Unified Quiz 4F
April 4, 2002

NAME Solutions

Unified Quiz 4F

April 4, 2002

One 81/2" x 11" sheet (both sides) for notes
Calculators allowed.
No books allowed.

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the two pages provided.
- Show intermediate results.
- Explain your work --- don't just write equations.
- Partial credit will be given, but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Box your final answers.

Exam Scoring

#1 (15 %)	
#2 (35 %)	
#3 (50 %)	
Total Joe B	75

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QUIZ Q4F
April 4, 2002

Problem I [15 points]

For an initial Mach number of 2.35, **(a)** find the final Mach number after a Prandtl-Meyer turn of 45°. **(b)** Find the corresponding pressure ratio across the Prandtl-Meyer fan. **(c)** Sketch the flow field including Mach waves, Prandtl-Meyer fan angle, and streamlines. Assume $\gamma = 1.4$.

$M_1 = 2.35 \quad \delta_1 = 0 \quad \delta_2 = 45°$

Use NACA report 1135 (Appendices)

(5)

$\nu_1 = 35.53$

$\mu_1 = 25.18$

$\therefore \nu_2 = \nu_1 + (1)\delta_2 - (1)\delta_1 = 35.53° + 45° = 80.53°$

A) $M_2 = 5.41$

$\mu_2 = 10.65$

$\left(\frac{P_1}{P_{0_1}}\right)_{M_1 = 2.35} = 0.07396$

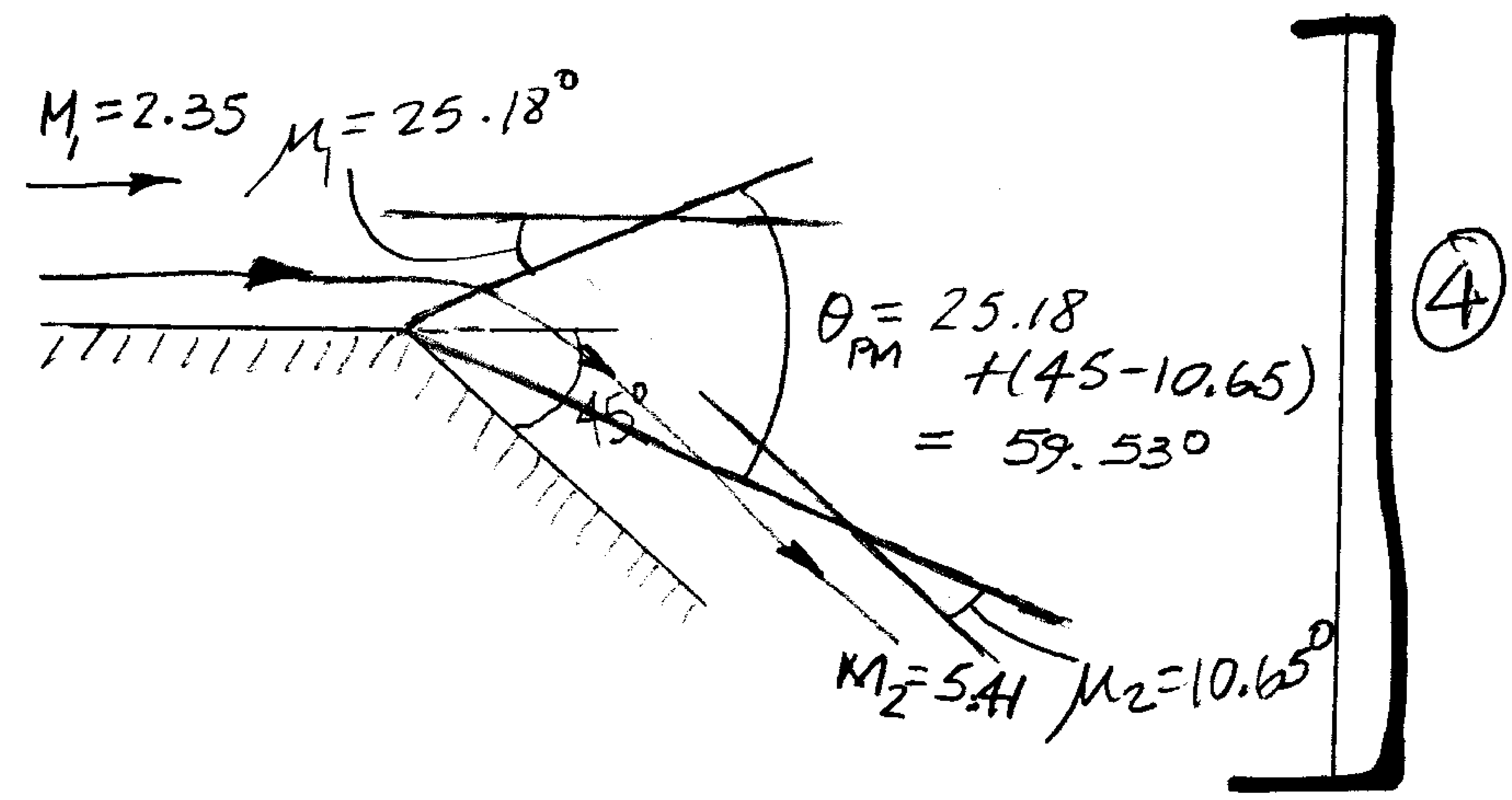
(6)

