}{\rho_1} = (1.55)(1.20)^{-1} = \underline{1.29}$$

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NAME: SCHOOL SOLUTION

$$\frac{T_2}{T_1} = \frac{1+0.34(1.2)^2}{1+0.34(0.85)^2} = \frac{1.49}{1.24} = \underline{1.20}$$

$$\frac{p_2}{p_1} = \frac{1+1.67(1.2)^2}{1+1.67(0.85)^2} = \frac{3.40}{2.20} = \underline{1.55}$$

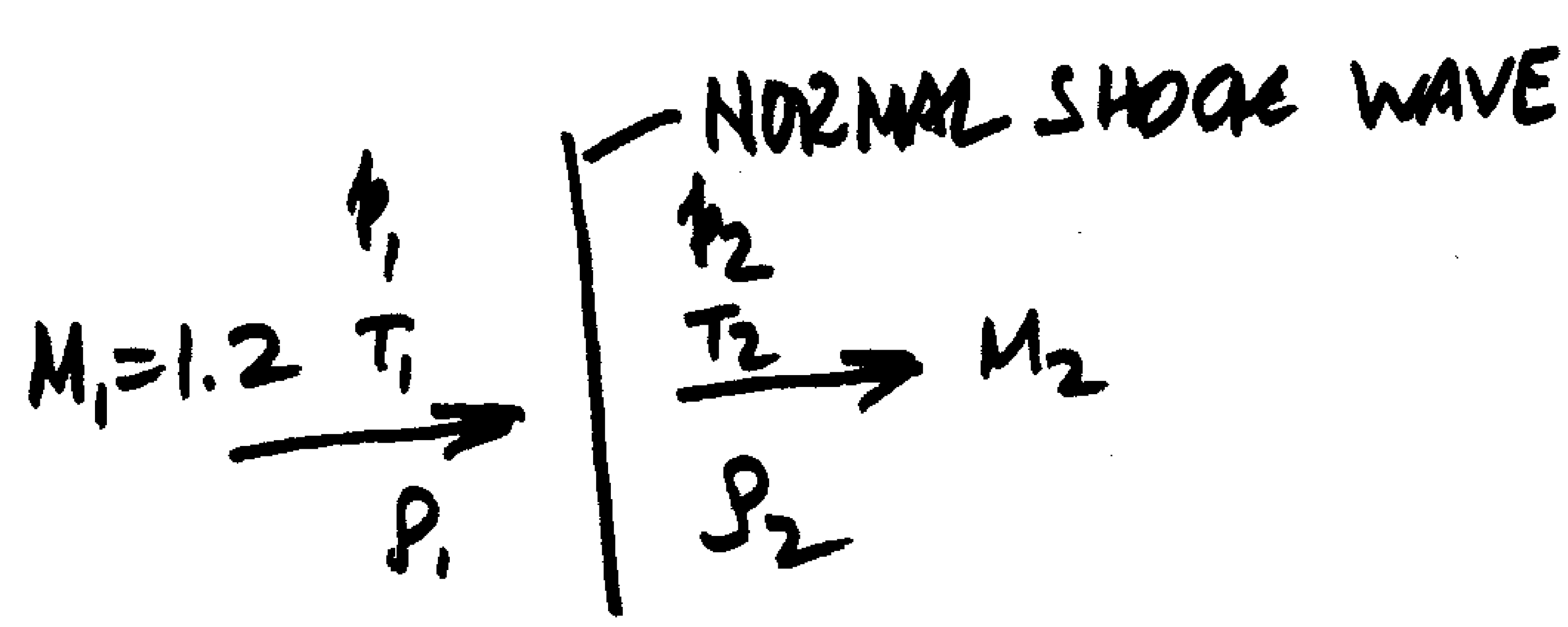
COMPARISON

$\boxed{M_1 = 1.2}$

γ	M_2	ρ_2/ρ_1	T_2/T_1	p_2/p_1
1.3	0.84	1.35	1.1	1.49
1.67	0.85	1.29	1.20	1.55

γ IS A MEASURE OF MOLECULAR COMPLEXITY $\rightarrow$ NUMBER OF DEGREES OF FREEDOM (TO FIRST ORDER). SMALL VALUES OF γ IMPLY GREATER MOLECULAR COMPLEXITY. DATA SHOWS THAT IT IS MORE DIFFICULT TO COMPRESS COMPLEX (MOLECULAR) GASES.

DIAGRAM



NAME: SCHOOL SOLUTION

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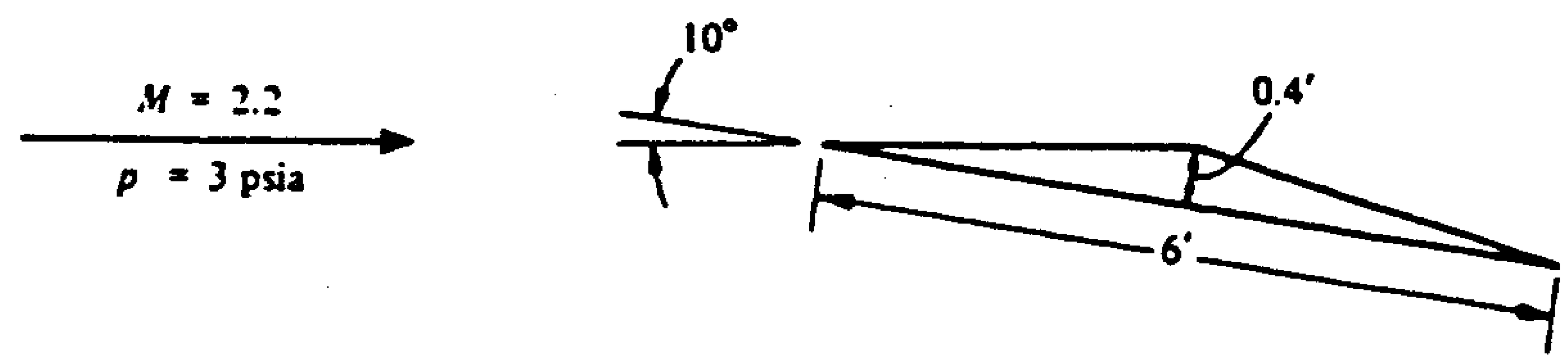
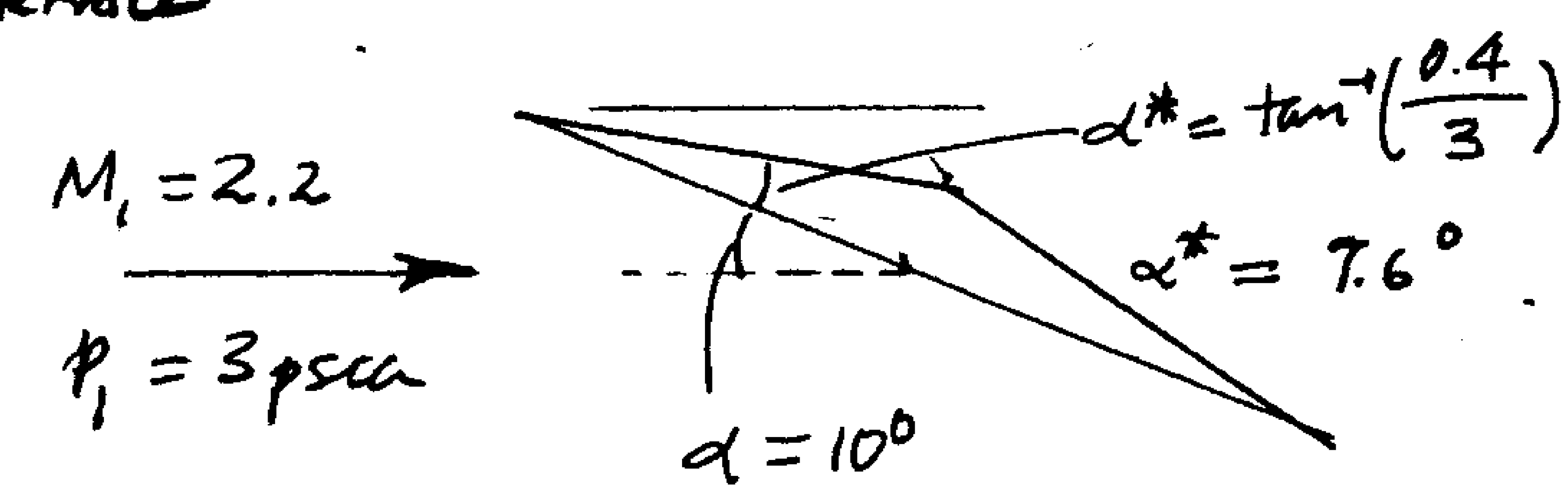
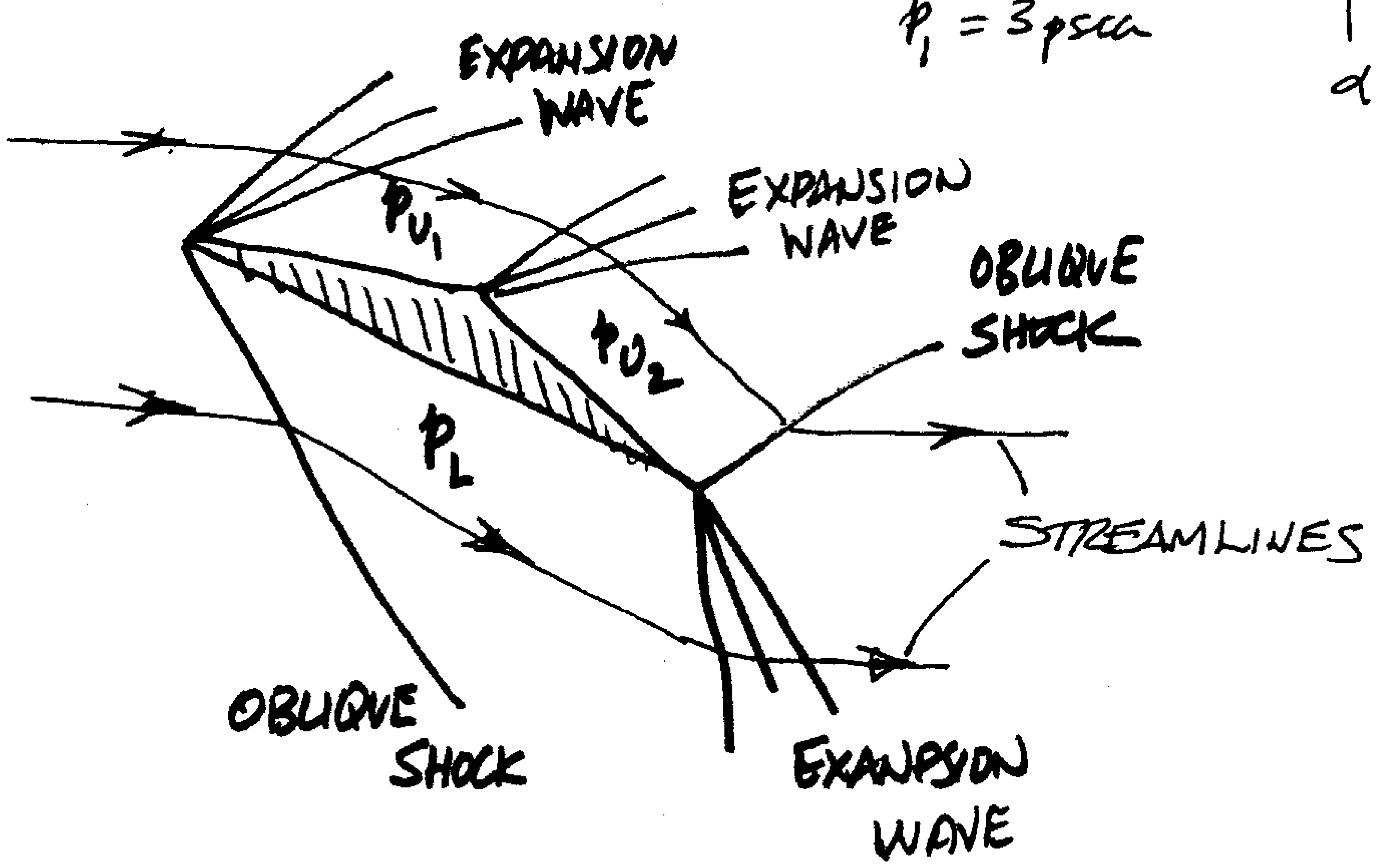
Problem III

A two-dimensional airfoil is shown below. The various dimensions and Mach numbers are shown. The airfoil is flying at an angle of attack of 10°. **Find**, per foot of span width, the (a) lift and (b) drag of this airfoil. **Draw** a picture of the flow over the airfoil showing the various types of waves formed as the flow interacts with the airfoil.



