

# Fluids – Lecture 12 Notes

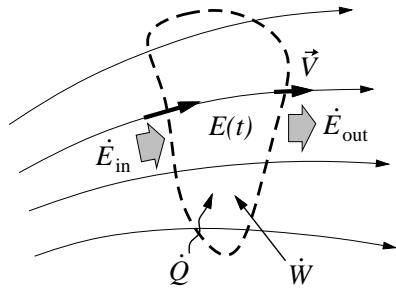
## 1. Energy Conservation

Reading: Anderson 2.7, 7.4, 7.5

### Energy Conservation

#### Application to fixed finite control volume

The first law of thermodynamics for a fixed control volume undergoing a process is



$$dE = \delta Q + \delta W$$

$$\frac{dE}{dt} + \dot{E}_{\text{out}} - \dot{E}_{\text{in}} = \dot{Q} + \dot{W} \quad (1)$$

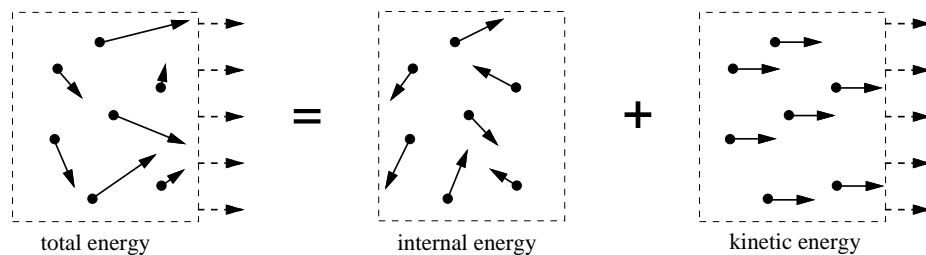
where the second rate form is obtained by dividing by the process time interval  $dt$ , and including the contribution of flow in and out of the volume.

#### Total energy

In general, the work rate  $\dot{W}$  will go towards the kinetic energy as well as the internal energy of the fluid inside the CV, in some unknown proportion. This ambiguity is resolved by defining the *total specific energy*, which is simply the sum of internal and kinetic specific energies.

$$e_o = e + \frac{1}{2}V^2 = c_v T + \frac{1}{2}V^2$$

This is the overall energy/mass of the fluid seen by a fixed observer. The  $e$  part corresponds to the molecular motion, while the  $V^2/2$  part corresponds to the bulk motion.



We now define the overall system energy  $E$  to include the kinetic energy

$$E = \iiint \rho e_o dV$$

so that  $\dot{W}$  can now include all work.

#### Energy flow

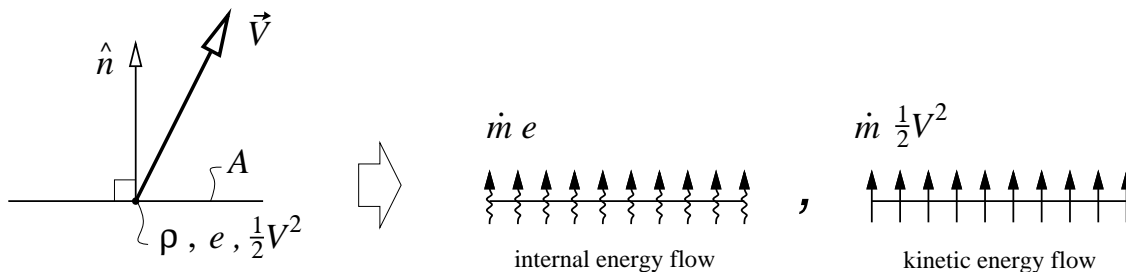
The  $\dot{E}_{\text{out}}$  and  $\dot{E}_{\text{in}}$  terms in equation (1) account for mass flow through the CV boundary, which carries not only momentum, but also thermal and kinetic energies. The *internal energy flow* and *kinetic energy flow* can be described as

$$\begin{aligned} \text{internal energy flow} &= (\text{mass flow}) \times (\text{internal energy/mass}) \\ \text{kinetic energy flow} &= (\text{mass flow}) \times (\text{kinetic energy/mass}) \end{aligned}$$