$\left(\frac{P_2}{P_{0_1}}\right)_{M_2 = 5.41} = 0.00119$

$\frac{P_2}{P_1} = \left(\frac{P_2}{P_{0_1}}\right)_{M=5.41} \left(\frac{P_{0_1}}{P_1}\right)_{M=2.35} = (0.00119) \cdot \frac{1}{0.07396}$

B) $\frac{P_2}{P_1} = 0.016$

$$\theta_{PM} = 25.18 + (45 - 10.65) = 59.53$$



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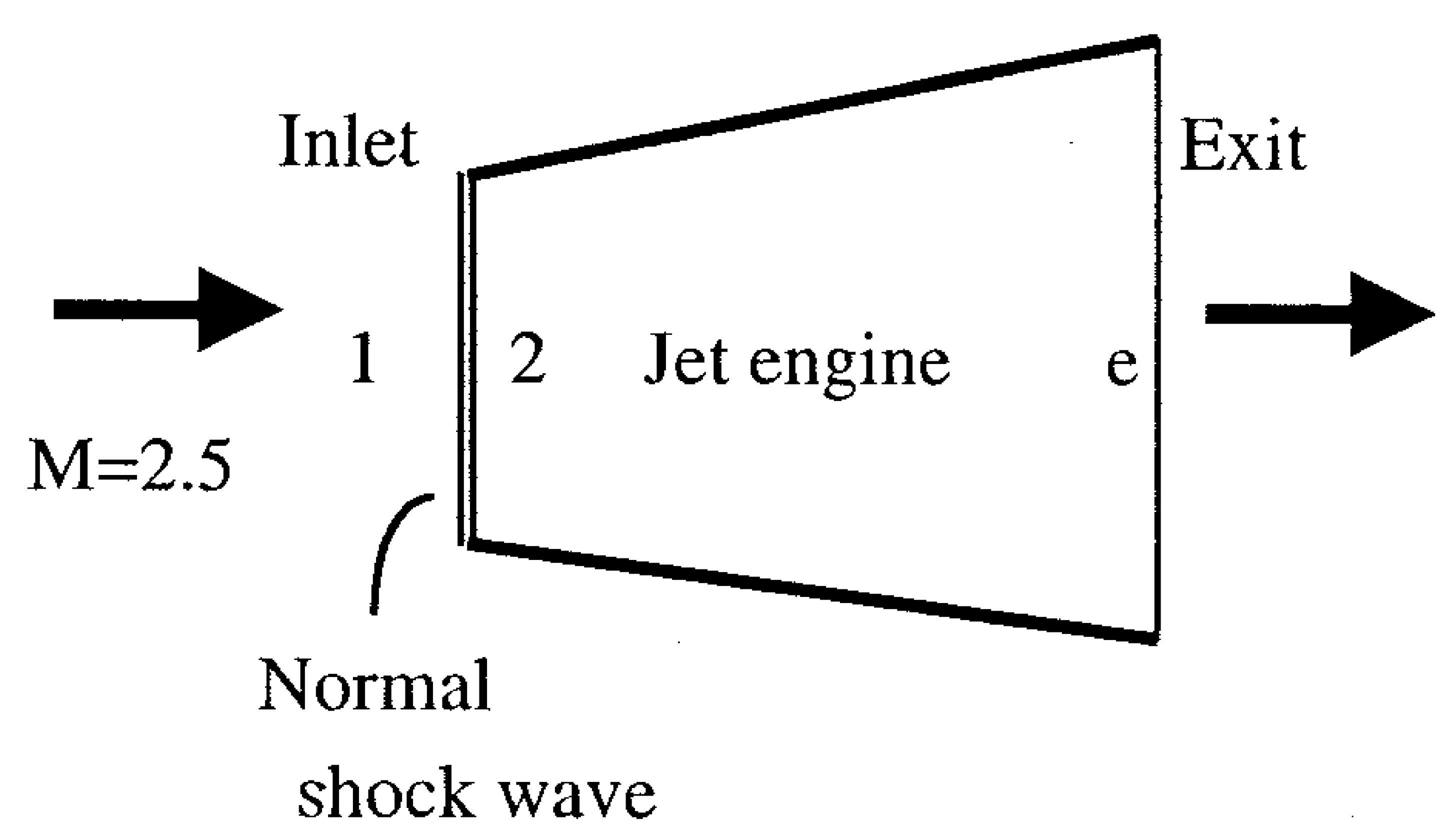
QUIZ Q4F
April 4, 2002