NOTE THAT $\alpha^* < \alpha$. HENCE WE WILL HAVE AN EXPANSION OVER LE OF TOP SURFACE AND COMPRESSION (OBLIQUE SHOCK) OVER LE OF BOTTOM SURFACE.





FIRST, COMPUTE p_L:

$$M_1 = 2.2, \ \theta = 10°, \ \gamma = 1.4$$

K & C TABLE 12, page. 228: $\beta = 36°$

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$$M_{1_n} = M_1 \sin\beta = (2.2)(\sin 36°) = (2.2)(0.59) = \underline{1.29}$$

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_{1_n}^2 - \frac{\gamma-1}{\gamma+1} = \frac{(2)(1.4)}{1.4+1.0}(1.29)^2 - \frac{1.4-1.0}{1.4+1.0}$$

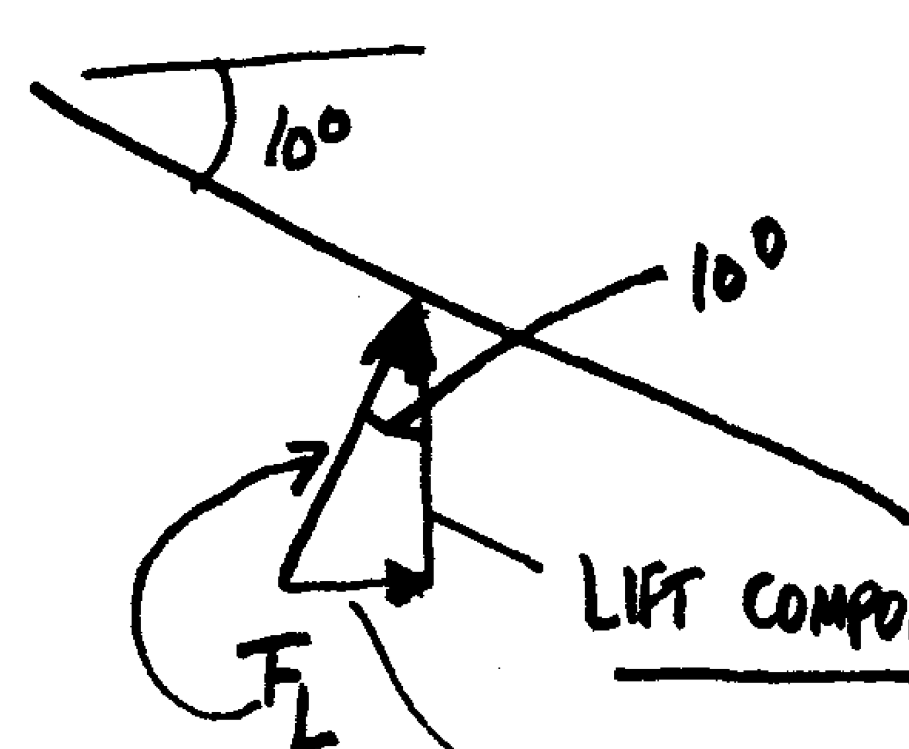
$$\frac{p_2}{p_1} = (1.17)(1.66) - 0.17 = \underline{1.77}$$

$$\therefore\ p_2 = p_L = (p_1)(1.77) = (3)(1.77)\,\text{psia} = \underline{5.32\,\text{psia}}$$

NET FORCE ACTING ON BOTTOM SURFACE:

$$F_L = (p_L)(c)(1) = (5.32)(6')(12)(1) = 382.8\ \frac{\text{lbs}}{\text{in}}$$

$$\underline{F_L} = 382.8\ \frac{\text{lb}}{\text{in}^2} = \underline{4593.54}\ \frac{\text{lb}}{\text{ft}}$$



LIFT COMPONENT $= F_L \cos 10° = 4523.75\ \frac{\text{lbs}}{\text{ft}}$

DRAG COMPONENT $= F_L \sin 10°\ \ 797.66\ \frac{\text{lbs}}{\text{ft}}$

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Problem III

NAME: SCHOOL SOLUTION

COMPUTE p_{U_1}

$$\nu(M_1 = 2.2) = 31.73^\circ \quad ; \quad \Delta\theta_1 = \alpha - \alpha^* = 10^\circ - 7.6^\circ = 2.4^\circ$$

$$\therefore \nu_2 = 31.73 + 2.4 = 34.13^\circ \longrightarrow M_{U_1} \cong 2.3$$

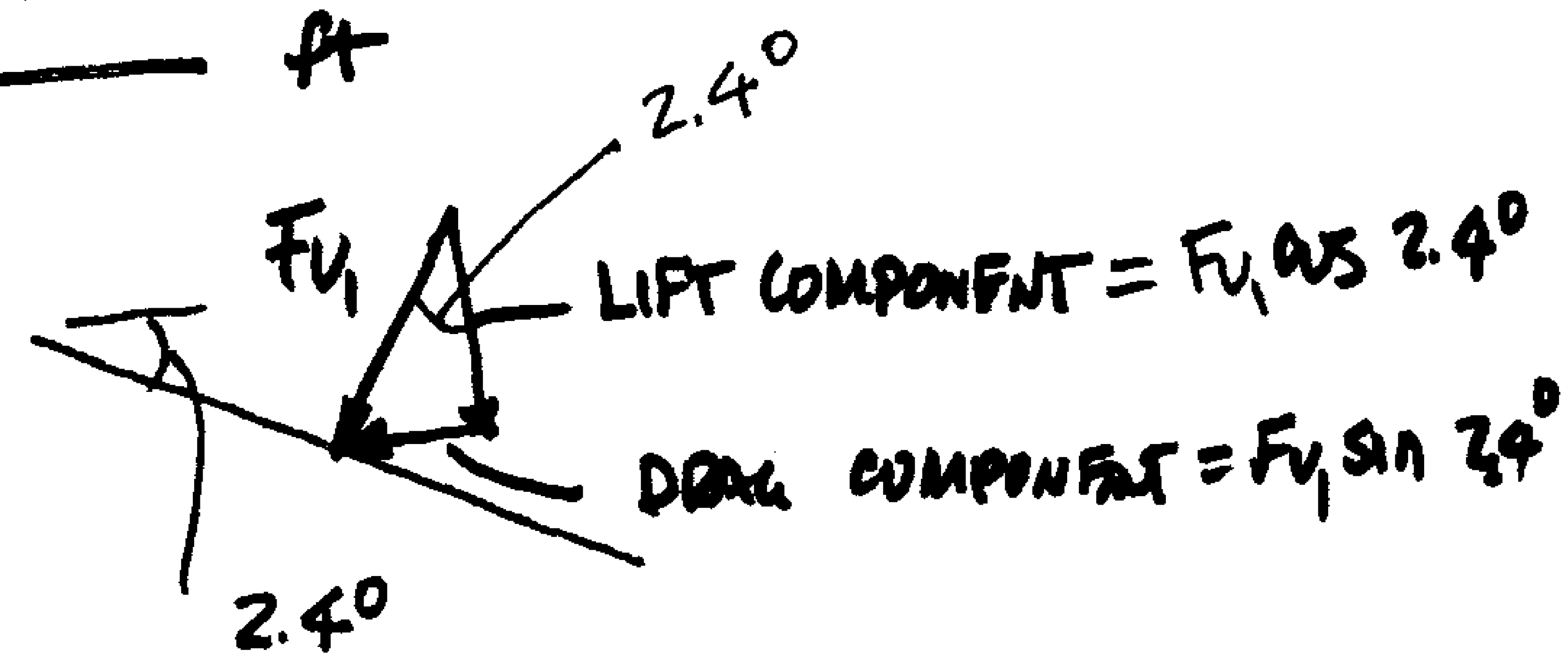
$$p_{u_1} = \left(\frac{p_{u_1}}{p_{0_{U_1}}}\right)\left(\frac{p_{0_1}}{p_1}\right)p_1 = \left(\frac{p_{u_1}}{p_{0_1}}\right)\left(\frac{p_{0_1}}{p_1}\right)p_1 = (0.08)\left(\frac{1}{0.094}\right)(3) \text{ psia}$$

$$p_{u_1} = 2.55 \text{ psia}$$

NET FORCE ACTING ON U_1-SURFACE:

$$F_{U_1} = (p_{u_1})\left(\frac{0.4}{\sin 7.6^\circ}\right)(12)(1) = (2.55)(.4)(12)/0.132 = 92.73 \ \frac{\text{lbs}}{\text{in}}$$

$$\underline{F_{U_1}} = 92.73 \ \frac{\text{lbs}}{\text{in}} = \underline{1,112.76} \ \frac{\text{lbs}}{\text{ft}}$$



LIFT COMPONENT

$$= F_{U_1}\cos 2.4^\circ$$
$$= (1112.8)(0.999)$$
$$= 1111.8 \ \frac{\text{lbs}}{\text{ft}}$$

DRAG COMPONENT

$$= F_{U_1}\sin 2.4^\circ$$
$$= (1112.8)(0.042)$$
$$= 46.6 \ \frac{\text{lbs}}{\text{ft}}$$

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Problem III

NAME: SCHOOL SOLUTION

COMPUTE p_{v_2}

$\nu_2 = 34.13$; $M_{v_1} = 2.3$; $\Delta\theta_2 = 180^\circ - 164.8^\circ = 15.2^\circ$

