Fluids – Lecture 10 Notes

1. Aircraft Performance Analysis
2. Parasite Drag Estimation

Reference: Hoerner,S.F., "Fluid-Dynamic Drag", Ch 3.

Aircraft Performance Analysis

Drag breakdown

The drag on a subsonic aircraft can be broken down as follows.

$$D \;=\; D_o \,+\, D_p \,+\, D_i$$

where
D_o = "parasite" drag of fuselage + tail + landing gear + ...
D_p = wing profile drag
D_i = induced drag

We now use the wing airfoil drag polar $c_d(c_\ell; Re)$ to give the wing profile drag, and use lifting line to give the induced drag. The nondimensional total drag coefficient is then

$$\frac{D}{\frac{1}{2}\rho V^2 S} \;\equiv\; C_D \;=\; \frac{CDA_o}{S} \,+\, c_d(C_L; Re) \,+\, \frac{C_L^2}{\pi e AR} \tag{1}$$

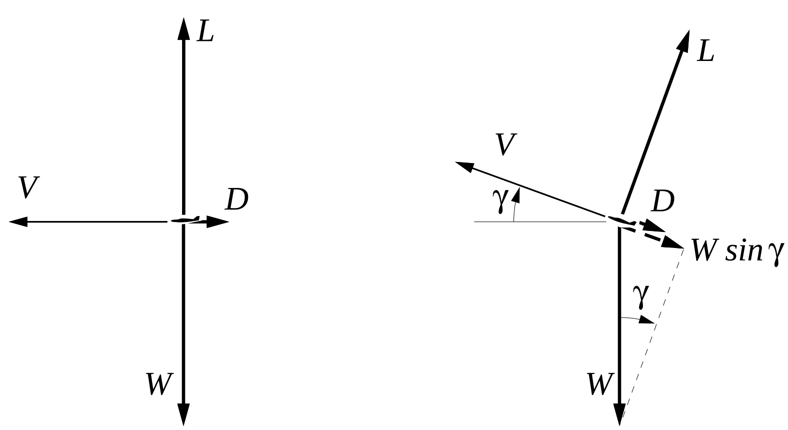
where the "∞" subscript on the flight speed V has been dropped. The parasite drag area CDA_o will be considered later.

Flight power

The mechanical power P needed for constant-velocity flight is given by

$$\eta_p P \;=\; V\,(D + W \sin\gamma) \tag{2}$$

where W is the weight, γ is the *climb angle*, and η_p is the propulsive efficiency. If P is defined as the motor shaft power, then η_p is the propeller efficiency.



In level flight, $\gamma = 0$, and the power is

$$\eta_p P \;=\; V\,D \;=\; \frac{1}{2}\rho V^3 S\, C_D \tag{3}$$

The flight speed V is given by the Lift = Weight condition, together with the definition of the lift coefficient C_L.

$$L = \frac{1}{2}\rho V^2 S C_L = W$$
$$V = \left(\frac{2\,W/S}{\rho\, C_L}\right)^{1/2}$$

The ratio W/S is called the *wing loading*, and has the units of force/area, or pressure. The level-flight power equation (3) then takes the following form.

$$P = \frac{1}{\eta_{\rm p}} \left(\frac{2\,W/S}{\rho}\right)^{1/2} W \frac{C_D}{C_L^{3/2}} \tag{4}$$

Power dependencies

Equation (4) indicates how the level flight power depends on the quantities of interest. The following dependencies are particularly worthy to note:

$$P \sim \frac{1}{\eta_{\rm p}}$$
$$P \sim \frac{1}{S^{1/2}}$$
$$P \sim W^{3/2}$$
$$P \sim \frac{C_D}{C_L^{3/2}}$$

Note the very strong effect of overall weight W. This indicates that to reduce flight power, considerable effort should be directed towards weight reduction if that's possible.

An important consideration is that the proportionality with each parameter assumes that all the other parameters are held fixed, which is difficult if not impossible to do in practice. For example, increasing the wing area S is likely to also increase the weight W, so the net influence on the flight power requires a closer analysis of the area-weight relation.

