Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
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16.003/16.003 Unified Engineering III, IV
Spring 2009

Problem Set 1

Name: ______________________

Due Date: 2/13/2009

	Time Spent (min)
M1	
M2	
M3	
F1	
F2	
T1	
T2	
T3	
Study Time	

Announcements:

Unified Engineering Problem Set 1 **Lectures:** **M1, M2, M3, M4** ***(look-ahead)***
Week 2 **Spring, 2009** **Units: M4.1, M4.2** **(part** ***w/look-ahead*****)**

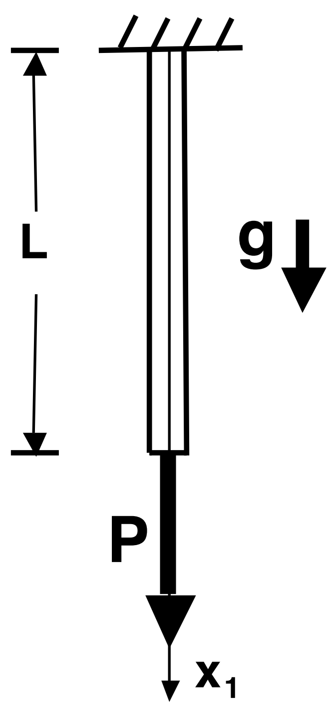
M1 **(M2.1)** ***(5 points)*** This problem serves as a means to review the key concepts and governing equations in general elasticity.

(a) Write out fully, in tensor notation, the governing *independent* equations of elasticity for a solid body. Identify the key assumptions associated with each set of these equations and the underlying fundamental(s) upon which these are based.

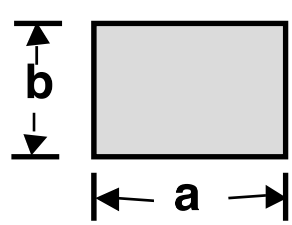
(b) There are also several other equations, known as "compatibility equations". Describe what they are and from where they come. Why aren't these also independent equations?

(c) If engineering notation is used, do these equations change? How? Indicate this either through careful and complete description or by writing out the equations that change. In either case, highlight any key differences.

M2 **(M2.2)** ***(15 points)*** A rod with a rectangular cross-section is suspended from an overhead support as indicated in the accompanying figure. The rod is of length L, and has sides of lengths a and b. The rod is made of an isotropic material with a longitudinal modulus of E, Poisson's ratio of ν, shear modulus of G, and density of ρ. A tip load of magnitude P is applied at the end of the rod. This load can be either tensile or compressive. The entire arrangement is subjected to a gravity field of value g.



CROSS-SECTION



For this problem, ignore the effects of the mass of the rod and only consider the effects of the applied load.

(a) What are the boundary conditions for this configuration?

(b) Determine the stress and strain states throughout the rod.

(c) Determine the displacements throughout the rod.

(d) Comment on the applicability of the rod model for this configuration.

(e) Can the rod model still be used if the cross-section of the rod tapers such that the aspect ratio stays constant at 2 with the area varying by a factor of two with the maximum at the bottom along the length of the rod? Why or why not? Does this explanation change for other variations of area? Be sure to explain clearly using equations if/as needed.

M3 (M2.3) ***(10 points)*** Consider the same configuration as in problem M2.2, but now give consideration to the effects of the mass of the rod. Repeat parts (a) through (d) including the effects of the mass of the rod using the results from the previous problem as reference.

(a) What are the boundary conditions for this configuration?

(b) Determine the stress and strain states throughout the rod.

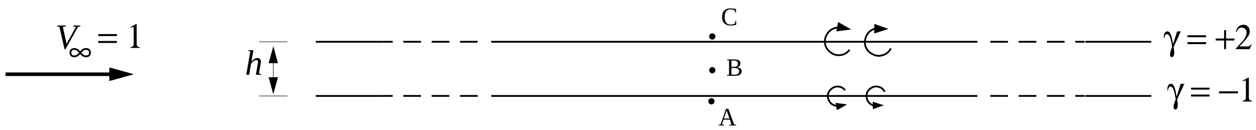
(c) Determine the displacements throughout the rod.

(d) Comment on the applicability of the rod model for this configuration.

(e) Now make an overall comparison of the solutions for the two cases (i.e., including and ignoring the mass of the rod) and make comments. In particular, answer the question as to when the mass of the rod becomes important. How does this change with the direction of the tip load?

(f) The final objective is to design this rod as a structural fuse with failure occurring at the overhead attachment. Knowing that failure occurs when the stress in the rod reaches a material ultimate value of σ_{ult}, describe how this can be done. Comment on how this changes with direction and magnitude of the tip load. Use equations and solve for pertinent values as appropriate.

a) Two long vortex sheets of strengths $\gamma = +2$, $\gamma = -1$, as shown, are located at distance h apart. A horizontal freestream velocity $V_\infty = 1$ is also present. Determine the velocities at points A, B, C.



b) From very far away where the small gap h is nearly invisible, the two sheets will look like a single sheet placed on the center point B. Determine the required strength γ_{app} of this apparent single sheet, such that the velocities at A,C do not change.