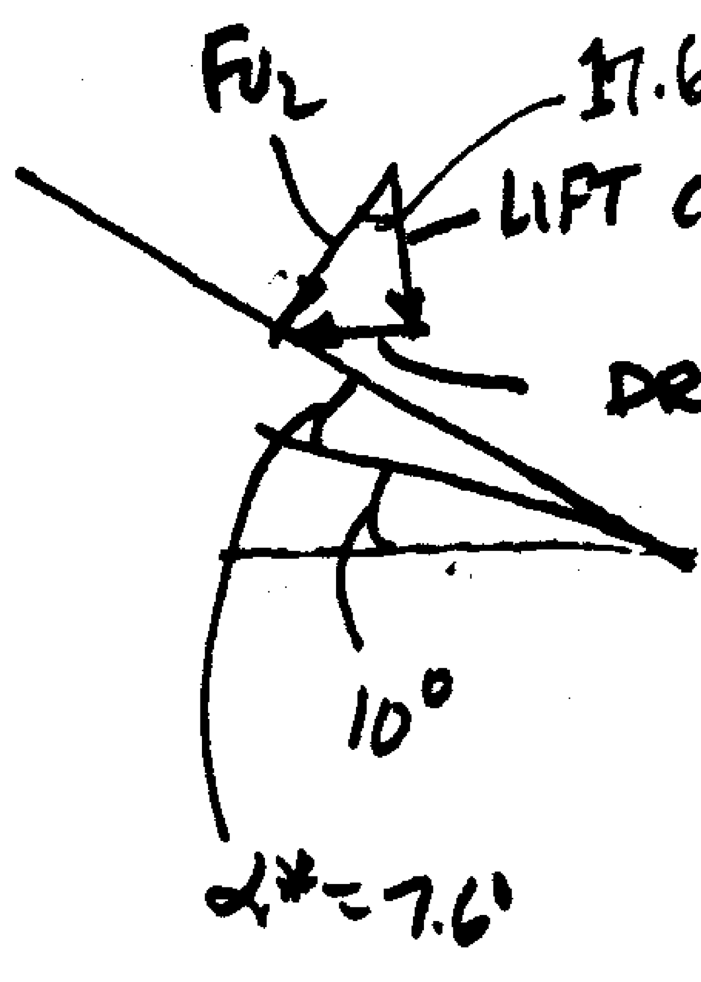
$\therefore \nu_3 = 34.13 + 15.2^\circ = 49.3^\circ \Rightarrow M_{v_2} = 2.98$

$$p_{v_2} = \left(\frac{p_{v_2}}{p_{0_{v_2}}}\right)\left(\frac{p_{0_1}}{p_{v_1}}\right) p_{v_1} = (0.028)\left(\frac{1}{0.08}\right)(2.55) = 0.89 \text{ psia}$$

NET FORCE ACTING ON U_2-SURFACE:

$$F_{U_2} = (p_{v_2})\left(\frac{0.4}{\sin 7.6^\circ}\right)(12)(1) = (0.89)(0.4)(12)/0.132$$

$$\underline{F_{U_2}} = 32.36 \frac{\text{lbs}}{\text{in}} = \underline{388.36} \frac{\text{lbs}}{\text{ft}}$$



LIFT COMPONENT $= F_{U_2} \cos 17.6^\circ = (388.36)(0.953) = 370.2 \frac{\text{lbs}}{\text{ft}}$

DRAG COMPONENT $= F_{U_2} \sin 17.6^\circ = (388.36)(0.30) = 117.4 \frac{\text{lbs}}{\text{ft}}$

NET LIFT: $(4523.75 - 1111.8 - 370.2) \frac{\text{lbs}}{\text{ft}} = \underline{3041.75} \frac{\text{lbs}}{\text{ft}}$

NET DRAG: $(797.66 - 46.6 - 117.4) \frac{\text{lbs}}{\text{ft}} = \underline{633.66} \frac{\text{lbs}}{\text{ft}}$

NAME:	SCHOOL SOLUTIONS

UNIFIED ENGINEERING
Spring Semester 1999
QUIZ Q4F
April 1, 1999
10:05 AM - 10:55 AM
37-212

RESOURCES ALLOWED

- Class notes
- School solutions
- Calculator
- Kuethe and Chow

GENERAL REQUIREMENTS

- Show the logical path of your work.
- Show all of your work, including intermediate steps.
- State clearly all assumptions invoked in each problem solution.
- Underscore your answers.
- Put your name on each page of the quiz.

QUIZ SCORING

PROBLEM I 20%	
PROBLEM II 25%	
PROBLEM III 55%	
TOTAL 100%	

NAME:

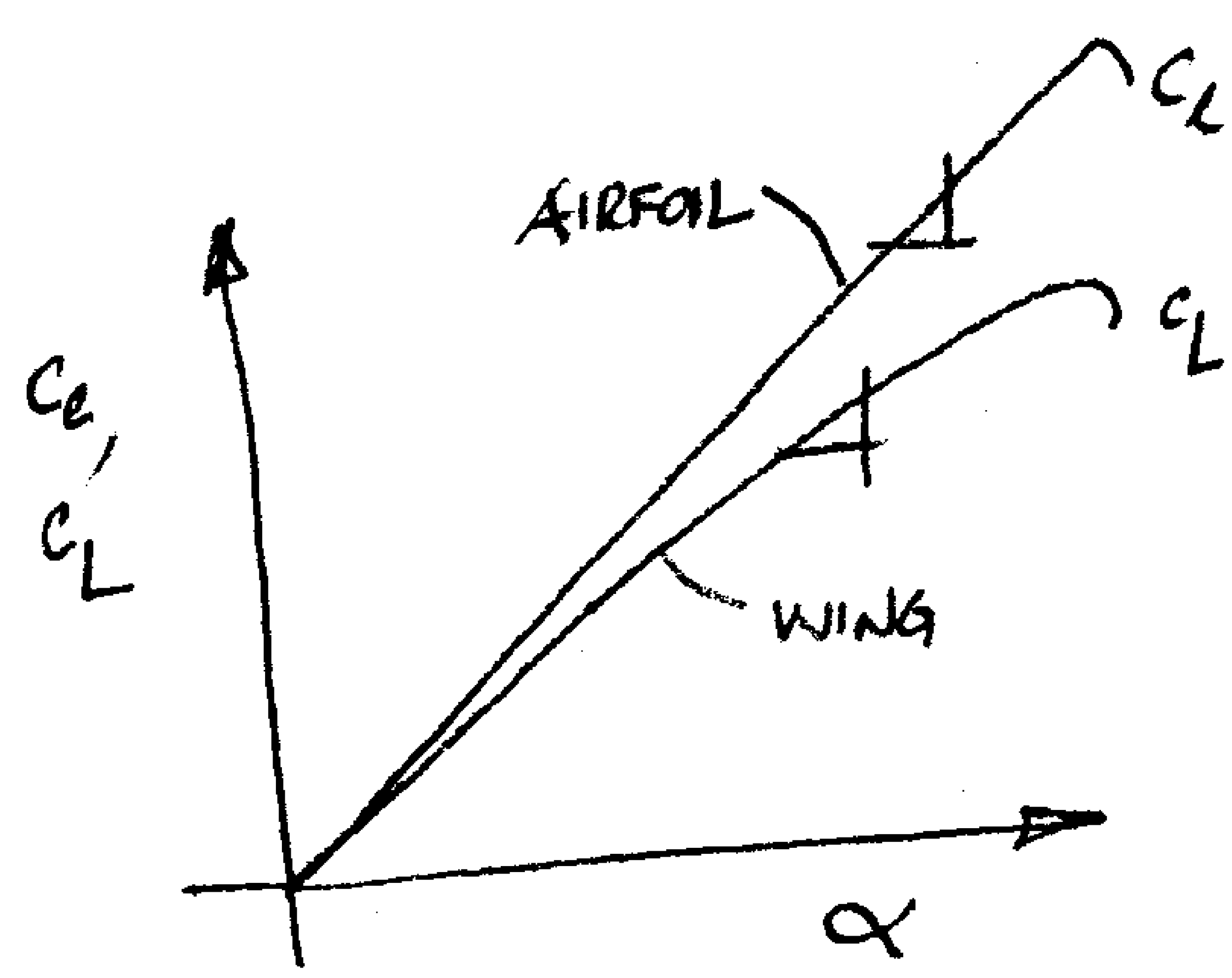
UNIFIED ENGINEERING QUIZ Q4F
Spring Semester 1999
04/01/99

Problem I

Consider an aircraft with a NACA 0009 airfoil, $e = 0.95$, $C_{D_0} = 0.01$, and AR = 10. If the aircraft is flying at 5-deg angle of attack, calculate C_L and C_D:

e = the Oswald span efficiency factor

ASSUMPTIONS:
SEA LEVEL FLIGHT
AIRFOIL AERODYNAMIC PROPERTIES ARE NOT AFFECTED BY REYNOLDS NUMBER (FLOW SPEED IS NOT SPECIFIED).
AIRFOIL DOES NOT HAVE A FLAP.
OR FLAP DEFLECTION IS ZERO



DATA FROM NACA 0009 C_ℓ VS α PLOT

-5- AT $\alpha = 5°$, $C_\ell = 0.54$

$\alpha_{L=0} = 0$

$$C_{L_\alpha} = \frac{C_{\ell_\alpha}}{1 + \dfrac{57.3\, C_{\ell_\alpha}}{\pi e AR}}$$

-5-

$$C_{\ell_\alpha} = \frac{0.54 - 0}{5° - 0} = 0.108/\text{deg}$$

$$C_{L_\alpha} = \frac{0.108}{1 + \dfrac{(57.3)(0.108)}{(3.14)(0.95)(10)}} = \frac{0.108}{1 + 0.207} = 0.089/\text{deg}$$

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Problem I (continued)

$$C_L = C_{L_\alpha}(\alpha - \alpha_{L=0}) = C_{L_\alpha}\alpha$$

$$C_L = (0.089)(5) = 0.445$$

-5-

$$\boxed{C_L = 0.445}$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e AR}$$

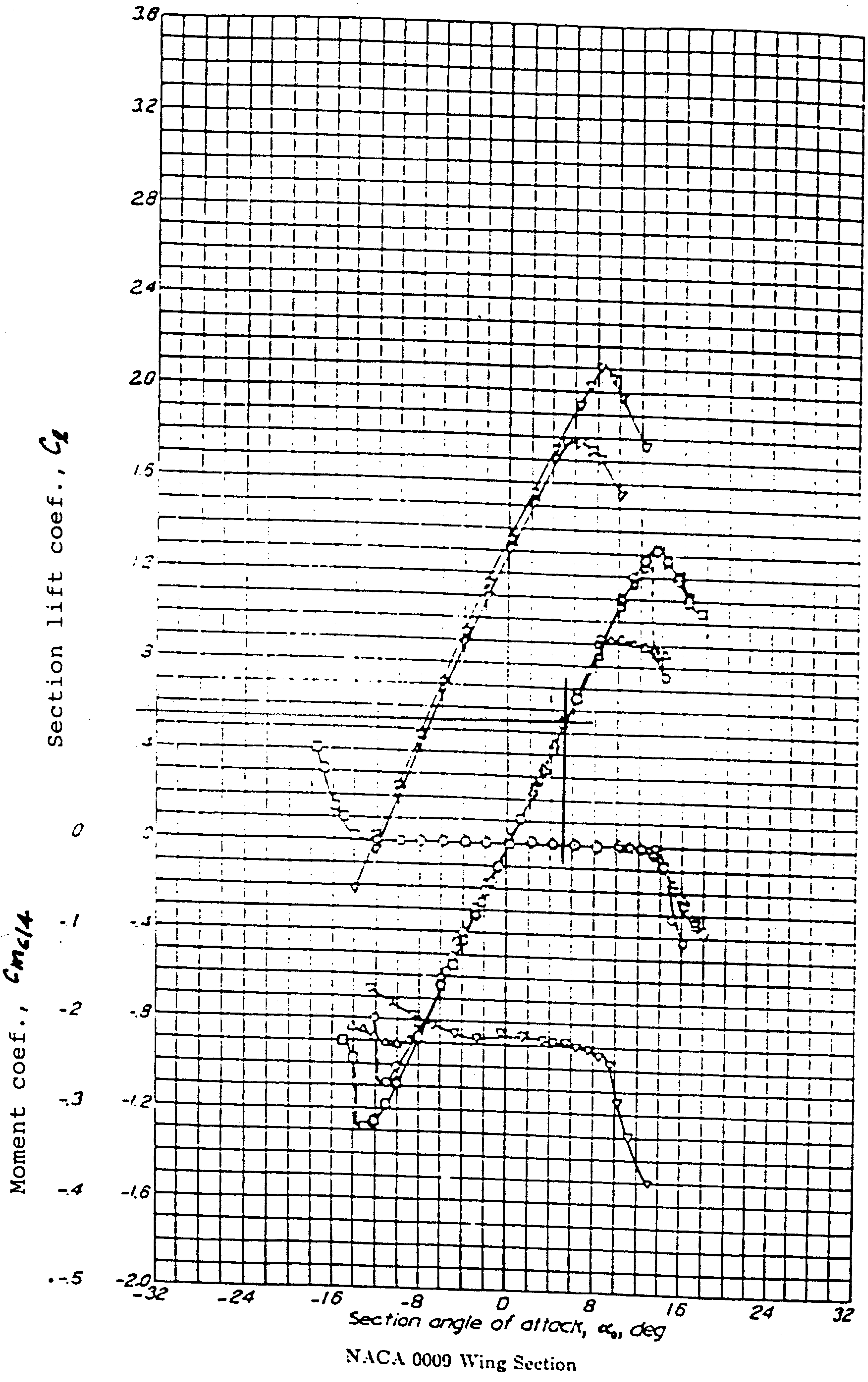
NOTE: $C_{D_0} = 0.01$ (GIVEN)

$$C_D = 0.01 + \frac{(0.445)^2}{(3.14)(0.95)(10)}$$

-5-

$$C_D = 0.01 + 6.638 \times 10^{-3}$$

$$\boxed{C_D = 0.01664}$$



NACA 0009 Wing Section

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Problem II

A nozzle is properly designed to pass air isentropically from a stagnation state of 200 psia, 100°F, to an exit pressure of 150 psia. The minimum area is known to be 0.5 sq. in. Find:

(a) The exit Mach number.

