Unified Engineering Problem Set 10
Week 12 Spring, 2009

Solutions

\([1/8(1/2.1)]\)

**Condition A:**
\[
\begin{align*}
\sigma_{11} &= 6f \\
\sigma_{22} &= 2f \\
\sigma_{33} &= 2f \\
\sigma_{12} &= 0 \\
\sigma_{13} &= 0 \\
\sigma_{23} &= 0
\end{align*}
\]

**Condition B:**
\[
\begin{align*}
\sigma_{11} &= -2f \\
\sigma_{22} &= -2f \\
\sigma_{33} &= -2f \\
\sigma_{12} &= 0 \\
\sigma_{13} &= 0 \\
\sigma_{23} &= 0
\end{align*}
\]

**Condition C:**
\[
\begin{align*}
\sigma_{11} &= 9f \\
\sigma_{22} &= 0.5f \\
\sigma_{33} &= 2f \\
\sigma_{12} &= 0 \\
\sigma_{13} &= 0 \\
\sigma_{23} &= 0
\end{align*}
\]

**Condition D:**
\[
\begin{align*}
\sigma_{11} &= -f \\
\sigma_{22} &= 2f \\
\sigma_{33} &= 0.5f \\
\sigma_{12} &= 0 \\
\sigma_{13} &= 0 \\
\sigma_{23} &= 0
\end{align*}
\]
(a) Application of the Tresca condition requires knowledge of the principal stresses.

For Conditions B, C, and D, there are no applied shear stresses, so the applied normal stresses are the principal stresses.

Put these in appropriate order based on magnitude:

\[
\begin{align*}
\text{Condition B} & : \\
\sigma_1 &= \sigma_1 = -2\bar{q} \\
\sigma_2 &= \sigma_2 = -2\bar{q} \\
\sigma_3 &= \sigma_3 = -2\bar{q} \\
\text{Condition C} & : \\
\sigma_1 &= \sigma_3 = 2\bar{q} \\
\sigma_2 &= \sigma_1 = \bar{q} \\
\sigma_3 &= \sigma_2 = 0.5\bar{q} \\
\text{Condition D} & : \\
\sigma_1 &= \sigma_2 = 2\bar{q} \\
\sigma_2 &= \sigma_1 = -\bar{q} \\
\sigma_3 &= \sigma_3 = 0.5\bar{q}
\end{align*}
\]

For Condition A, there is no applied shear stress in the 3-axis since \( \sigma_3 = 0 \) and \( \sigma_{23} = 0 \). Thus, \( \sigma_3 \) is a principal stress.

However, \( \sigma_2 \) is nonzero, so the principal stresses in the 1-2 plane need to be determined.

Call \( \sigma_{33} = \sigma_{33}^{\text{eff}} \) for now and label the two in the 1-2 plane as \( \sigma_1 \) and \( \sigma_2 \). Actual "order" will be determined after...
Determining principal stresses in 1-2 plane.

For the case of planar stress, the principal stresses are the roots (τ) of the equation:

\[ \tau^2 - \tau (\sigma_{11} + \sigma_{22}) + (\sigma_{11} \sigma_{22} - \sigma_{12}^2) = 0 \]

Applies here since we have principal shearing τ. Using the values for Condition A:

\[ \tau^2 - \tau(8q + 2q) + [(8q)(2q) - (4q)^2] = 0 \]

\[ \Rightarrow \tau^2 - 10q \tau + [16q^2 - 16q^2] = 0 \]

\[ \tau(2 - 10q) = 0 \]

\[ \Rightarrow \tau = \sigma_{11} = 10q \]

\[ \tau = \sigma_{12} = 0 \]

Finally, in order for Condition A:

\[ \sigma_{11} = 10q \]

\[ \sigma_{12} = q \]

\[ \sigma_{13} = 0 \]

→ Now apply the Tresca criterion where yield occurs if:
\[
\begin{align*}
|\sigma_I - \sigma_II| &= \sigma_Y \\
\sigma Y &= \sigma Y \\
|\sigma_II - \sigma_III| &= \sigma Y \\
\sigma Y &= \sigma Y
\end{align*}
\]

In addition, the directionality associated with this is that yielding occurs via a shear on the plane of maximum shear stress corresponding to the difference in those two principal stress.