where the mass flow was defined earlier. The internal energy/mass is by definition the specific internal energy  $e$ , and the kinetic energy/mass is simply  $V^2/2$ . Therefore

$$\begin{aligned} \text{internal energy flow} &= \dot{m} e = \rho (\vec{V} \cdot \hat{n}) A e = \rho V_n A e \\ \text{kinetic energy flow} &= \dot{m} \frac{1}{2} V^2 = \rho (\vec{V} \cdot \hat{n}) A \frac{1}{2} V^2 = \rho V_n A \frac{1}{2} V^2 \end{aligned}$$

where  $V_n = \vec{V} \cdot \hat{n}$ . Note that both types of energy flows are scalars.



The net total energy flow rate in and out of the volume is obtained by integrating the internal and kinetic energy flows over its entire surface.

$$\dot{E}_{\text{out}} - \dot{E}_{\text{in}} = \iint \rho (\vec{V} \cdot \hat{n}) \left( e + \frac{1}{2} V^2 \right) dA = \iint \rho (\vec{V} \cdot \hat{n}) e_o dA$$

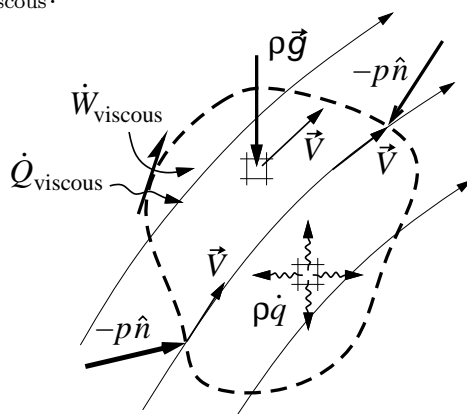
## Heat addition

Two types of heat addition can occur.

Body heating. This acts on fluid inside the volume. Examples are combustion, and internal absorption of radiation (e.g. microwaves). Whatever the source, the body heating is given by a specific heating rate  $\dot{q}$ , with units of power/mass.

$$\dot{Q}_{\text{body}} = \iiint \rho \dot{q} dV$$

Surface heating. The surface can transmit *heat flux* into or out of the volume. This is usually associated with viscous action, and like the viscous work rate it is complicated to write out. It will be simply called  $\dot{Q}_{\text{viscous}}$ .



## Work by applied forces

The work rate (or power) performed by a force  $\vec{F}$  applied to fluid moving at velocity  $\vec{V}$  is

$$\dot{W} = \vec{F} \cdot \vec{V}$$

We will consider two types of applied forces.

Body forces. These act on fluid inside the volume. The most common example is the gravity force, along the gravitational acceleration vector  $\vec{g}$ . The work rate done on the fluid inside the entire CV is

$$\dot{W}_{\text{gravity}} = \iiint \rho \vec{g} \cdot \vec{V} d\mathcal{V}$$

Surface forces. These act on the surface of the volume, and can be separated into pressure and viscous forces. The work rate done by the pressure is

$$\dot{W}_{\text{pressure}} = \oint -p \hat{n} \cdot \vec{V} dA$$

The work rate done by the viscous force is complicated to write out, and for now will simply be called  $\dot{W}_{\text{viscous}}$ .

### Integral Energy Equation

Substituting all the energy flow, heating, and work term definitions into equation (1) gives the *Integral Energy Equation*.

$$\begin{aligned} \frac{d}{dt} \iiint \rho e_o d\mathcal{V} + \oint \rho (\vec{V} \cdot \hat{n}) e_o dA = & \iiint \rho \dot{q} d\mathcal{V} + \oint -p \hat{n} \cdot \vec{V} dA + \iiint \rho \vec{g} \cdot \vec{V} d\mathcal{V} \\ & + \dot{Q}_{\text{viscous}} + \dot{W}_{\text{viscous}} \end{aligned} \quad (2)$$

### Total enthalpy

We now define the *total enthalpy* as

$$h_o = h + \frac{1}{2}V^2 = e + \frac{p}{\rho} + \frac{1}{2}V^2 = e_o + \frac{p}{\rho}$$

We can now combine the two surface integrals in equation (2) into one enthalpy-flow integral. The result is the simpler and more convenient *Integral Enthalpy Equation* which does no longer explicitly contains the pressure work term.