(b) The exit area.

(c) The mass flow rate.

Is this flow choked? Explain your answer.

Draw a sketch of the pressure distribution along the centerline of the nozzle.

GIVEN STAGNATION CONDITIONS: $p_0 = 200$ psia ; $T_0 = 100°F = 560°R$
GIVEN EXIT CONDITIONS: $p_e = 150$ psia
IDEAL GAS: AIR, $\gamma = 1.4$

$\underline{M_e}$

$$\frac{p_e}{p_0} = \frac{150}{200} = 0.75$$

-5-

USING TABLE 4, K&C:

$$\frac{p_e}{p_0} = 0.75 \Rightarrow M_e = 0.653$$

(a) $\boxed{M_e = 0.653}$

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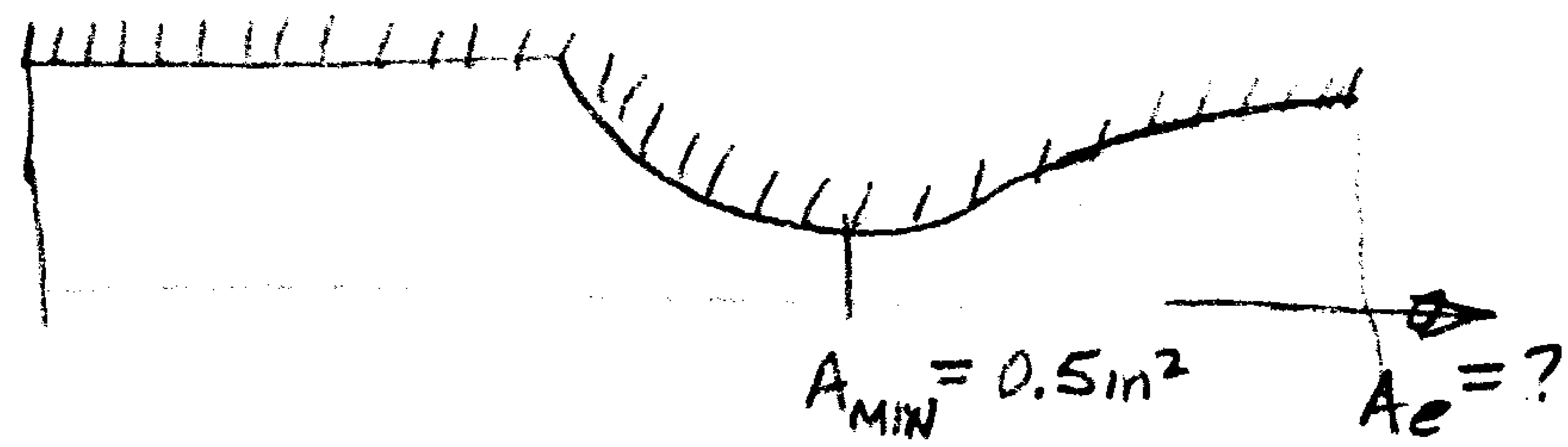
Problem II (continued)

$\underline{A_e}$



<u>ASSUME CHOKED FLOW.</u>

$$A_{MIN} = A^* = 0.5 in^2$$

USING TABLE 4, K+C:

$$\left(\frac{A_e}{A^*}\right)_{M_e = 0.653} = (0.883)^{-1} = 1.133$$

-10-

$$\therefore A_e = (0.5)(1.133)$$

(b) $\boxed{A_e = 0.567 \ in^2}$

$\underline{\dot{m}}$

$$\dot{m} = \rho_e A_e V_e = \rho_t A_t V_t$$

$$\frac{\dot{m}_c}{A_t} = \rho_t V_t = \frac{p_o}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}} \left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$= \frac{(200)(144)}{\sqrt{560}} \sqrt{\frac{1.4}{53.3}} \left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{2(1.4-1)}}$$

$$= \frac{(200)(144)}{\sqrt{560}} \left(\frac{1.4}{53.3}\right)^{1/2} (0.578)$$

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Problem II (continued)

$$\frac{\dot{m}_c}{A_t} = \frac{(200)(144)}{\sqrt{560}}(0.162)(0.578) = \frac{(200)(144)}{\sqrt{560}}(0.093)$$

$$\dot{m}_c = \frac{(200)(144)}{\sqrt{560}}(0.093)(0.5)\left(\frac{1}{144}\right)$$

-5-

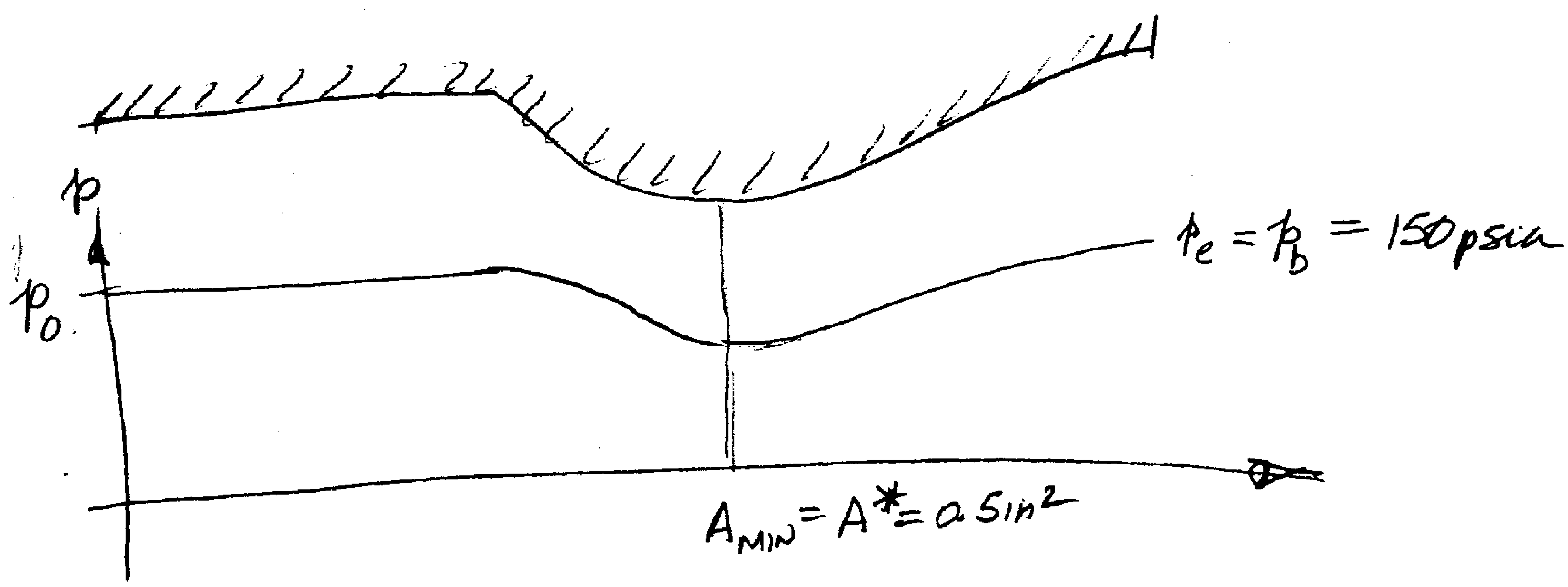
$$= \frac{(200)(0.093)(0.5)}{\sqrt{560}} = \frac{9.34}{\sqrt{560}} = \frac{9.34}{23.66}$$

$$\boxed{\dot{m}_c = 0.395 \text{ lbm/sec.}}$$

FLOW MUST BE CHOKED SINCE $\dot{m}$ IS NOT GIVEN. CANNOT EVALUATE A_e UNLESS $\dot{m}$ OR M_t IS SPECIFIED.

MASS IS CONSERVED, HENCE $\dot{m}_c = \dot{m}_t =$ constant.

-5-



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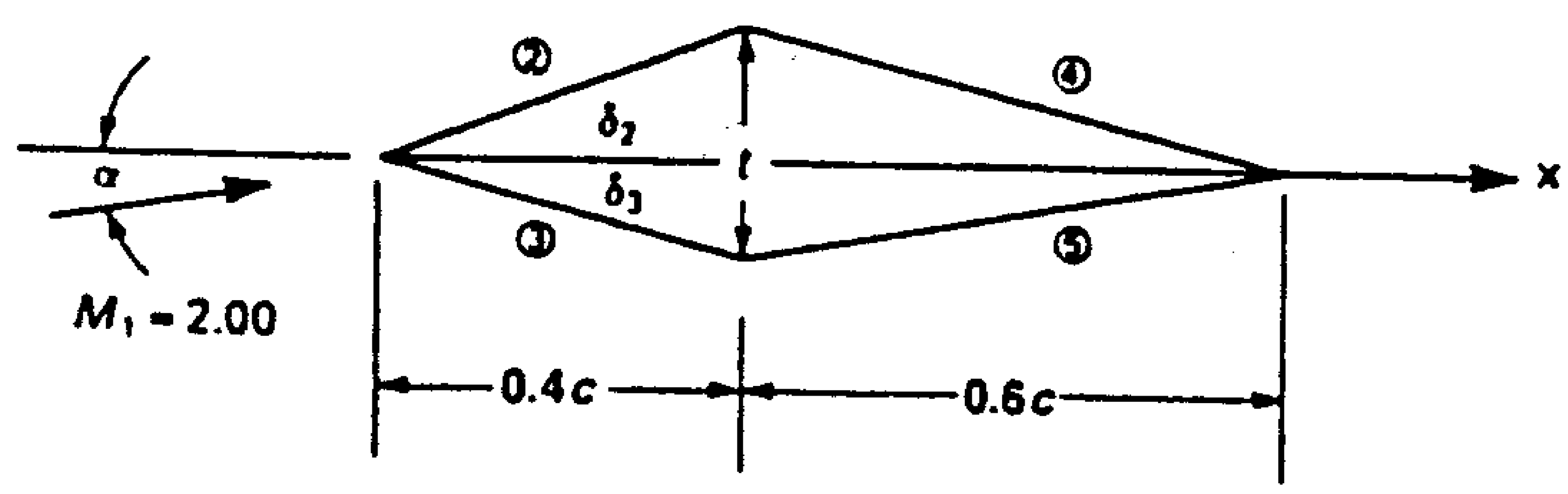
Problem III

Consider the "cambered," diamond-shaped airfoil exposed to a Mach 2.00 air stream in a wind tunnel. For the wind tunnel, p_{o_1} = 125 psia, T_o = 650°R. The air foil is such that $\delta_2 = 8°$, $\delta_3 = 2°$, the maximum thickness t equals to 0.07c and is located at x = 0.40c. The angle of attack is 10°. Using the shock - expansion method, calculate (a) the pressures in regions 2 through 5, (b) the net lift on the airfoil, and (c) the lift coefficient.

