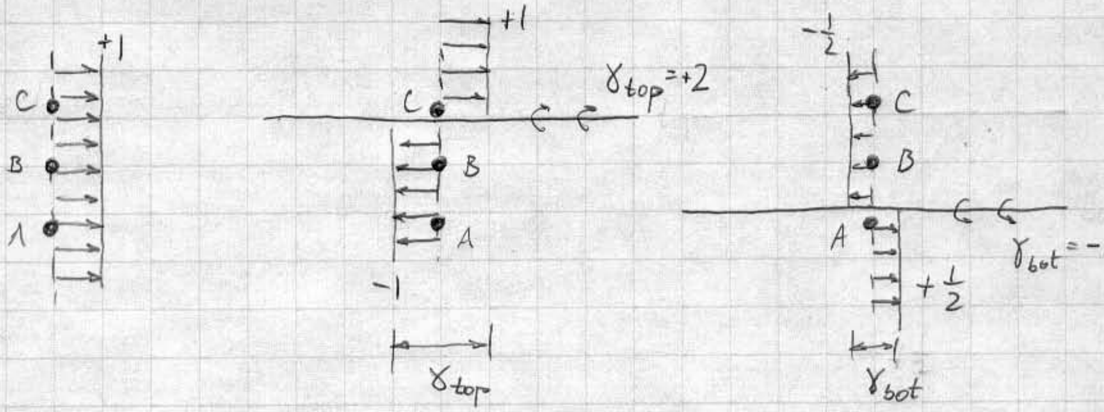
UE Fluids Problem F1 Solution S '09

a) Velocities of $\vec{V}_\infty$ and sheets in isolation:

Sheet velocities satisfy $\Delta V_t = \gamma$:

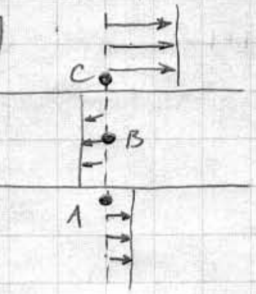


All velocity fields superimposed:

$$\boxed{V_C = 1 + 1 - \tfrac{1}{2} = +\tfrac{3}{2}}$$

$$\boxed{V_B = 1 - 1 - \tfrac{1}{2} = -\tfrac{1}{2}}$$

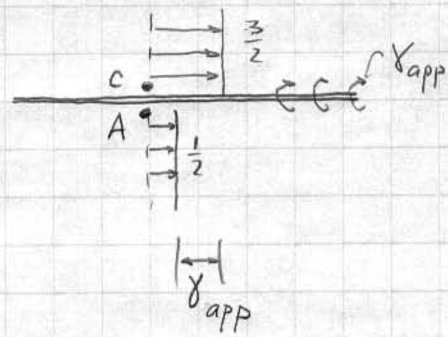
$$\boxed{V_A = 1 - 1 + \tfrac{1}{2} = +\tfrac{1}{2}}$$



size of h does not matter

b) From far away:

$$\boxed{\gamma_{app} = \Delta V_t = \tfrac{3}{2} - \tfrac{1}{2} = 1}$$



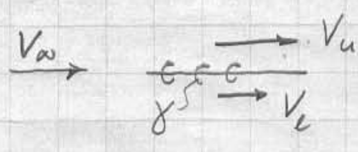
Can also simply add (superimpose) the two sheets:

$$\gamma_{app} = \gamma_{top} + \gamma_{bot} = 2 - 1 = 1$$ same result

UE Fluids — Problem F2 Solution — S'09

a) This is a flat camberline ($Z=0$, $\frac{dZ}{dx}=0$) airfoil at $\alpha = 4° = 0.070$ rad

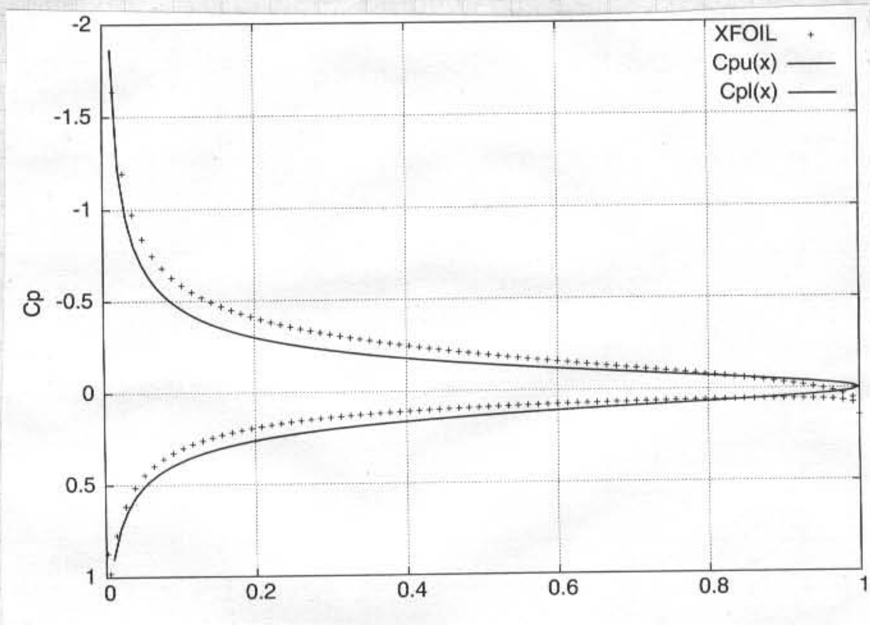
From F2 notes: $\frac{\gamma}{V_\infty} = 2\alpha\sqrt{\frac{c-x}{x}} = 0.14\sqrt{\frac{1-x/c}{x/c}}$



Using $\Delta V_t = \gamma$: $\boxed{\begin{aligned} V_u/V_\infty &= 1 + \tfrac{1}{2}\gamma/V_\infty = 1 + 0.07\sqrt{\tfrac{1-x/c}{x/c}} \\ V_\ell/V_\infty &= 1 - \tfrac{1}{2}\gamma/V_\infty = 1 - 0.07\sqrt{\tfrac{1-x/c}{x/c}} \end{aligned}}$

$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2$ (general)

So $\boxed{\begin{aligned} C_{p_u} &= 1 - \left(\tfrac{V_u}{V_\infty}\right)^2 \\ C_{p_\ell} &= 1 - \left(\tfrac{V_\ell}{V_\infty}\right)^2 \end{aligned}}$



b) $\boxed{\begin{aligned} &\text{TAT result: } C_\ell = 2\pi\alpha = 0.4386 \\ &\text{XFOIL result: } C_\ell = 0.4489 \end{aligned}}$

TAT result is too small by $100\% \times \left(\frac{0.4386}{0.4489} - 1\right) = 2.28\%$ not too bad!

c) $C_\ell = \int_0^1 \left(C_{p_\ell} - C_{p_u}\right) d\left(\frac{x}{c}\right)$ (integrated pressure force)

From the plot we see that C_{p_u} and C_{p_ℓ} differ between TAT and XFOIL.

However, the difference $C_{p_\ell} - C_{p_u}$, which is what matters for C_ℓ, is very nearly the same.

$\boxed{\text{So TAT is better at predicting } C_\ell \text{ rather than } C_p(x)}$