XFOIL is a program which computes the flow about airfoils using the panel method, which is effectively exact for our purposes here. It is available on Athena — see the "XFOIL Instructions" link on the UE Fluids web page. A Linux or Windows version can also be downloaded at `http://raphael.mit.edu/xfoil/`

Use XFOIL to compute the "exact" inviscid $C_{p_u}(x)$ and $C_{p_\ell}(x)$ distributions for the NACA 0003 airfoil, at angle of attack $\alpha = 4°$. The $C_p(x)$ distributions can be written to a text file with the `CPWR` command. Note also the c_ℓ which is predicted by XFOIL for this case. By default, XFOIL assumes a unit chord $c = 1$.

a) Using Thin Airfoil Theory (TAT), determine $\gamma(x)$ for a zero-camber (symmetric) airfoil at $\alpha = 4°$. Also determine the corresponding $V_u(x)/V_\infty$, $V_\ell(x)/V_\infty$, and then $C_{p_u}(x)$ and $C_{p_\ell}(x)$. Plot and compare these two $C_p(x)$ distributions with the "exact" ones from XFOIL.

b) Determine the c_ℓ value predicted by TAT for $\alpha = 4°$, and compare with the "exact" c_ℓ from XFOIL.

c) Explain why the TAT results differ somewhat from those of XFOIL. Is TAT better at predicting $C_p(x)$, or the overall lift c_ℓ?

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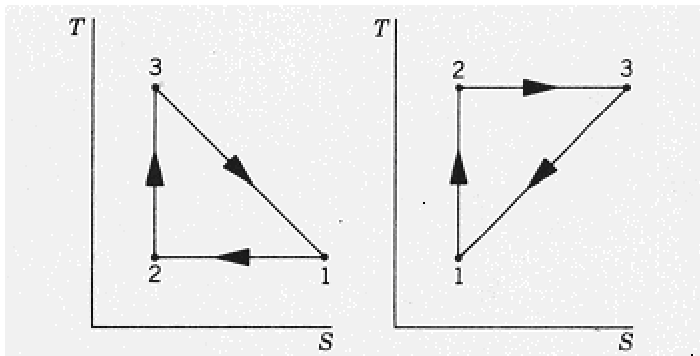
(Add a short summary of the concepts you are using to solve the problem)

Problem T1

The two cycles shown below are composed of internally reversible processes. Determine an expression for the thermal efficiency of each cycle in terms of the temperatures T_1 and T_3. Which of the two cycles has a higher thermal efficiency? Why?



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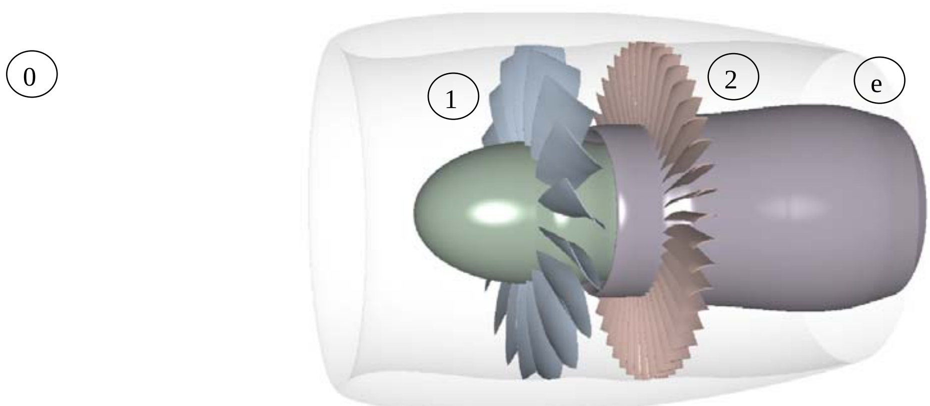
(Add a short summary of the concepts you are using to solve the problem)

Problem T2

Modern airliners are powered by turbo-fan engines, consisting of a gas turbine engine driving a single-stage axial flow fan. At take-off static conditions, the fan shaft power is 25 MW. The static pressure and temperature of the air at the fan face are $p_1 = 0.8$ bar and $T_1 = 285$ K. The fan stagnation pressure ratio is 1.4 and the adiabatic efficiency of the fan is 0.85.The fan inlet and exit states have the same kinetic energy, such that $c_1 = c_2 = 200$ m/s. The flow expands to the atmosphere at the nozzle exit. Ambient pressure and temperature are 1bar and 300K respectively, and air can be assumed an ideal gas with $\gamma = 1.4$ and R = 287 J/kg-K. The mass flow through the core engine can be neglected in this problem.



a) Sketch the process in an h-s diagram and indicate both static and stagnation states for stations 0, 1, 2, and e in one diagram.

b) What is the mass flow through the fan?

c) What are the static temperature and Mach number at station 2?

d) How much entropy per kg of air is generated in the fan?

e) What is the thrust force produced by the fan?

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(Add a short summary of the concepts you are using to solve the problem)

Problem T3

A turbojet engine on an aircraft takes in air at *static temperature* T_0 from the atmosphere and exhausts air at a *static temperature* of T_1 at its exit. The aircraft is in level steady flight at velocity u_0. The exhaust velocity of the engine, relative to the aircraft is u_1. Assume air and the combustion gas to be perfect gases with the same specific heat c_p.

a) Sketch the thermodynamic cycle in an h-s diagram and indicate work and heat transfer in each segment of the cycle.

b) What is the heat rejected to the atmosphere per unit of mass flow?

c) What is the *net heat input* to the engine per unit of mass flow?

d) What is the thermal efficiency $\eta_{thermal}$ of the engine?

e) What is the overall efficiency ($\eta_{overall} = \eta_{thermal} \times \eta_{propulsive}$) of the engine?

Express your answers in terms of the given quantities.