Problem II [35 points]

As shown in the attached figure, an F-16 supersonic fighter aircraft is traveling at a Mach number of 2.5 where the free stream atmospheric pressure is 50 kPa. At the inlet of the jet engine stands a normal shock. Assume isentropic flow everywhere except across the shock, **(a)** find the pressure and **(b)** the Mach number of the air leaving the jet engine at the exit plane e if $A_e / A_2 = 4$

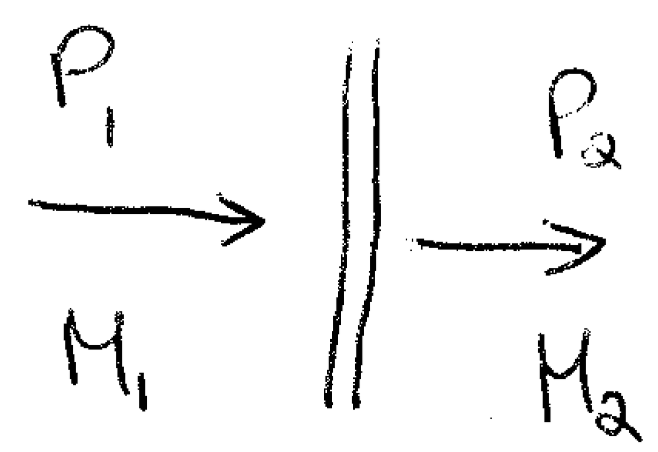


(5)

$M_1 = 2.5$

Normal Shock wave: $\boxed{M_2 = 0.5130}$ (NACA Report 1135 Appendix)

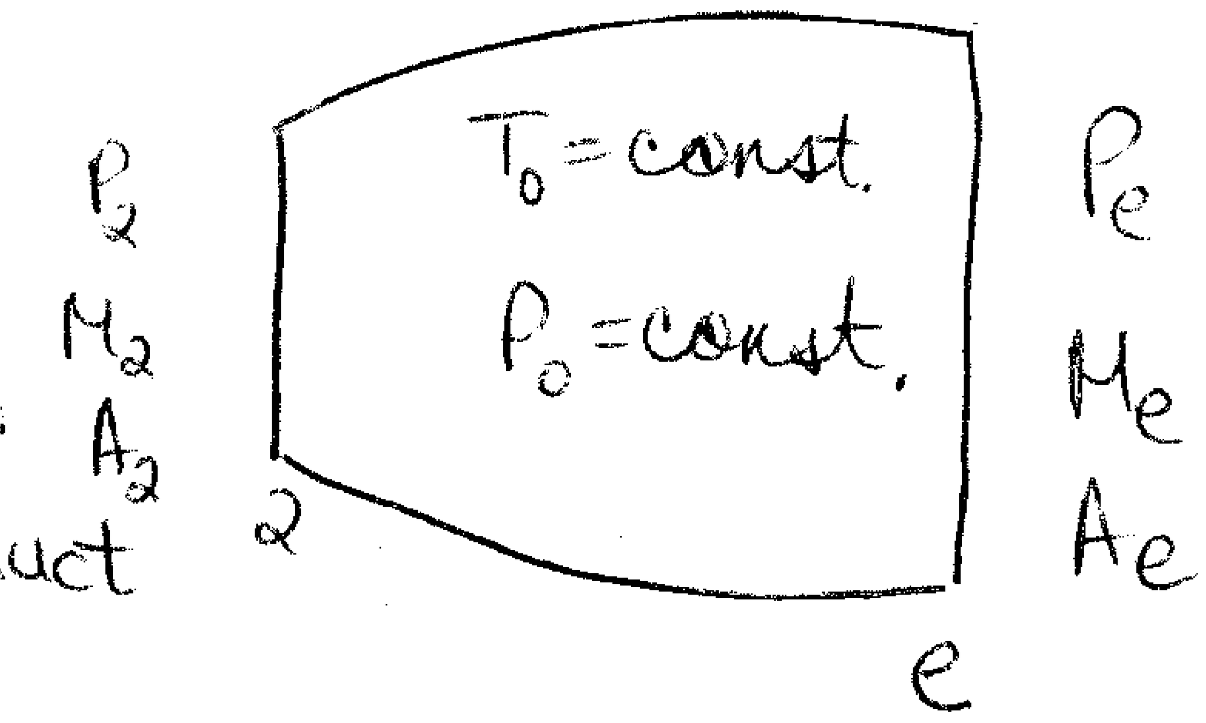
(remember, flow is subsonic after a normal shock)