$$\frac{d}{dt} \iiint \rho e_o d\mathcal{V} + \oint \rho (\vec{V} \cdot \hat{n}) h_o dA = \iiint \rho \dot{q} d\mathcal{V} + \iiint \rho \vec{g} \cdot \vec{V} d\mathcal{V} + \dot{Q}_{\text{viscous}} + \dot{W}_{\text{viscous}} \quad (3)$$

In most applications we can assume steady flow, and also neglect gravity and viscous work. Equation (3) then simplifies to

$$\oint \rho (\vec{V} \cdot \hat{n}) h_o dA = \iiint \rho \dot{q} d\mathcal{V} + \dot{Q}_{\text{viscous}} \quad (4)$$

Along with the Integral Mass Equation and the Integral Momentum Equation, equation (4) can be applied to solve many thermal flow problems involving finite control volumes. The  $\dot{Q}_{\text{viscous}}$  term typically represents heating or cooling of the fluid by a solid wall.

### Differential Enthalpy Equation

The enthalpy-flow surface integral in equation (3) can be converted to a volume integral using Gauss's Theorem.

$$\oint \rho (\vec{V} \cdot \hat{n}) h_o dA = \iiint \nabla \cdot (\rho \vec{V} h_o) d\mathcal{V}$$

Omitting the viscous terms for simplicity, the integral enthalpy equation (3) can now be written strictly in terms of volume integrals.

$$\iiint \left[ \frac{\partial(\rho e_o)}{\partial t} + \nabla \cdot (\rho \vec{V} h_o) - \rho \dot{q} - \rho \vec{g} \cdot \vec{V} \right] d\mathcal{V} = 0$$

This relation must hold for *any* control volume whatsoever, and hence the whole quantity in the brackets must be zero at every point.

$$\frac{\partial(\rho e_o)}{\partial t} + \nabla \cdot (\rho \vec{V} h_o) = \rho \dot{q} + \rho \vec{g} \cdot \vec{V}$$

which applies at every point in the flowfield. Adding  $\partial p/\partial t$  to both sides and combining it on the left side gives the *Differential Enthalpy Equation* in “divergence form”.

$$\frac{\partial(\rho h_o)}{\partial t} + \nabla \cdot (\rho \vec{V} h_o) = \frac{\partial p}{\partial t} + \rho \dot{q} + \rho \vec{g} \cdot \vec{V} \quad (5)$$

We can recast this equation further by expanding the lefthand side terms.

$$h_o \left[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) \right] + \rho \left( \frac{\partial h_o}{\partial t} + \vec{V} \cdot \nabla h_o \right) = \frac{\partial p}{\partial t} + \rho \dot{q} + \rho \vec{g} \cdot \vec{V}$$

The quantity in the brackets is exactly zero by the mass continuity equation, and the quantity in the braces is simply  $Dh_o/Dt$ , so that the result can be written compactly as the *Differential Enthalpy Equation* in “convective form”.

$$\frac{Dh_o}{Dt} = \frac{1}{\rho} \frac{\partial p}{\partial t} + \dot{q} + \vec{g} \cdot \vec{V} \quad (6)$$

More specifically, equation (6) gives the rate of change of total enthalpy along a pathline, in terms of three simple terms.

### Steady, adiabatic flows

Assume the following three conditions are met:

Steady Flow. This implies that  $\partial p/\partial t = 0$

Adiabatic Flow. This implies that  $\dot{q} = 0$

Negligible gravity. This implies that we can ignore  $\vec{g} \cdot \vec{V}$

With these assumptions, equation (6) becomes simply

$$\frac{Dh_o}{Dt} = 0 \quad (7)$$

so that the total enthalpy is constant along a streamline. If all streamlines originate at a common upstream reservoir or uniform flow, which is the usual situation in external aerodynamic flows, then

$$h_o = h + \frac{1}{2}V^2 = \text{constant} \quad (8)$$

throughout the flowfield. Equation (8) in essence replaces the full differential enthalpy equation (5), which is an enormous simplification. Note also its similarity to the incompressible Bernoulli equation. However, equation (8) is more general, since it is valid for compressible flows, and is also valid in the presence of friction.