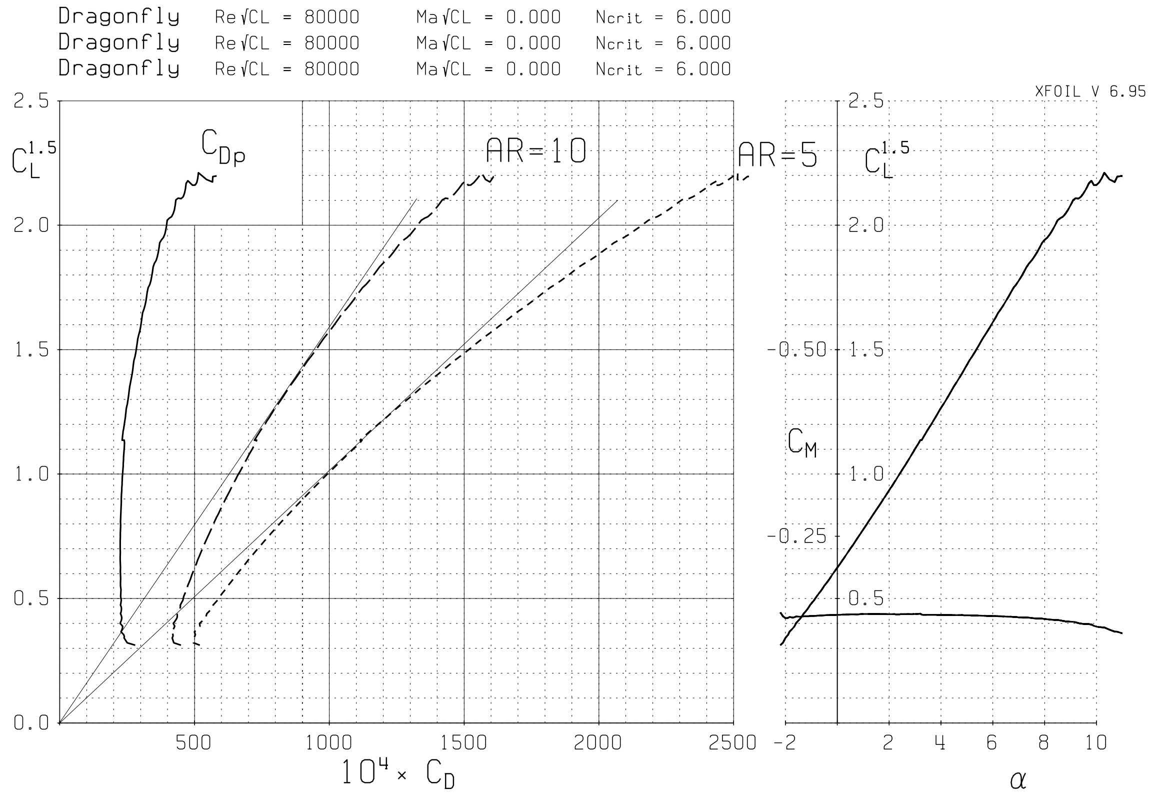
The flight power is seen to vary as the inverse of the ratio $C_L^{3/2}/C_D$, must be maximized to obtain minimum-power flight, or maximum-duration flight. But again, increasing the maximum achievable $C_L^{3/2}/C_D$ will typically require changes in the other aircraft parameters.

Drag and power polars

One convenient way to examine the aircraft's power-requirement characteristics is in a *power polar*. This is a variation on the more common drag polar, with the vertical C_L axis being replaced by $C_L^{3/2}$. An example power polar is shown below, showing three curves:

Only C_{Dp}, which assumes $C_{Do} = 0$ and $AR = \infty$.
Total C_D, assuming $C_{Do}S_o/S = 0.01$, and $e\,AR = 10$.
Total C_D, assuming $C_{Do}S_o/S = 0.01$, and $e\,AR = 5$.