$$\left(\frac{P_2}{P_1}\right)_{M_1=2.5} = 7.125 \quad \text{(NACA report 1135 Appendix)}$$

(23)

subsonic isentropic flow in diverging duct



$$\left(\frac{A_2}{A_*}\right)_{M_2=0.5130} = 1.31 \quad \text{(NACA report 1135 Appendix)}$$

$$\left(\frac{A_e}{A_*}\right) = \left(\frac{A_e}{A_2}\right)\left(\frac{A_2}{A_*}\right) = (4)(1.31) = 5.24$$

B) $\therefore M_e = 0.113$ (NACA report 1135 Appendix)

*anyone who had a supersonic M_e or assumed $A_* = A_2$ received a maximum of 7 points for this part.

(7)

$$\frac{P_2}{P_e} = \left(\frac{P_2}{P_{0_2}}\right)_{M=0.5130}\left(\frac{P_{0_2}}{P_e}\right)_{M_e=0.113} = (0.835)\left(\frac{1}{0.9913}\right)$$

$$\frac{P_2}{P_e} = 0.842$$

$$P_e = \frac{P_2}{0.842} = \frac{(7.125)P_1}{0.842} = 8.459P_1$$

$$P_e = (8.459)(50)\,kPa$$

A) $P_e = 422.95\ kPa$

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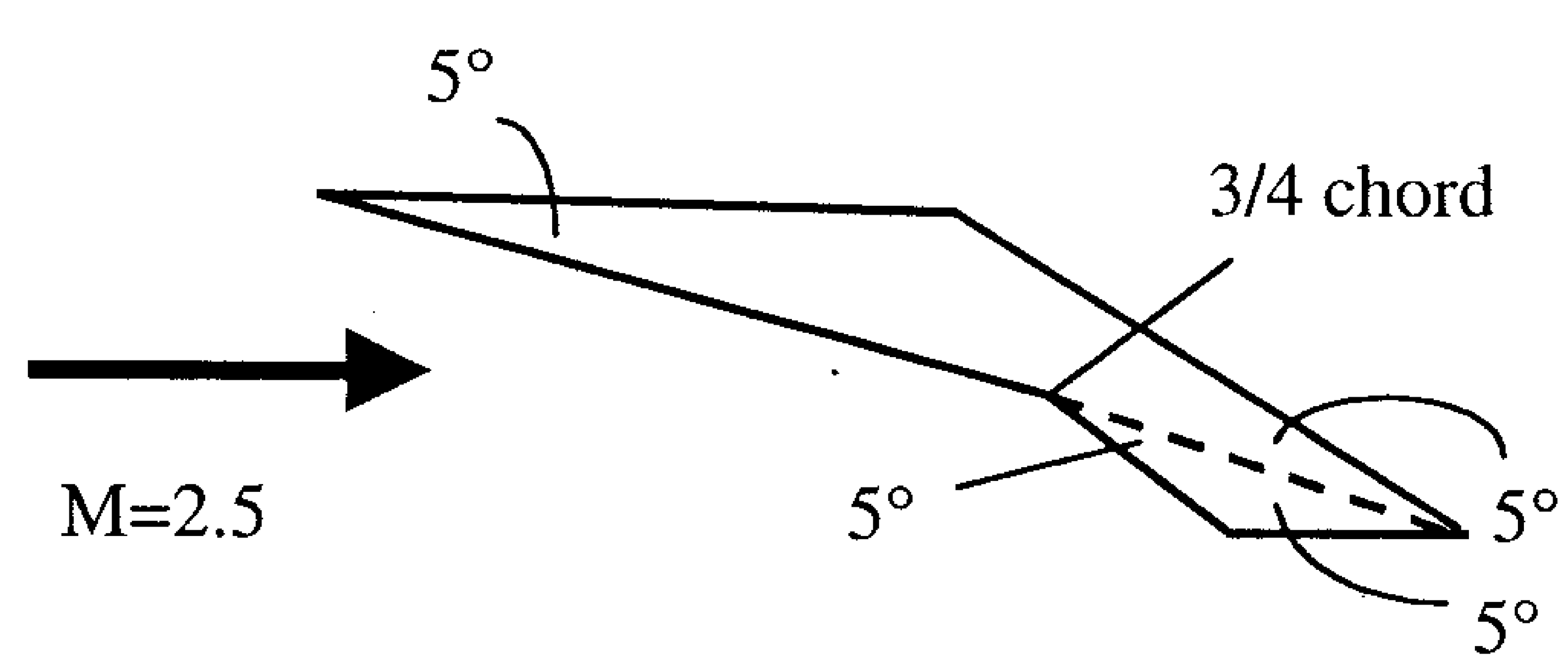
Problem III [50 points]