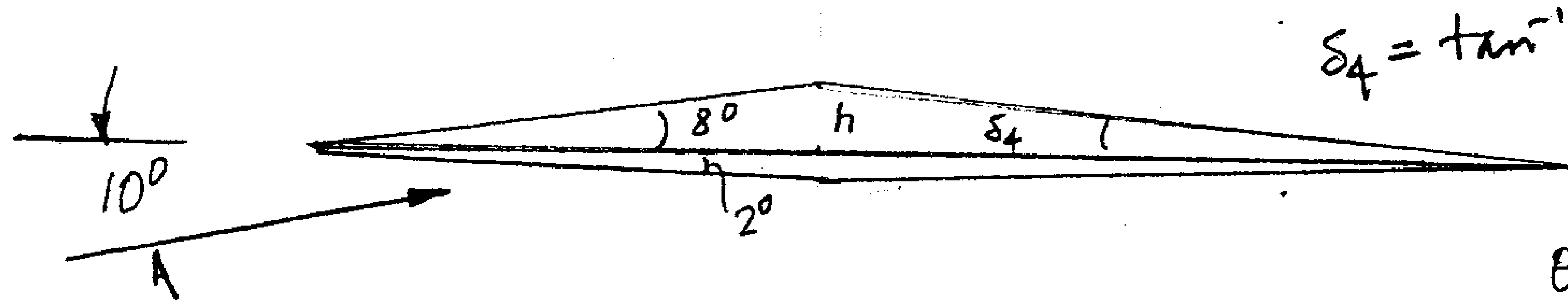


$$\tan 8° = \frac{h}{0.4c}$$

$$h = 0.4\,c \tan 8° = 0.056c$$

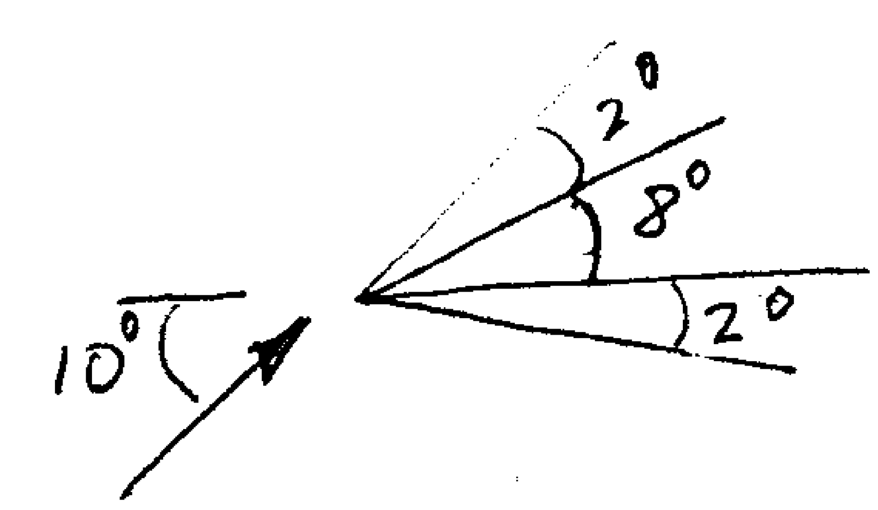
$$\delta_4 = \tan^{-1}\left(\frac{h}{0.6c}\right) = \tan^{-1}\left(\frac{0.056}{0.6}\right)$$

$$\delta_4 = 5.33°$$



$$\theta_4 = 180° - (180° - 90° - 8°) - (180° - 90° - 5.33°)$$

$$\theta_4 = 13.33°$$



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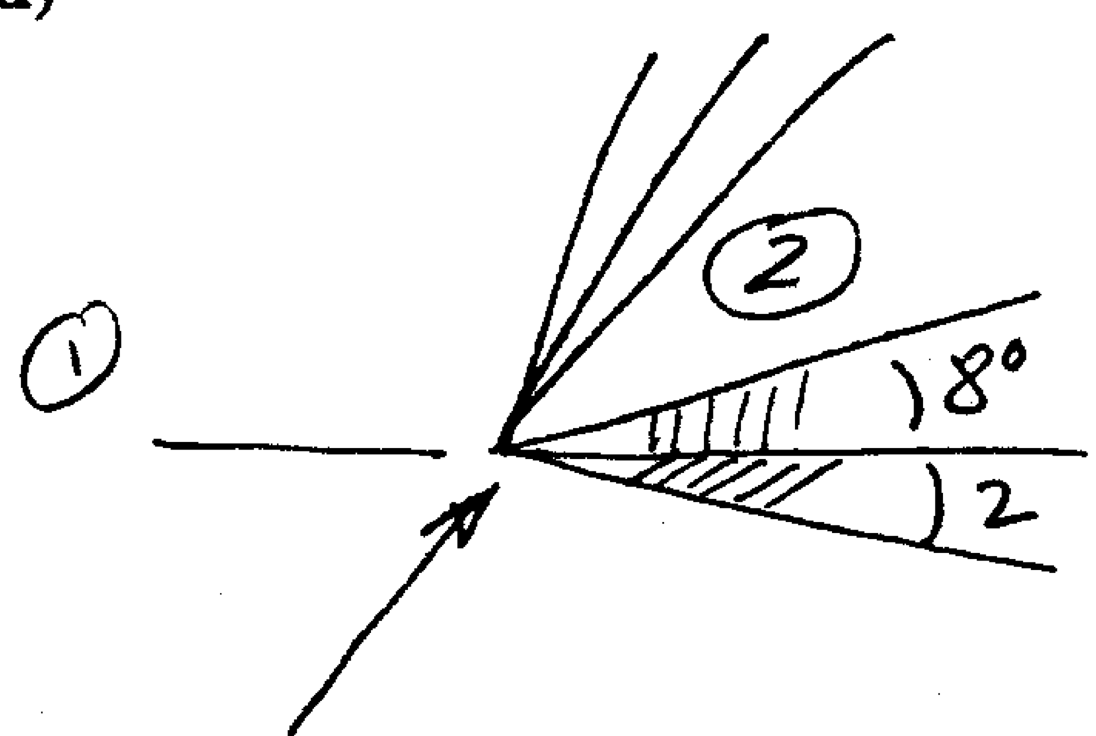
Problem III (continued)

REGIONS ② & ④

PRANDTL-MEYER FUNCTION, $\nu(M)$



– FLOW TURNS THROUGH EXPANSION ANGLE OF 2° ; FOLLOWED BY A TURN THROUGH 13.33° IN REGION ④

$\theta_1 = 0;\ M_1 = 2.0;\ \nu_1 = 26.38$

$\theta_2 = 2°;\ \nu_2 = 26.38 + 2 = 28.38;\ \therefore M_2 = 2.075$

$\theta_4 = 13.33°;\ \nu_4 = 28.38 + 13.33° = 41.71°;\ \therefore M_4 = 2.615$

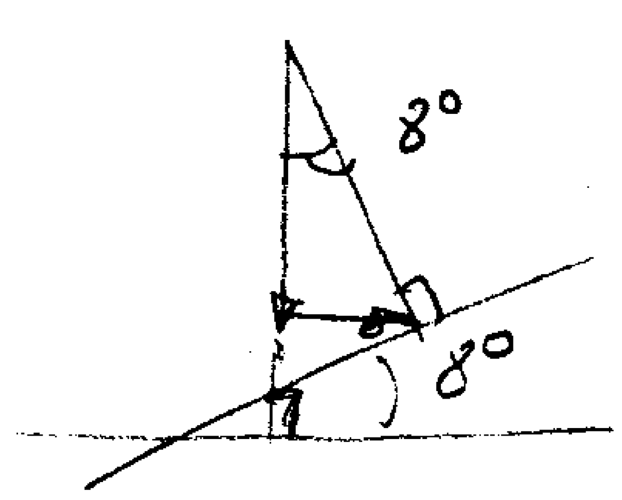
$$p_2 = p_1\left(\frac{p_2}{p_{02}}\right)\left(\frac{p_{01}}{p_1}\right) = p_{01}\left(1+\frac{\gamma-1}{2}M_1^2\right)^{-\gamma/(\gamma-1)}\left(\frac{p_2}{p_{02}}\right)\left(\frac{p_{01}}{p_1}\right)$$

$$= (125)(0.1278)(0.1137)(0.1278)^{-1}\ \text{psia}$$

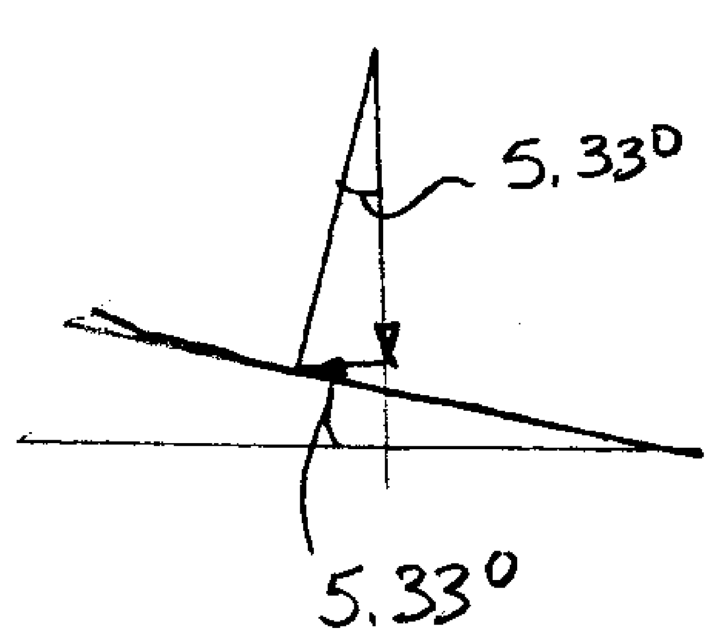
$$p_2 = \underline{14.2\ \text{psia}}$$

– 20 – $$p_4 = p_2\left(\frac{p_4}{p_{04}}\right)\left(\frac{p_{02}}{p_2}\right) = 14.2\,(0.0489)(0.1137)^{-1}$$

$$= \underline{6.11\ \text{psia.}}$$



$$L_2 = p_2 \cos 8°\left(\frac{0.4c}{\cos 8°}\right) = 0.4c\,p_2 = 5.68c\ \text{pia}$$



$$L_4 = p_4 \cos 5.33°\left(\frac{0.6c}{\cos 5.33°}\right) = 0.6c\,p_4 = 3.67c\ \text{pia}$$

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Problem III (continued)

REGION (3)

EFFECTIVE DEFLECTION ANGLE: $\theta_{d_3} = 12^\circ$; $M_1 = 2.00$; $\therefore M_3 = 1.58$ (WEAK SHOCK WAVE)

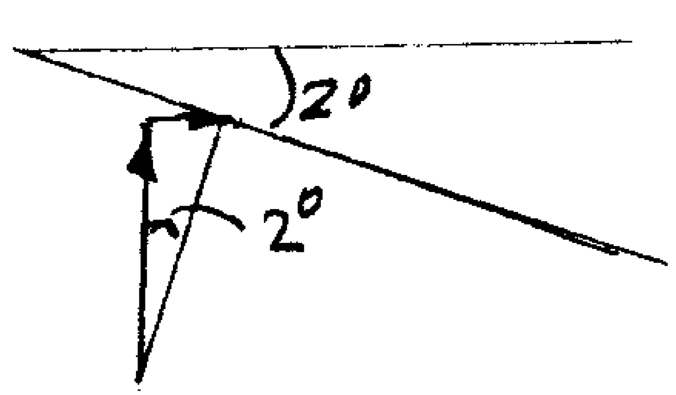
$\beta_3 = 41.5^\circ$

$\therefore M_{1n} = M_1 \sin\beta_3 = (2.00)\sin 41.5^\circ = (2.00)(0.6626) = 1.325$

$p_3 = p_1\left(1 + \frac{2\gamma}{\gamma+1}(M_{1n}^2 - 1)\right) = p_{0_1}\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{-\gamma}{(\gamma-1)}}\left(1+\frac{2(1.4)}{1.4+1}(1.325^2-1)\right)$

− 10 − $p_3 = (125)(0.1278)\left(1+\frac{2.8}{2.4}(1.756-1)\right) = (125)(0.1278)(1+(1.167)(0.756))$

$p_3 = (125)(0.1278)(1.882) = \underline{30.06 \text{ psia}}$



$L_3 = p_3 \cos 2^\circ\left(\frac{0.4c}{\cos 2^\circ}\right) = 0.4c\,p_3 = 12.02c$ pia

REGION (5)

FLOW EXPANDS ISENTROPICALLY FROM REGION (3) TO REGION (5).

