Fluids – Lecture 8 Notes

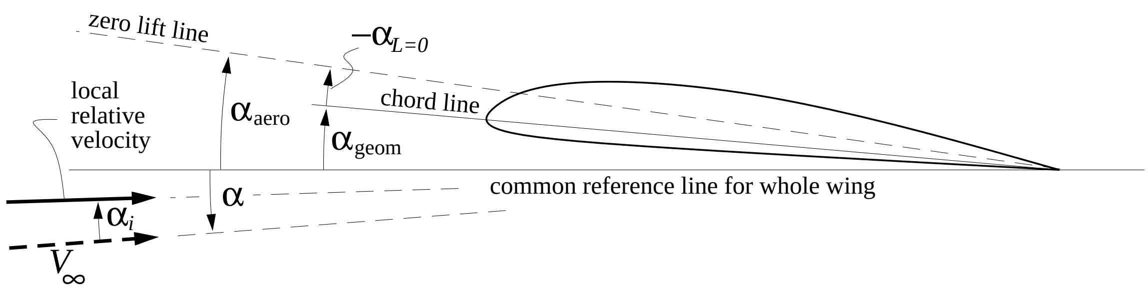
1. Wing Geometry
2. Wing Design Problem

Reading: Anderson 5.3.2, 5.3.3

Wing Geometry

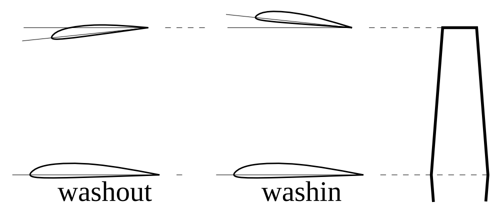
Chord and twist

The chord distribution is given by the $c(y)$ function. Each spanwise station also has a local *geometric twist* angle $\alpha_{\text{geom}}(y)$, measured from some reference line which is common to the whole wing. The freestream angle α is also defined from this same common reference line. The choice of the reference line for all these angles is arbitrary. A typical choice might be the fuselage axis line, or the the wing-center chord line.



How the geometric twist varies across the span is loosely described by the terms *washout* and *washin*:

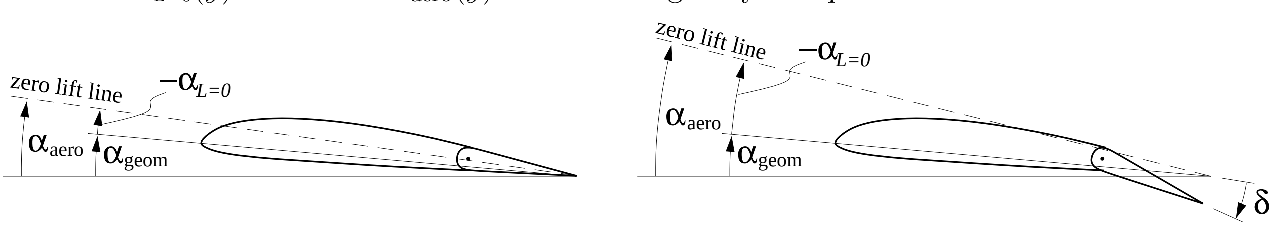
Washout: $\alpha_{\text{geom}}(y)$ decreases towards the tip.
Washin : $\alpha_{\text{geom}}(y)$ increases towards the tip.



If the wing has a spanwise-varying camber, the local zero-lift angle $\alpha_{L=0}(y)$ will also vary. It is useful to define an overall *aerodynamic twist* angle as

$$\alpha_{\text{aero}}(y) \;\equiv\; \alpha_{\text{geom}}(y) \;-\; \alpha_{L=0}(y)$$

The local $\alpha_{L=0}(y)$ and hence $\alpha_{\text{aero}}(y)$ can be changed by a flap deflection δ.