Consider a wing section (airfoil) of a special mission jet fighter aircraft as shown in the attached figure. The jet fighter aircraft is traveling at a Mach number of 2.5 where the free stream atmospheric pressure is 50 kPa. The airfoil chord is 5 meters. The angle of attack is 5°. **The leading edge of the airfoil under container is at the 3/4 chord.**

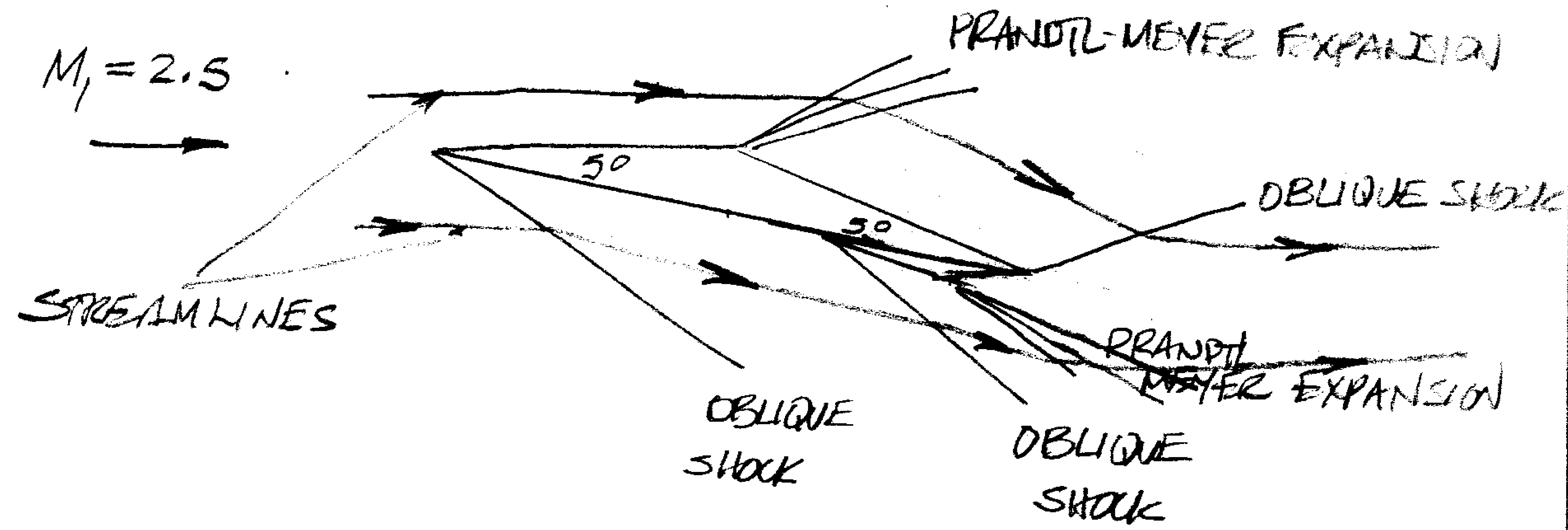
(a) **Sketch and label the wave structure and streamlines** for the supersonic, inviscid flow over this airfoil.
(b) **Calculate the lift and drag** produced by the supersonic, inviscid flow over this airfoil.



