UNIFIED HANDOUT

MATERIALS AND STRUCTURES - #M-15

Spring, 2009

Concept Review Sheet

for Unified Q5M: M4.6, M4.7, M4.8, M5.1-M5.4 *(latter as per pages noted)*

TORSION

- A shaft is long compared to its other two dimensions (L >> b,h).
- The load applied is a pure torque about the axis of the long direction.
- The cross-section rotates as a rigid body (i.e. no distortion of the cross-section).
- All displacements and deformations can be defined through the twist angle ϕ.
- The pertinent internal resultant is the torque, T.
- The torque resultant at any location along the shaft is equipollent to the torque caused by the shear stresses acting and integrated over the "cut" surface.
- The change in torque along the shaft is equal to the negative of the applied torsional load.
- The polar moment of inertia, J, defines the geometric contribution of the structural configuration to the resistance to torsional deformation.
- The shear modulus defines the material contribution of the structural configuration to the resistance to torsional deformation.
- The total torsional stiffness is GJ.
- Maximum magnitude of shear stress occurs at a point farthest from the centerline of the shaft.
- The model gives exact solutions for circular cross-sections (including tubes), but approximations for other sections.

COLUMNS AND BUCKLING

- "*Effective structural stiffness*" is a linear change in restoring force with deflection.
- "A system becomes unstable when a negative stiffness overcomes the natural stiffness of the structure"
- Terminology: *eigenvalue* is the value of the load for static instability; *eigenvector* is the displacement shape/mode of the structure in the static instability.
- **Bifurcation** is a mathematical concept -- the manifestations in actual systems are altered due to physical realities/imperfections.
- A column is long compared to its other two dimensions (L >> b,h) and is loaded in compression along its long axis (x_1).
- The transverse deflection is governed by the bending equation.
- Basic equation is:

$$EI\,(d^2u_3/dx_1^2) + Pu_3 = 0 \;;$$

this is a reduction of a fourth order equation for the case of EI and P not dependent on x_1.

- General homogenous solution to equation is:
  $u_3 = A \sin \lambda x_1 + B \cos \lambda x_1 + C + D\, x_1$ where $\lambda = (P/EI)^{1/2}$
- Obtain constants by using boundary conditions (2 at each end needed); Solve the resulting set of equation for roots of the set of equations -- this results in eigenvalues and associated eigenvectors; Smallest eigenvalue is the buckling load.
- General form of critical buckling load is:
  $P_{cr} = c\pi^2 EI/L^2$
- c = coefficient of edge fixity (dependent on boundary conditions)
- If there are eccentricities and imperfections, an inhomogeneous solution also arises and this provides a transverse deflection versus load that approaches the buckling bifurcation line as an asymptote.
- Columns can fail by yielding/squashing/crushing prior to reaching critical buckling load -- this is governed by yield and ultimate stresses in compression.
- The more slender a column is, the more likely it is to buckle (characterized by *slenderness ratio*).
- A column buckles in the direction with the smallest moment of inertia.

GENERAL ITEMS WITH REGARD TO "1-D" STRUCTURAL MODELS

- The assumptions of the model result in limitations for application of the models.
- The internal resultant(s), or expressions for such, can be determined by applying equilibrium for a virtual "cut" cross-section.
- The pertinent deformation is related to loading divided by a structural stiffness which has a material parameter and a geometrical parameter of the cross-section.
- The pertinent stress is related to loading times a cross-sectional distance divided by a geometrical parameter of the cross-section.
- The use of these models involves six steps: 1. Drawing the Free Body diagram; 2. Determining the reactions; 3. Determining the internal stress resultant(s); 4. Determining the major deflection parameter as related to the pertinent structural stiffness; 5. Determining the pertinent stresses; 6. Determining related strains and deflections/deformations as needed.
- The six parts can be solved sequentially for a statically determinate case or simultaneously for a statically indeterminate case.
- The solutions are linear and superposition can be used.
- The models can be "extended" slightly to allow for variations in dimensions and directions not strictly allowed.
- The key in the use of all models is **consistency**.

FAILURE: THE MATERIAL'S ROLE

- *Structural failure* is the inability of the structure to perform as intended.
- Strain energy per unit volume is the area under the stress-strain curve.
- *Proportional stress* is the stress where the stress-strain behavior deviates from linear.
- *Yield* stress is the stress where the stress-strain behavior deviates from elastic.
- *Ultimate* stress is the stress where the material breaks or the material holds its highest stress.

- Stress-strain behavior is characterized as *linear* (if modulus/stiffness stays the same), *softening* (if modulus/stiffness decreases), or *stiffening* (if modulus/stiffness increases).
- Stress-strain behavior is characterized as *elastic* (if unloading occurs along the same path as loading and energy is conserved); *plastic* (if unloading occurs along a different path from loading and energy is expended).
- Material can have an increase in strain under stress over a period of time. This phenomenon is known as creep (this is a viscoelastic response).
- The creep phenomenon increases with increased temperature.
- Material can *relax* over time -- requiring less load to maintain a given displacement (this is also a manifestation of viscoelastic response).

- Failure is a weakest link phenomenon and occurs at imperfections, etc. such as dislocations in materials with crystal structure.
- There are multiple lengthscales at which these imperfections can occur.

- In crystals, failure occurs by shear stress along planes of slippage.
- Hydrostatic stress does not cause failure.
- The Tresca criterion is based on failure by maximum shear stresses and thereby compares the difference of principal stresses.
- The von Mises criterion also considers the difference of principal stresses but adds up the effects (interactive criterion) rather than looking at each of the three differences separately.

- Stress concentrations cause failure load to be lowered due to the increase in stress local to the stress concentration.
- Failure in fracture mechanics is governed by the length of the crack and the critical stress intensity factor (also known as *fracture toughness*).
- The (strain) energy expended in the fracture process is consumed by the surface energy of the new crack area created.