Local loading/angle relations

The local lift/span can be given either in terms of the local circulation $\Gamma(y)$, or the local chord-c_ℓ product.

$$L'(y) \;=\; \rho V_\infty \Gamma(y)$$

$$L'(y) \;=\; \frac{1}{2}\rho V_\infty^2 \, c(y)\, c_\ell(y)$$

Equating these gives the circulation in terms of the chord and c_ℓ.

$$\Gamma(y) \;=\; \frac{1}{2} V_\infty \, c(y) \, c_\ell(y) \tag{1}$$

Assuming the airfoils are not stalled, the local c_ℓ is proportional to the angle between the local relative velocity and the zero lift line.

$$c_\ell(y) \;=\; a_o \left[\alpha \,+\, \alpha_{\text{aero}}(y) \,-\, \alpha_i(y)\right] \tag{2}$$

The constant of proportionality $a_o = dc_\ell/d\alpha$ is nearly 2π for thin airfoils, but is somewhat larger for thick airfoils. It can be obtained from either experimental data or calculation.

Finally, the relation between α_i at any one location y_o, and the entire spanwise circulation distribution, was obtained previously using the Biot-Savart law.

$$\alpha_i(y_o) \;=\; \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \frac{d\Gamma}{dy} \frac{dy}{y_o - y} \tag{3}$$

Equations (1), (2), and (3) form the basis of wing analysis and design. The general analysis problem is rather complicated and is somewhat beyond scope here. However, the design problem is simpler, and an example design problem is considered next.

Wing Design Problem

Elliptic loading

A typical basic wing design problem is to determine the geometry of a wing so that it will have some specified load (or circulation) distribution $\Gamma(y)$. As an example, consider the simple case where the span b and flight speed V_∞ are given, and the circulation is to be elliptic.

$$\Gamma(y) \;=\; \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

Although we don't yet know what the wing looks like, we do already know its lift and induced drag from $\Gamma(y)$ alone. From previous results,

$$L \;=\; \frac{\pi}{4} \rho V_\infty \Gamma_0 \, b \qquad , \qquad D_i \;=\; \frac{(L/b)^2}{\frac{1}{2}\rho V_\infty^2 \, \pi}$$

Planform definition

The wing chord distribution, or planform, is partially given by equation (1). This now becomes

$$c(y) \, c_\ell(y) \;=\; \frac{2\Gamma_0}{V_\infty} \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

which states only that the $c \times c_\ell$ product must be elliptic. How c or c_ℓ vary individually is not determined, but rather must be chosen by the designer. The possibilities are unlimited, but it's useful to consider two particularly simple choices.

Choice 1: Pick a spanwise constant c_ℓ. In this case the chord distribution must be elliptic,

$$c(y) \;=\; c_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

$$c_0 \;=\; \frac{2\Gamma_0}{V_\infty \, c_\ell}$$

where c_0 is the center chord. Such a wing is aerodynamically attractive, but the curved outlines may be impractical for construction.

Choice 2: Pick a simple constant wing chord, $c(y) = c$. Using equation (1) again we have

$$
\begin{aligned}
c_\ell(y) &= c_{\ell_0} \sqrt{1 - \left(\frac{2y}{b}\right)^2} \\
c_{\ell_0} &= \frac{2\Gamma_0}{V_\infty \, c}
\end{aligned}
$$

where c_{ℓ_0} is the wing-center lift coefficient.