→ For the titanium under consideration, \( \sigma_Y = 98.0 \text{ ksi} \).

→ Apply each condition...

- **Condition A**: \( |\sigma_I - \sigma_II| = |100 - q| = \sigma_Y \)
  
  \[\Rightarrow |9q| = 98.0 \text{ ksi} \]
  
  \[\Rightarrow q = \pm 10.9 \text{ ksi} \]

\[|\sigma_II - \sigma_III| = |q - 0| = \sigma_Y \]

\[\Rightarrow q = \pm 98.0 \text{ ksi} \]
\[
\left| \sigma_{III} - \sigma_{I} \right| = \sigma_y \\
\Rightarrow \quad 10 - 10 \beta = 98.0 \text{ksi} \\
\Rightarrow \quad \beta = \pm 9.8 \text{ksi} \\
\]

→ Critical case is the last one

So, for (A):

Yielding at \( \beta = \pm 9.8 \text{ksi} \)
on plane at 45° to direction of principal stress in r-2 plane* and \( \sigma_{33} \)

**Note**: Find angle in r-2 plane by using:

\[
\Theta_p = \frac{1}{2} \tan^{-1} \left( \frac{2 \sigma_{12}}{\sigma_{11} - \sigma_{22}} \right) \\
= \frac{1}{2} \tan^{-1} \left( \frac{\beta \sigma}{\beta - \sigma} \right) \\
= \frac{1}{2} \tan^{-1} \left( \frac{\beta}{\beta - \sigma} \right) \\
\Rightarrow \quad \Theta_p = \frac{1}{2} (53.10) \\
\text{Only} \quad \Theta_p = 26.6°
Condition B: No shear stresses and all equal extensional stresses
\[ \Rightarrow \text{hydrostatic stress state} \]

Giving all differences = 0

So, for \( B \):
\[ \text{No yielding} \]

Condition C:
\[ |\sigma_{II} - \sigma_{III}| = |2\sigma - 8| = |8| = \sigma_y \]
\[ \Rightarrow \sigma = \pm 98.0 \text{ ksi} \]

\[ |\sigma_{I} - \sigma_{III}| = |4 - 0.5\sigma| = 10.5|\sigma| = \sigma_y \]
\[ \Rightarrow \sigma = \pm 196 \text{ ksi} \]

\[ |\sigma_{III} - \sigma_{I}| = |0.5\sigma - 2\sigma| = 1.5|\sigma| = \sigma_y \]
\[ \Rightarrow \sigma = \pm 65.3 \text{ ksi} \]

\[ \rightarrow \text{critical case is the last one.} \]
So, for $\sigma$:

Yielding at $\sigma = \pm 65.3 \text{ ksi}$

on plane at $45^\circ$ between $\sigma_{22}$ and $\sigma_{33}$

Condition D:

\[ |\sigma_I - \sigma_{II}| = |2\sigma - (-\sigma)| = \sigma_y \]

\[ = 13\sigma = 98.0 \text{ ksi} \]

\[ \Rightarrow \sigma = \pm 32.7 \text{ ksi} \]

\[ |\sigma_{II} - \sigma_{III}| = |\sigma - 0.5\sigma| = \sigma_y \]

\[ = 1.5\sigma = 98.0 \text{ ksi} \]

\[ \Rightarrow \sigma = \pm 65.3 \text{ ksi} \]

\[ |\sigma_{III} - \sigma_I| = |0.5\sigma - 2\sigma| = \sigma_y \]

\[ = 1.5\sigma = 98.0 \text{ ksi} \]

\[ \Rightarrow \sigma = \pm 65.3 \text{ ksi} \]

→ Critical case is the first
So, for D:

\[ \text{yielding at } q = \pm 32.7 \text{ ksi} \]
on plane at +50° between \( \sigma_1 \) and \( \sigma_{22} \)

(b) The von Mises criterion is:

\[
(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2
\]

→ Look at each condition again.