$\tan 2^\circ = \frac{h'}{0.4c}$; $h' = 0.4c \tan 2^\circ = 0.014c$; $\delta_5 = \tan^{-1}\left(\frac{h'}{0.6c}\right) = \tan^{-1}\left(\frac{0.014}{0.6}\right)$

$\delta_5 = 1.34^\circ$

EXPANSION ANGLE: $\theta_5 = 180^\circ - (180^\circ - 90^\circ - 2^\circ) - (180^\circ - 90^\circ - 1.34^\circ) = 3.34^\circ$

$\theta_3' = 0$; $M_3 = 1.58$, $\nu_3 = 14.27^\circ$

− 10 − $\theta_5' = 3.34^\circ$; $\nu_5 = 14.27^\circ + 3.34^\circ = 17.61^\circ$; $M_5 = 1.695$

$p_5 = p_3\left(\frac{p_{0_3}}{p_3}\right)\left(\frac{p_5}{p_{0_5}}\right) = (30.06)(0.2423)^{-1}(0.2042) = \underline{25.33 \text{ psia}}$

$L_5 = p_5 \cos 1.34^\circ\left(\frac{0.6c}{\cos 1.34^\circ}\right) = 0.6c\,p_5 = 15.2c$ pia

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Problem III (continued)

NET LIFT: $L'/c = (L_3 + L_5) - (L_2 + L_4) = (12.02 + 15.2) - (5.68 + 3.67)$

$L/c = 17.87$ psic

– 10 —

$\underline{L = 17.87c \text{ pcia}}$; $\underline{L' = L\cos 10° = 17.6c \text{ pia}}$ (relative to flow direction)

LIFT COEFFICIENT: $C_L = \dfrac{L}{\frac{1}{2}\rho_1 V_1^2 c} = \dfrac{17.87c}{\frac{1}{2}\rho_1 V_1^2 c} = \dfrac{17.87}{\frac{1}{2}\rho_1 V_1^2}$

$\frac{1}{2}\rho_1 V_1^2 = \frac{\gamma}{2} p_1 M_1^2 = \frac{\gamma}{2} p_{0_1} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{-\gamma}{(\gamma-1)}} M_1^2$

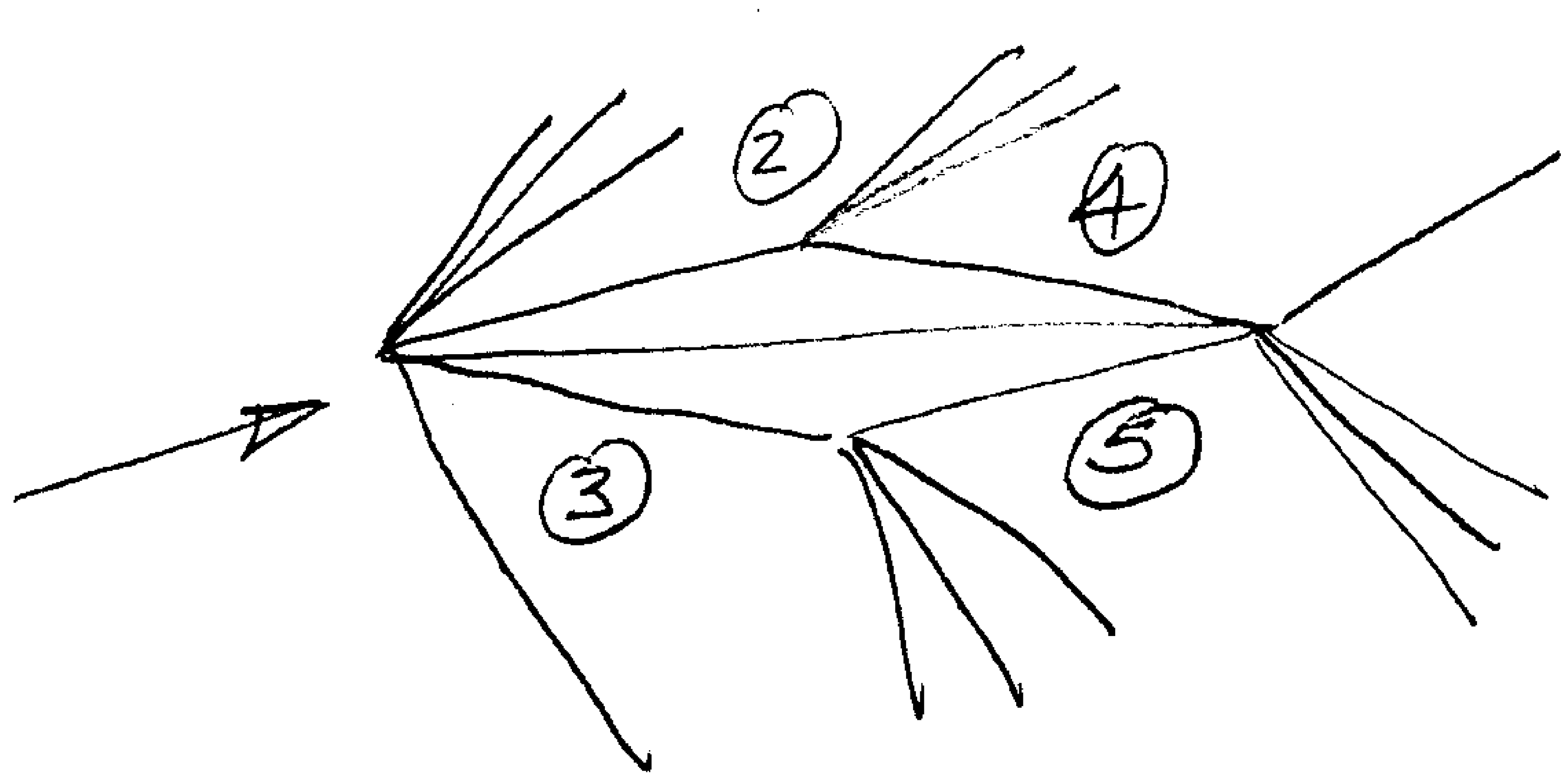
– 5 —

$= \frac{1.4}{2}(125)(0.1728)(2.0)^2 = (1.4)(125)(2.0)(0.1728)$

$= 60.48$ psia

$\therefore\ C_L = \dfrac{17.87}{60.48} = 0.295$

$\underline{C_L = 0.295}$; $\underline{C_L' = 0.291}$



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3:30 PM -- 4:30 PM
Walker

GENEERAL REQUIREMENTS
Put your name on each page of this quiz.
Show all your work, especially intermediate steps.
Show the logical path of your work.
Underscore your answers.

RESOURCES ALLOWED
Kuethe & Chow
Calculator
School Solutions to Homework Problems
Class Notes

QUIZ SCORING

PROBLEM I (60% max.)	
PROBLEM II (40% max.)	
TOTAL (100% max.)	

NAME: SCHOOL SOLUTIONS

UNIFIED ENGINEERING QUIZ Q9F
Spring Semester 2000
05/15/00

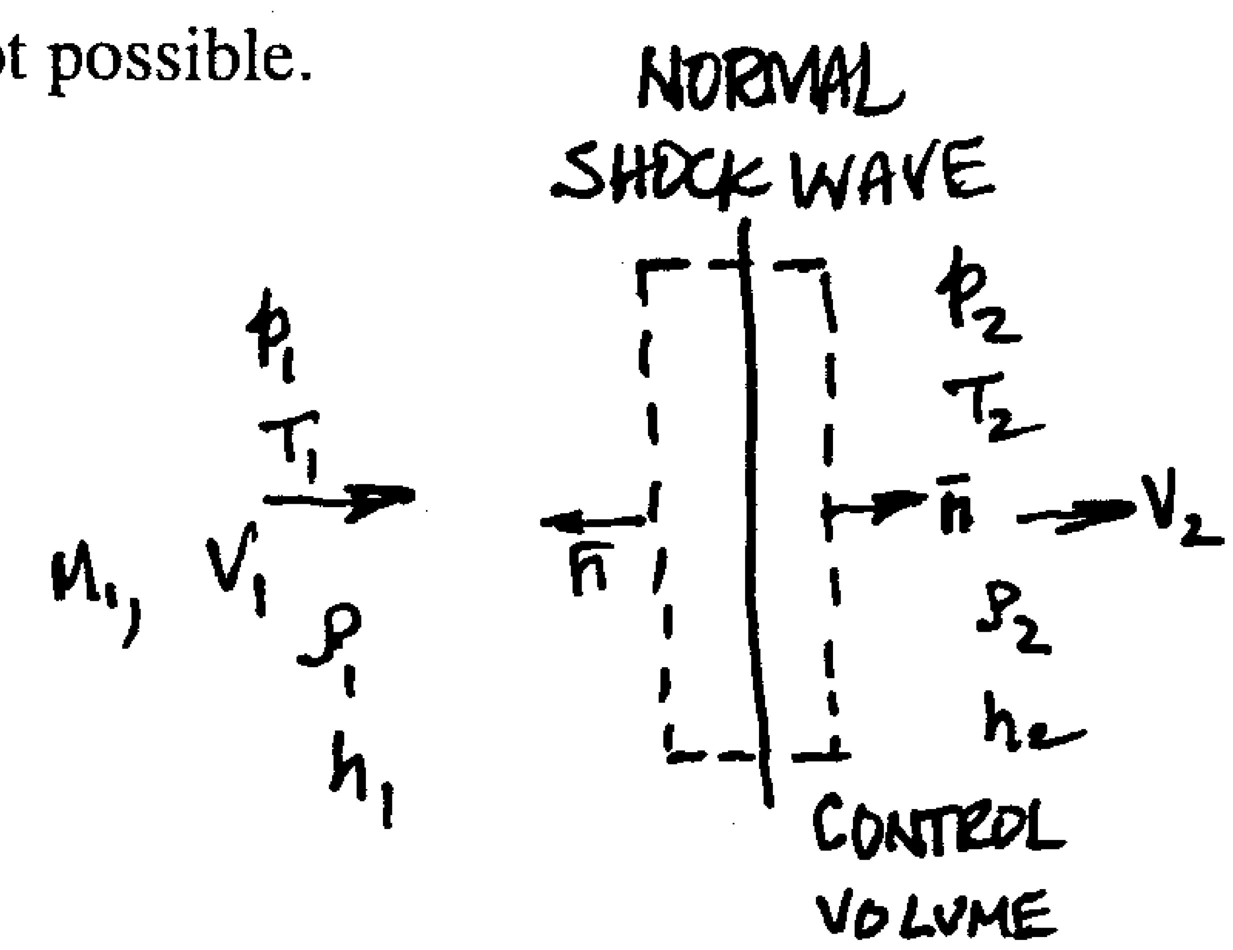
Problem I

(a) Starting with the conservation principles, fundamental laws of thermodynamics, and an equation of state, express the governing equations for flow through a normal shock wave. Please state all assumptions and approximations consistent with your set of governing equations.