Aerodynamic twist definition

Whatever choice is made for either $c(y)$ or $c_\ell(y)$, the required corresponding wing twist distribution can now be obtained. We have determined earlier that for the elliptic loading case, equation (3) produces a spanwise-constant induced angle, given by

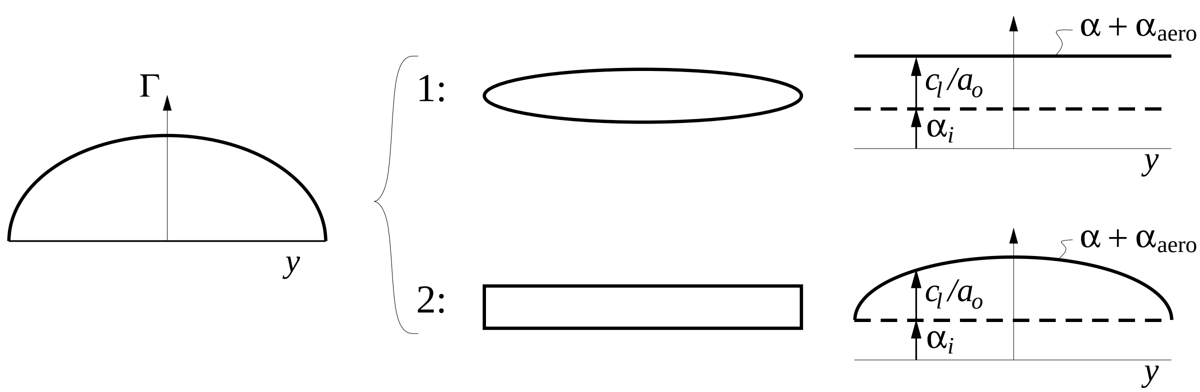
$$
\alpha_i(y) = \frac{\Gamma_0}{2bV_\infty} \qquad \text{(spanwise constant)}
$$

although a non-constant α_i does not present any complications. The aerodynamic twist distribution is then given by rearranging equation (2).

$$
\alpha + \alpha_{\text{aero}}(y) = \frac{c_\ell(y)}{a_o} + \alpha_i(y)
$$

Only the sum $\alpha + \alpha_{\text{aero}}(y)$ is defined at this point, since it is the relevant angle which directly affects the lift. How this sum is split up between $\alpha_{\text{aero}}(y)$ and α is arbitrary, and will be defined as a final step.

The figure shows the two design choices described here, and the resulting wing shapes and aerodynamic twist distributions. Note that the elliptic-planform wing is *aerodynamically flat*, meaning that it has a constant aerodynamic twist (i.e. the zero-lift lines at all spanwise locations are parallel). In contrast, the "simple" constant chord wing has not turned out so simple after all, with an elliptic aerodynamic twist distribution.



Geometric twist definition

Definition of the geometric wing twist requires that the airfoil be selected, possibly varying across the span. This selection fixes the $\alpha_{L=0}(y)$ distribution,

$$
\text{given airfoil} \longrightarrow \alpha_{L=0}(y)
$$

which in turn determines the required geometric twist, apart from the constant α offset.

$$\alpha + \alpha_{\text{geom}}(y) = \frac{c_\ell(y)}{a_o} + \alpha_i(y) + \alpha_{L=0}(y) \tag{4}$$

Reference line selection
The overall angle of attack α is finally determined by selection of a common angle reference line for the whole wing. A convenient choice is to set the geometric twist angle at the wing center $y = 0$ to take on some specific value, say $\alpha_{\text{geom}}(0) = \alpha_0$. Applying equation (4) at $y = 0$ then defines α.

$$\alpha = \frac{c_\ell(0)}{a_o} + \alpha_i(0) + \alpha_{L=0}(0) - \alpha_0$$

When this α is substituted into equation (4), the geometric twist distribution is finally obtained.

$$\alpha_{\text{geom}}(y) = \alpha_0 + \frac{c_\ell(y) - c_\ell(0)}{a_o} + \alpha_i(y) - \alpha_i(0) + \alpha_{L=0}(y) - \alpha_{L=0}(0)$$

It must be stressed that the choice of α_0 does not alter the wing twist, or the wing's orientation relative to the freestream velocity (i.e. the physical flow situation is unaffected). As shown in the figure, changing α_0 merely sets the arbitrary reference line at a different orientation on the wing/freestream combination. The flowfields are identical.