- **Condition A**:

  \[
  (\sigma_1 - \sigma_2)^2 + (\sigma_2 - 0)^2 + (0 - 10q)^2 = 2\sigma_y^2
  \]

  \[
  \Rightarrow 8q^2 + 9q^2 + 100q^2 = 2\sigma_y^2
  \]

  \[
  \Rightarrow 182q^2 = 2\sigma_y^2
  \]

  \[
  \Rightarrow q = \pm \sqrt{\frac{1}{91}} \sigma_y
  \]

  \[
  \Rightarrow \text{for A: } q = \pm 10.3 \text{ ksi}
  \]

- **Condition B**:

  All differences equal zero.
So, again for $\mathbb{B}$:

\[ \text{No yield} \]

- **Condition (C):**

\[ (2q - q)^2 + (q - 0.5q)^2 + (0.5q - 2q)^2 = 20y^2 \]

\[ \Rightarrow q^2 + 0.25q^2 + 2.25q^2 = 20y^2 \]

\[ 3.5q^2 = 20y^2 \]

Finally:

\[ q = \pm \sqrt{\frac{2}{3.5}} y \]

\[ \Rightarrow \text{for (C):} \quad q = \pm 7.41 \text{ ksi} \]

- **Condition (D):**

\[ (2q - (-q))^2 + (-q - 0.5q)^2 + (0.5q - 2q)^2 = 20y^2 \]

\[ \Rightarrow 9q^2 + 2.25q^2 + 2.25q^2 = 20y^2 \]

\[ 13.5q^2 = 20y^2 \]

Finally:

\[ q = \pm \sqrt{\frac{2}{13.5}} y \]

\[ \Rightarrow \text{for (D):} \quad q = \pm 37.7 \text{ ksi} \]
(c) First summarize the overall results:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Critical/t/ [E0, J]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tresca</td>
</tr>
<tr>
<td>A</td>
<td>9.8</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>65.3</td>
</tr>
<tr>
<td>D</td>
<td>32.7</td>
</tr>
</tbody>
</table>

- For each condition, the Tresca Criterion gives a more conservative estimate of the yielding load characteristic \( \phi \), except for the hydrostatic case (Condition B) where both criteria predict no yield load as this is a fundamental basic for each.

- The Tresca Criterion considers yielding on a single plane and thus only the two principal stresses acting on that plane. In contrast, the von Mises Criterion involves and interacts all the applied stresses and thus slightly higher values.
M19 (M12.2)  
Airplane fuselage

\[ R = 8 \text{ ft} \]

\[ t = 0.045 \text{ in} \]

At limit, \( p = 10 \text{ psi} \) (pressure differential)

Pressure causes stress of:

\[ \sigma_{\text{hoop}} = \sigma_{22} = \frac{pR}{t} \]

\[ \sigma_{\text{long}} = \sigma_{11} = \frac{pR}{2t} \]

(a) Stress in skin of fuselage is sum of stress due to pressure differential and stress from other loads accounting for 40% load-carrying factor of skin.

So:

\[ \sigma_{11} = 0.4 \left( \sigma_{11, \text{due to } p} + \sigma_{11, \text{applied}} \right) \tag{1} \]

\[ \sigma_{22} = 0.4 \left( \sigma_{22, \text{due to } p} \right) \tag{2} \]

\[ \sigma_{12} = 0.4 \left( \sigma_{12, \text{applied}} \text{ (torsin)} \right) \tag{3} \]
Using the pressure equation at the limit condition:

\[
\sigma_{11} \text{ (due to } \frac{p}{\rho}) = \frac{(10 \text{ psi}) (8 \text{ feet}) (12 \text{ in}^2/4\text{ ft})}{2(0.045 \text{ in})} = 10.667 \text{ psi} = 10.7 \text{ ksi}
\]

\[
\sigma_{22} \text{ (due to } \frac{p}{\rho}) = \frac{(10 \text{ psi}) (8 \text{ feet}) (12 \text{ in}^2/4\text{ ft})}{2(0.045 \text{ in})} = 21.333 \text{ psi} = 21.3 \text{ ksi}
\]

Using the above (1)', (2)', (3)):

All \( \sigma_{ij} \) in [ksi] :

\[
\sigma_{11} = 4.3 + 0.4 \sigma_{11} \text{ (applied)} \quad (1')
\]

\[
\sigma_{22} = 8.5 \quad (2')
\]

\[
\sigma_{12} = 0.4 \sigma_{12} \text{ (applied)} \quad (3')
\]

Now use the Tresca Criterion:

\[
\begin{align*}
|\sigma_1 - \sigma_2| &= \sigma_y \\
|\sigma_2 - \sigma_3| &= \sigma_y \\
|\sigma_3 - \sigma_1| &= \sigma_y
\end{align*}
\]
The case is plane stress with $\sigma_{III} = 0$.