The slope of the line tangent to each polar curve indicates the maximum $C_L^{3/2}/C_D$ value, and the point of contact gives the C_L at which this condition occurs. For the two example polars, we have:



For $AR = 10$: $(C_L^{3/2}/C_D)_{\max} = 16.0$ at $C_L \simeq 0.90$

For $AR = 5$: $(C_L^{3/2}/C_D)_{\max} = 10.0$ at $C_L \simeq 0.70$

The $AR = 10$ aircraft can therefore be expected to have a power requirement which is lower by a factor of $10.0/16.0 = 0.625$. The expected duration is longer by the reciprocal factor $16.0/10.0 = 1.6$. But this assumes that the larger AR has no other adverse effect on the other important parameters affecting P, which is very unlikely.

Maximum Speed

Maximum speed can also be estimated using the drag or power relations developed here. The level-flight power relation (3) at maximum power becomes

$$\eta_p P_{\max} = \frac{1}{2} \rho V_{\max}^3 S \left[\frac{CDA_o}{S} + c_d(C_{L_{\min}}; Re_{\max}) + \frac{C_{L_{\min}}^2}{\pi e AR} \right]$$

$$\text{where} \quad C_{L_{\min}} = \frac{2W/S}{\rho V_{\max}^2}$$

This can be solved for $V_{\max}$, numerically or graphically if necessary. If one can assume that at maximum speed the induced drag is negligible, and c_d is some constant, then we have

$$V_{\max} \simeq \left[\frac{2 \eta_p P_{\max}}{\rho (CDA_o + S c_d)} \right]^{1/3}$$

Alternatively, if the maximum thrust $T_{\max}$ is known rather than the maximum power, the maximum velocity is obtained from the horizontal force balance.

$$T_{\max} = \frac{1}{2} \rho V_{\max}^2 S \left[\frac{CDA_o}{S} + c_d(C_{L_{\min}}; Re_{\max}) + \frac{C_{L_{\min}}^2}{\pi e AR} \right]$$

$$V_{\max} \simeq \left[\frac{2 T_{\max}}{\rho (CDA_o + S c_d)} \right]^{1/2}$$

Parasite Drag Estimation

To estimate the total parasite drag D_o, it is commonly assumed that it is simply a summation of the estimated parasite drags of the various drag-producing components on the aircraft. For example,

$$D_o \;=\; D_{\text{fuselage}} + D_{\text{tail}} + D_{\text{gear}} + \ldots$$

To make this summation applicable to any flight speed, we simply divide it by the flight dynamic pressure, to give a summation of the drag areas.

$$CDA_o \equiv \frac{D_o}{\frac{1}{2}\rho V^2} \;=\; CDA_{\text{fuselage}} + CDA_{\text{tail}} + CDA_{\text{gear}} + \ldots \tag{5}$$

$$\text{where} \quad CDA_{\text{fuselage}} = \frac{D_{\text{fuselage}}}{\frac{1}{2}\rho V^2}$$

$$CDA_{\text{tail}} = \frac{D_{\text{tail}}}{\frac{1}{2}\rho V^2} \qquad \ldots\text{etc}$$

The summation (5) is called a *drag area buildup*.

Each component's drag area in (5) is estimated separately by whatever means are available. Typically, this will be the product of a drag coefficient C_D, and the reference area S_{ref} corresponding to that drag coefficient.

$$CDA_{\text{object}} \;=\; (C_D)_{\text{object}}\,(S_{\text{ref}})_{\text{object}}$$

For streamlined objects such as the tail surfaces, S_{ref} is typically the planform area. For bluff objects such as nacelles, wheels, etc, S_{ref} is typically the frontal area. For complex streamlined shapes, another approach is to use an average *skin friction coefficient* and the *wetted area*.

$$\begin{aligned} CDA_{\text{tail}} &= c_{d_{\text{tail}}}\, S_{\text{tail}} \\ CDA_{\text{wheel}} &= C_{D_{\text{wheel}}}\, S_{\text{wheel}} \\ CDA_{\text{shape}} &= C_f\, S_{\text{wet}} \end{aligned}$$