(b) Is the stagnation temperature of the gas constant in the flow through a normal shock wave? Explain your answer.

(c) Is the stagnation pressure of the gas constant in the flow through a normal shock wave. Explain your answer.

(d) Show why an expansion normal shock wave is not possible.



(a) ASSUMPTIONS

- ONE-DIMENSIONAL FLOW, COMPRESSIBLE FLOW
- CONSTANT AREA
- ADIABATIC PROCESS
- STEADY FLOW
- IDEAL GAS W/ C_p = CONSTANT
- SHOCK THICKNESS IS NOT CONSIDERED
- INTERNAL SHOCK STRUCTURE IS NOT MODELED
- UPSTREAM AND DOWNSTREAM FLOWS ARE ISENTROPIC
- MASS, LINEAR MOMENTUM, AND ENERGY ARE CONSERVED IN FLOW THROUGH SHOCK WAVE.
- SECOND LAW OF THERMODYNAMICS IS VALID AND APPLICIABLE.

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Problem I

CONSERVATION OF MASS

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$A_1 = A_2$$

$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1}$$

EQUATION OF STATE

$$p_1 = \rho_1 R T_1$$

$$p_2 = \rho_2 R T_2$$

$$a^2 = \gamma R T$$

$$h = c_p T$$

CONSERVATION OF LINEAR MOMENTUM

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

CONSERVATION OF ENERGY

$$h_1 + \frac{1}{2} V_1^2 = h_2 + \frac{1}{2} V_2^2$$

SECOND LAW OF THERMODYNAMICS

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \int_{p_1}^{p_2} R \frac{dp}{p}$$

(b) $T_{0_1} = T_{0_2}$. THE STAGNATION TEMPERATURE IS CONSTANT IN THE FLOW OF A GAS THROUGH A NORMAL SHOCK WAVE. THE ENTIRE FLOW PROCESS IS ASSUMED TO BE ADIABATIC, i.e. NO THERMAL ENERGY IS ADDED TO THE FLOW.

(c) $p_{0_1} > p_{0_2}$. THE STAGNATION TEMPERATURE IS NOT CONSTANT IN THE FLOW OF A GAS THROUGH A NORMAL SHOCK WAVE. THE ABILITY OF THE GAS TO DO WORK HAS BEEN REDUCED BY THE SHOCK WAVE; THE RESULT IS A DECREASE IN STAGNATION TEMPERATURE ~~[illegible]~~ IN FLOWING THROUGH A NORMAL SHOCK WAVE.

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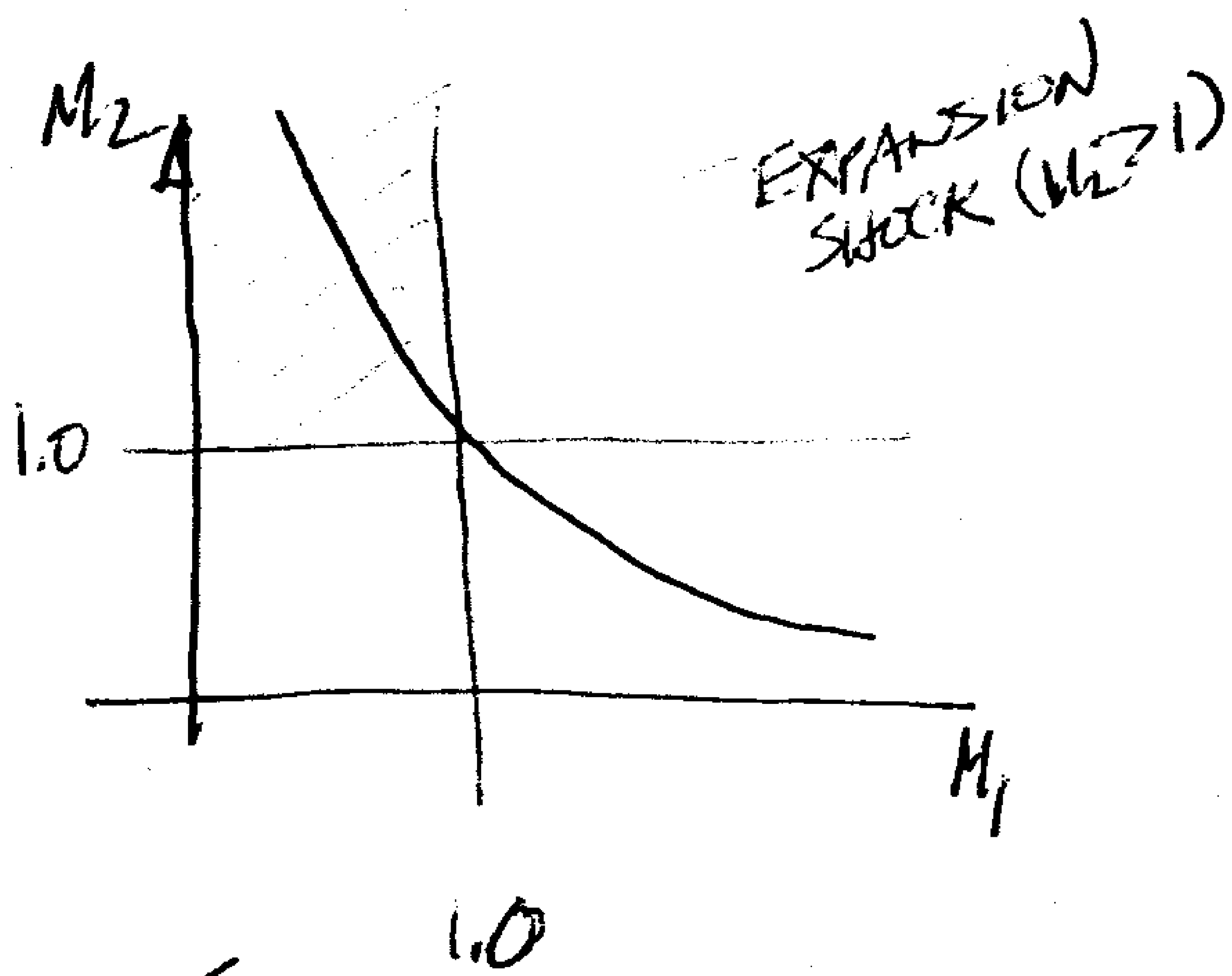
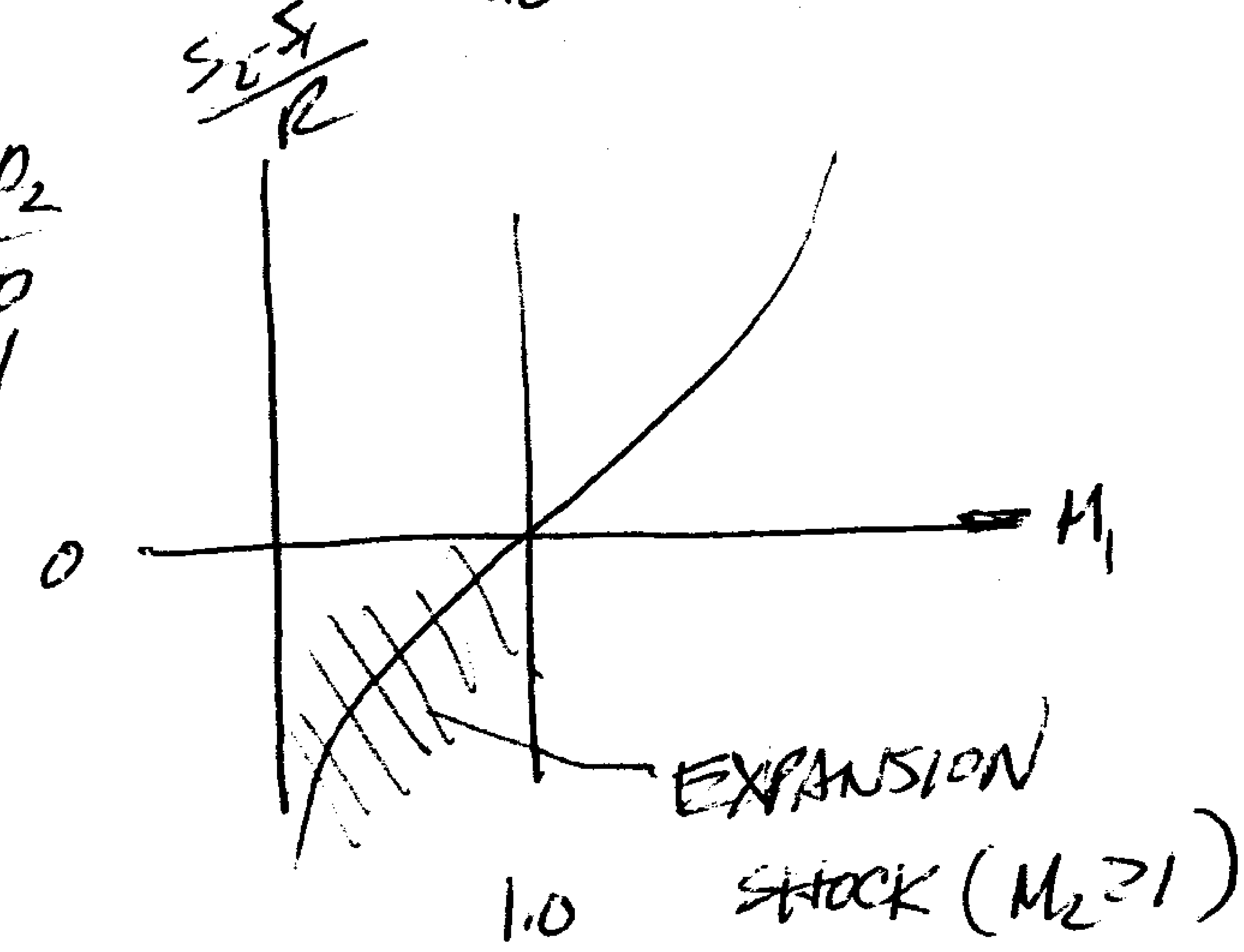
(d) EXPANSION NORMAL SHOCK WAVE IS NOT POSSIBLE. SUCH A NORMAL SHOCK WAVE WOULD CORRESPOND TO A VIOLATION OF THE SECOND LAW OF THERMODYNAMICS.