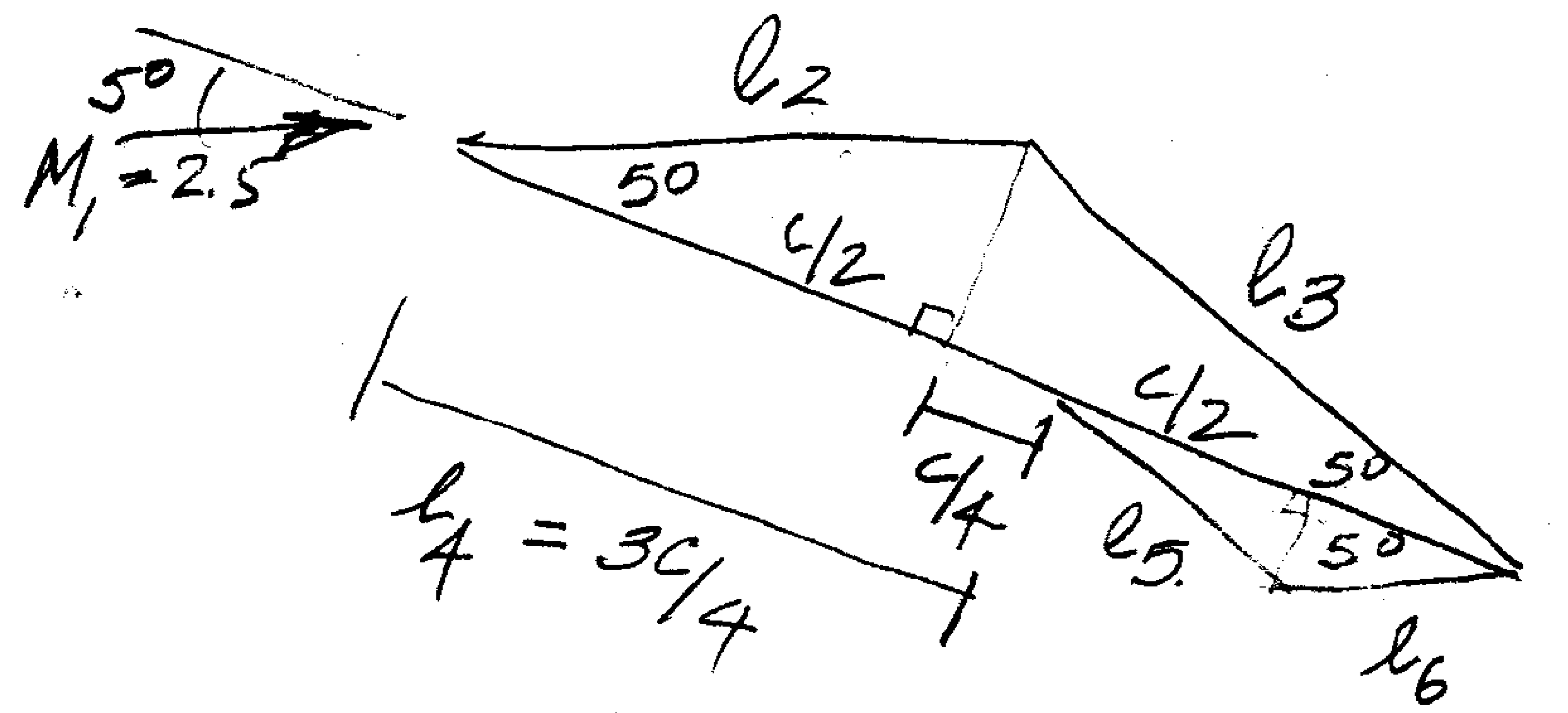
PROBLEM III

(15)

(a)



(b)



(3)

$$l_2 = l_3 = \frac{c/2}{\cos(5^\circ)} = 1.004\, c/2 \approx c/2$$

$$l_5 = l_6 = \frac{c/4}{\cos(5^\circ)} = 1.004\, c/4 \approx c/4$$

$p_2 = p_1 = 50\text{ kPa}$ (WHY?)

$M_2 = M_1 = 2.5$ (WHY?)

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p_3:

(5)

$M_2 = 2.5$, $\nu_2 = 39.12$, $\delta_2 = 0$, $\delta_3 = 10°$

$\therefore \nu_3 = \nu_2 + |\delta_3 - \delta_2| = 39.12 + 10 = 49.12°$

$$\boxed{M_3 = 2.97}$$

$$p_3 = p_2 \left(\frac{p_3}{p_{0_1}}\right)\left(\frac{p_{0_1}}{p_2}\right) = p_2 \left(\frac{p_3}{p_{0_1}}\right)_{M_3 = 2.97} \left(\frac{p_{0_1}}{p_2}\right)_{M_2 = 2.5}$$

$$p_3 = (50)(0.0285)\left(\frac{1}{0.0585}\right) = (50)(0.487)\ kPa$$

$$\boxed{p_3 = 23.4\ kPa}$$

p_4:

(5)

$M_1 = 2.5°$, $\delta = 5° \longrightarrow \beta_4 = \underline{27.5°}$

$M_2 = \underline{2.28}$

$M_{1n} = M_1 \sin\beta_4 = (2.5)\sin(27.5°) = (2.5)(0.462) = \underline{1.15}$

$$\left(\frac{p_4}{p_1}\right)_{M_{1n} = 1.15} = 1.376$$

$$p_4 = p_1(1.376) = (50)(1.376)$$

$$\boxed{p_4 = 68.8\ kPa}$$

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p_5:

(5)

$M_4 = 2.28$, $\delta = 5°$ → $\underline{\beta_5 = 30°}$

$M_5 = \underline{2.1}$

$M_{4n} = M_4 \sin\beta_5 = (2.28)\sin(30°) = (2.28)(0.500) = \underline{1.14}$

$$\left(\frac{p_5}{p_4}\right)_{M_{4n}=1.14} = 1.350$$

$$p_5 = p_4\,(1.350) = (68.8)(1.350)$$

$$\boxed{p_5 = 92.9 \text{ kPa}}$$

p_6:

$M_5 = 2.1$, $\nu_5 = 29.10$, $\delta_5 = 0$, $\delta_6 = 10°$

$\nu_6 = \nu_5 + |\delta_6 - \delta_5| = 29.10 + 10 = 39.10°$

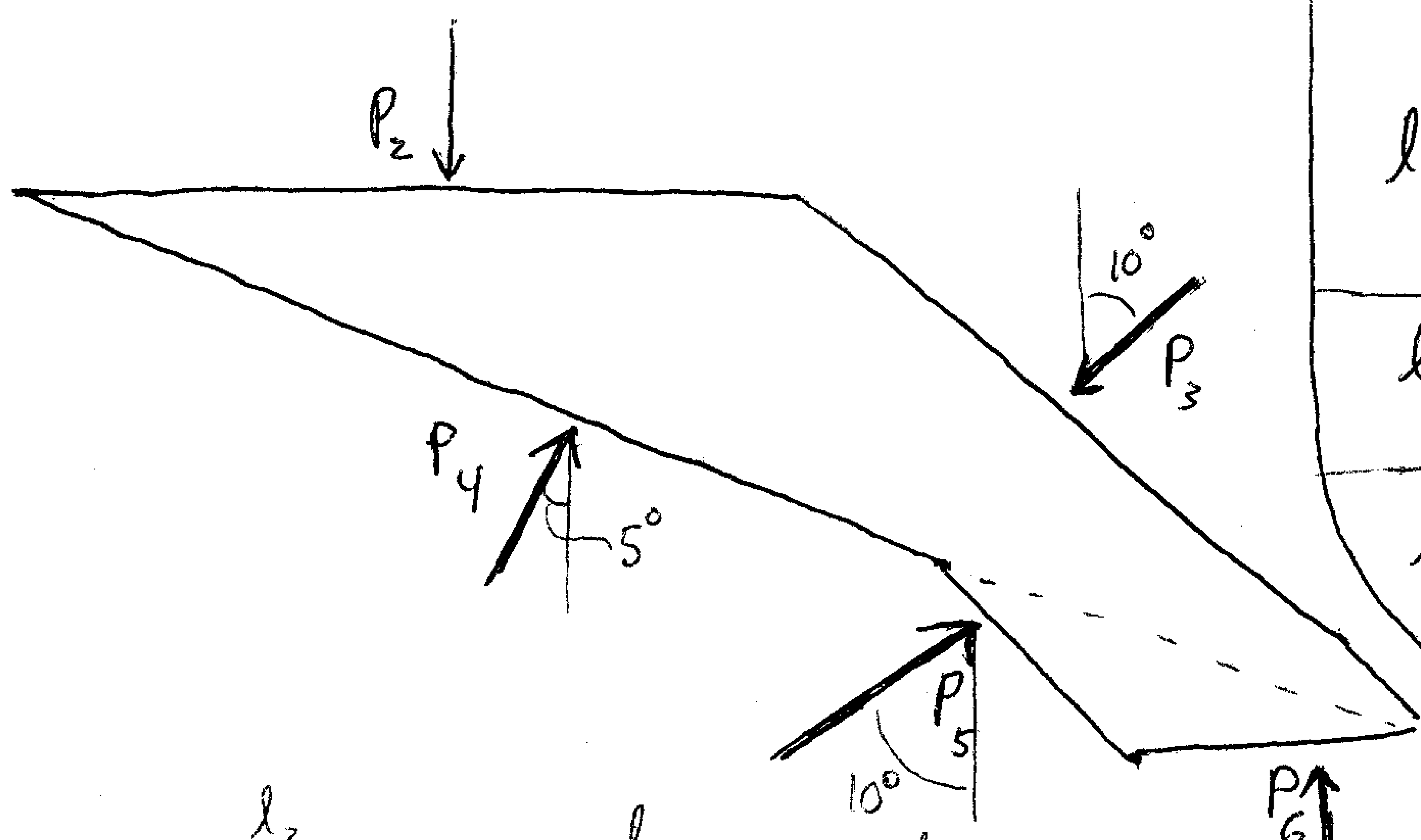
$$\boxed{M_6 = 2.50}$$

(5)

$$p_6 = p_5 \left(\frac{p_6}{p_{05}}\right)\left(\frac{p_{05}}{p_5}\right) = p_5 \left(\frac{p_6}{p_{05}}\right)_{M_6=2.50} \left(\frac{p_{05}}{p_5}\right)_{M_5=2.1}$$

$$p_6 = p_5\,(0.0585)\left(\frac{1}{0.1094}\right) = (92.9)(0.535)$$

$$\boxed{p_6 = 49.7 \text{ kPa}}$$



$l_2 = l_3 = \frac{c}{2} \cdot \frac{1}{\cos 5} = 2.51$

$l_4 = \frac{3}{4} \cdot 5 = 3.75$

$l_5 = l_6 = \frac{c/8}{\cos 5} = 0.627$

Lift

(6)

$$L' = -P_2\underbrace{\left(\frac{c/2}{\cos 5^\circ}\right)}_{l_2} + P_6\underbrace{\left(\frac{c/8}{\cos 5^\circ}\right)}_{l_6} - P_3\underbrace{\left(\frac{c/2}{\cos 5}\right)}_{l_3}\cos 10^\circ + P_4\left(\frac{3c}{4}\right)\cos 5^\circ + P_5\left(\frac{c/8}{\cos 5^\circ}\right)\cos 10^\circ$$

$$\frac{L'}{c} = \frac{-50}{2\cos 5} + \frac{49.7}{8\cos 5} + \left(\frac{92.9}{8\cos 5} - \frac{23.4}{2\cos 5}\right)(0.985) + (51.6)(0.996)$$

$$= 32.44 \text{ kPa} \Rightarrow$$

$$\boxed{L' = 32.44 \times 5\text{m} = 162.24 \text{ kN/m}}$$

Drag

(6)

$$D' = P_4 l_4 \sin 5^\circ + P_5 l_5 \sin 10^\circ - P_3 l_3 \sin 10^\circ$$

(P_2 & P_6 have no horizontal component)

$$\frac{D'}{c} = (51.6)(0.087) + (11.59 - 11.74)(0.174)$$

$$= 4.46 \text{ kPa} \Rightarrow$$

$$\boxed{D' = 4.46 \times 5 = 22.32 \text{ kN/m}}$$

$(L'/D' = 7.27)$