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M_1^2 - 1}$$



$$\frac{s_2-s_1}{R} = \frac{c_p}{R} \ln\frac{T_2}{T_1} - \ln\frac{p_2}{p_1}$$



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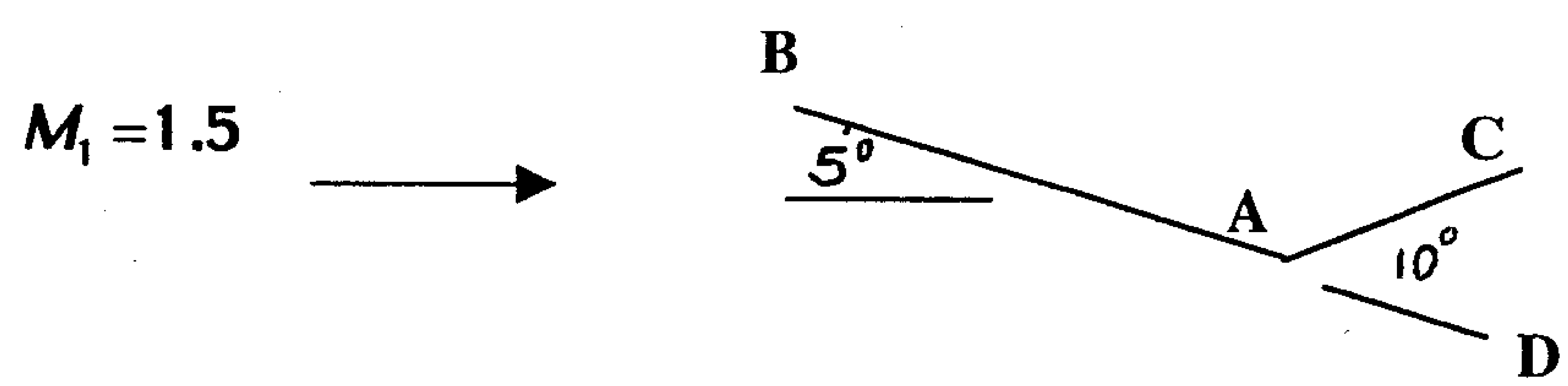
Problem II

(a) Compute the pressure on all surfaces of the flat plate airfoil in the given supersonic stream. AC is the airfoil flap.

(b) Draw the correct wave structure in the flow over the flat plate airfoil in the given supersonic stream.

(c) At what angle CAD will the wave structure not remain attached at point A on the upper surface?



The angle between the free stream velocity and the segment BA is 5°.
The angle CAD is 10°.

(a) ASSUME A FREE STREAM PRESSURE p_1 AND $\gamma = 1.4$.

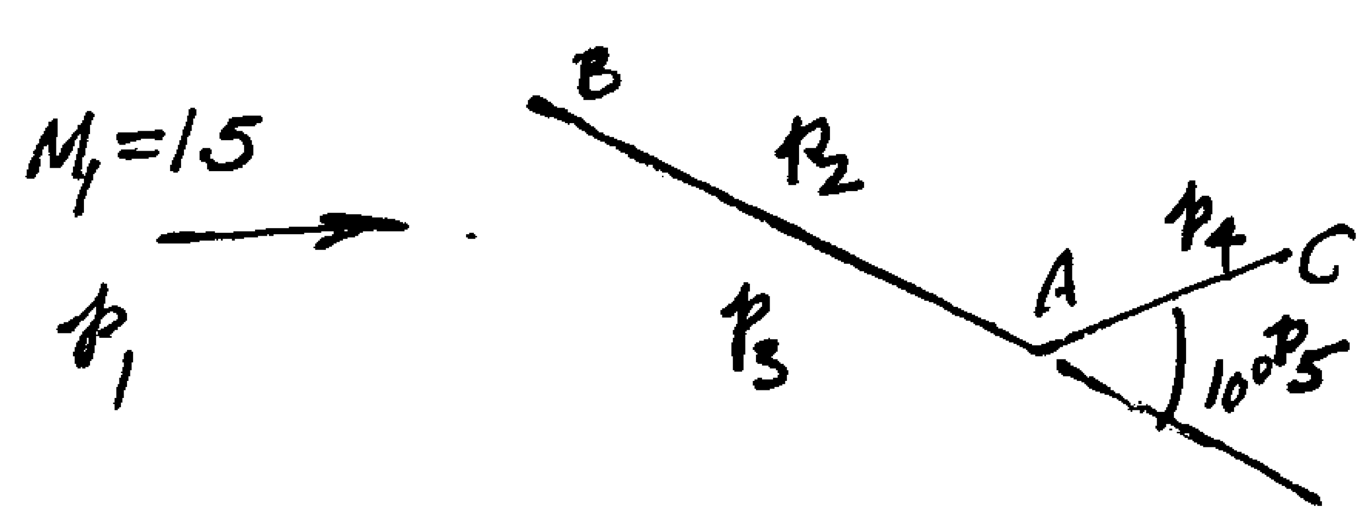
$M_1 = 1.5$

$\nu_1 = 11.91°$ (K&C, TABLE 5)

$\theta_1 = 0$

$\theta_2 = 5°$

$\nu_2 = \nu_1 + \theta_2 = 16.91° \rightarrow \underline{M_2 = 1.67}$



$$p_2 = p_1\left(\frac{p_{01}}{p_1}\right)\left(\frac{p_2}{p_{02}}\right) = p_1\left(\frac{p_0}{p}\right)_{M_1=1.5}\left(\frac{p}{p_0}\right)_{M=1.67} = p_1\left(\frac{0.2119}{0.2724}\right)$$

$$\underline{p_2 = 0.778\,p_1}$$

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$$\left.\begin{array}{l} M_2 = 1.67 \\ \theta_{WEDGE} = 10^\circ \end{array}\right\} \beta_2 = 48.5^\circ \ \{ K\&C, \text{ FIG. 10.11}$$

$$\underline{M_{2n}} = M_2 \sin\beta_2 = (1.67)\sin(48.5)^\circ = (1.67)(0.75) = \underline{1.25}$$

$$\left(\frac{p_4}{p_2}\right)_{M_{2n} = 1.25} = 1.656 \ \{ K\&C, \text{ TABLE 6}$$

$$\therefore \ p_4 = p_2 (1.656) = p_1 (0.778)(1.656)$$

$$\underline{p_4 = 1.29 p_1}$$

$$\left.\begin{array}{l} M_1 = 1.5 \\ \theta_{WEDGE} = 5^\circ \end{array}\right\} \beta_1 = 48^\circ \ \{ K\&C, \text{ FIG. 10.11}$$

$$\underline{M_{1n}} = M_1 \sin\beta_1 = (1.5)\sin(48^\circ) = (1.5)(0.743) = \underline{1.11}$$

$$\left(\frac{p_3}{p_1}\right)_{M_{1n} = 1.11} = 1.271 \ \{ K\&C, \text{ TABLE 6}$$

$$\underline{\therefore \ p_3 = p_1 (1.271) = 1.271 p_1}$$

$$M_{3n}^2 = \frac{(\gamma-1)M_{1n}^2 + 2}{2\gamma M_{1n}^2 - (\gamma-1)} = \frac{(1.4-1)(1.11)^2 + 2}{2(1.4)(1.11)^2 - (1.4-1)} = 0.819$$

$$\underline{M_{3n} = 0.905} \ ; \ M_3 = \frac{M_{3n}}{\sin(\beta-\theta)} = \frac{0.905}{\sin(48-5)} = \underline{1.327}$$

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$M_3 = 1.327 = 1.33$

$\nu_3 = 7.00^\circ$ (K&C, TABLE 5)

$\theta_3 = 0$

$\theta_5 = 10^\circ$

$\nu_5 = \nu_3 + \theta_5 = 17^\circ \rightarrow M_5 \cong 1.67$

$$p_5 = p_3\left(\frac{p_{0_3}}{p_3}\right)\left(\frac{p_5}{p_{0_5}}\right) = p_3\left(\frac{p_0}{p}\right)_{M_3=1.33}\left(\frac{p}{p_0}\right)_{M=1.67} = 1.271 p_1 \left(\frac{1}{0.3464}\right)(0.2119)$$

$$p_5 = 1.271 p_1 \left(\frac{0.2119}{0.3464}\right) = \underline{0.777 p_1}$$

REGION	PRESSURE	MACH NUMBER
2	$0.778 p_1$	1.67
3	$1.271 p_1$	1.327
4	$1.29 p_1$	—
5	$0.777 p_1$	1.67

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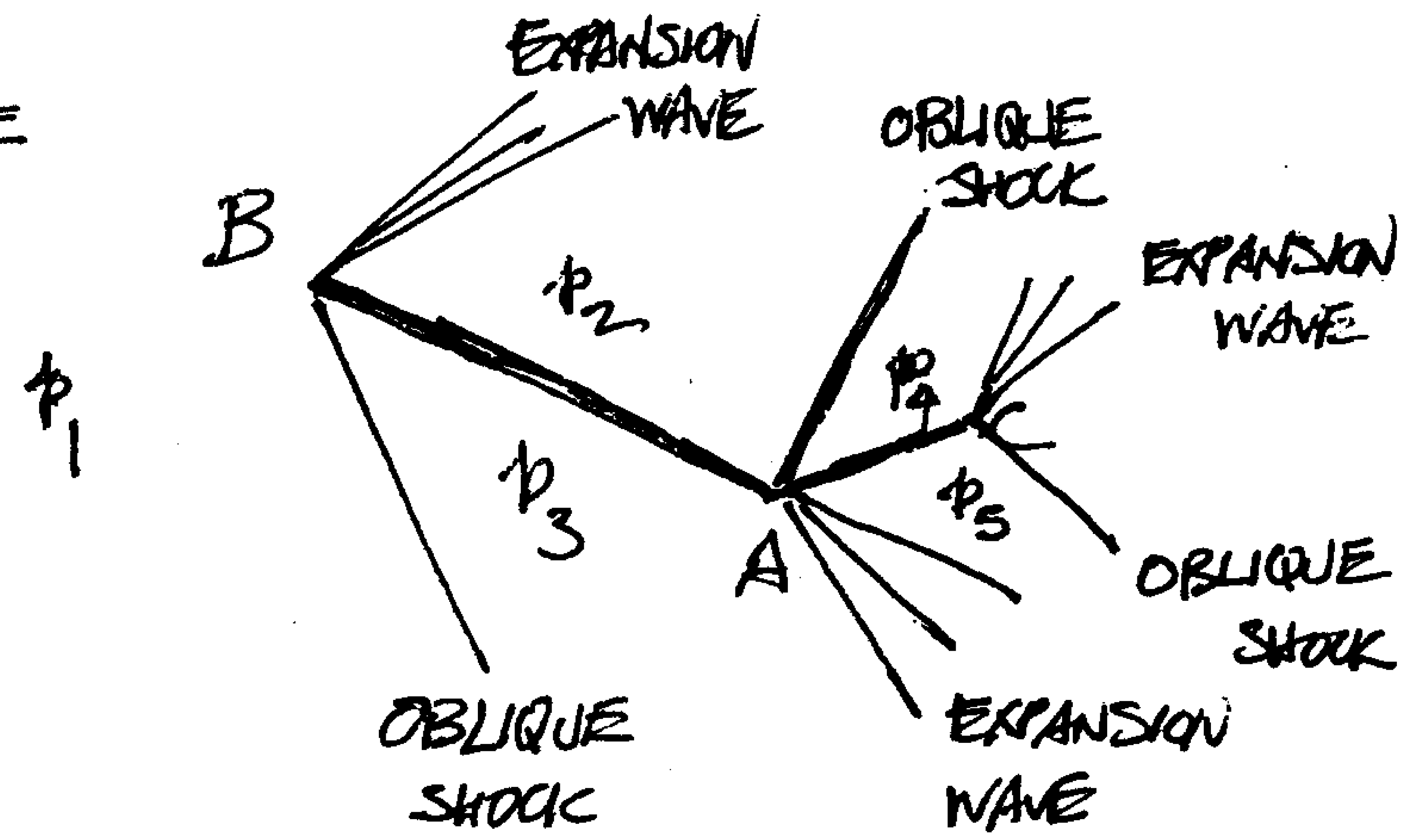
(b) WAVE STRUCTURE



(c)

$\theta_{CAD_{MAX}}$

$M_2 = 1.67$

FROM K&C, FIG. 10.11, $\underline{\theta_{CAD_{MAX}} \approx 17^\circ}$