It is necessary to find the in-plane principal stresses $\sigma_I$ and $\sigma_{II}$. Use:

$$\tau^2 - \tau (\sigma_{II} + \sigma_{22}) + (\sigma_{II} \sigma_{22} - \sigma_{22}^2) = 0$$

and find roots. Do so in the form with the quadratic solution:

$$\tau = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow \sigma_I, \sigma_{II} = \frac{1}{2} \left( \sigma_{II} + \sigma_{22} \right) \pm \sqrt{\left( \sigma_{II} + \sigma_{22} \right)^2 - 4 \left( \sigma_{II} \sigma_{22} - \sigma_{22}^2 \right)^2} \right)^{1/2}$

Working through gives:

$$= \frac{1}{2} \left( \sigma_{II} + \sigma_{22} \right) \pm \frac{1}{2} \sqrt{\sigma_{II}^2 + 2 \sigma_{II} \sigma_{22} + \sigma_{22}^2 - 4 \sigma_{II} \sigma_{22} + 4 \sigma_{22}^2} \right)^{1/2}$$

$$= \frac{1}{2} \left( \sigma_{II} + \sigma_{22} \right) \pm \frac{1}{2} \sqrt{\sigma_{II}^2 - 2 \sigma_{II} \sigma_{22} + \sigma_{22}^2 + 4 \sigma_{22}^2} \right)^{1/2}$$

$$= \frac{1}{2} \left( \sigma_{II} + \sigma_{22} \right) \pm \frac{1}{2} \sqrt{4 \sigma_{22}^2 + (\sigma_{II} - \sigma_{22})^2} \right)^{1/2}$$

Finally:

$$\sigma_I = \left( \frac{\sigma_{II} + \sigma_{22}}{2} \right) + \sqrt{\sigma_{22}^2 + \left( \frac{\sigma_{II} - \sigma_{22}}{2} \right)^2}$$

$$\sigma_{II} = \left( \frac{\sigma_{II} + \sigma_{22}}{2} \right) - \sqrt{\sigma_{22}^2 + \left( \frac{\sigma_{II} - \sigma_{22}}{2} \right)^2}$$

Rewrite the Tresca equations using these equations and $\sigma_{III} = 0$ and $\sigma_y = 50$ kpsi.
\[ 50 \cos i = \left| \sigma_{I} - \sigma_{II} \right| = 2 \sqrt{\sigma_{12}^2 + \left( \frac{\sigma_{11} + \sigma_{22}}{2} \right)^2} \]  
(4)

\[ 50 \cos i = \left| \sigma_{I} \right| = \left( \frac{\sigma_{11} + \sigma_{22}}{2} \right) + \sqrt{\sigma_{12}^2 + \left( \frac{\sigma_{11} - \sigma_{22}}{2} \right)^2} \]  
(5)

\[ 50 \cos i = \left| \sigma_{II} \right| = \left( \frac{\sigma_{11} + \sigma_{22}}{2} \right) - \sqrt{\sigma_{12}^2 + \left( \frac{\sigma_{11} - \sigma_{22}}{2} \right)^2} \]  
(6)

— Use the specific expressions of (4), (5), and (6) to express these for the particular example.

\[ \text{NOTE: ALL IN } \sigma_{Al} \]

\[ \text{from (4):} \quad 50 = \frac{1}{2} \sqrt{0.16 \sigma_{Al}^2 + \left( \frac{4.3 + 0.1 \sigma_{Al} - 8.5}{2} \right)^2} \]

\[ \Rightarrow \quad 25 = \sqrt{0.16 \sigma_{Al}^2 + (0.2 \sigma_{Al} - 2.1)^2} \]  
(4')

\[ \text{from (5):} \quad 50 = \frac{1}{2} \left( 4.3 + 0.1 \sigma_{Al} + 8.5 \right) + \sqrt{(0.2 \sigma_{Al} - 2.1)^2 + 0.16 \sigma_{Al}^2} \]

\[ \Rightarrow \quad 50 = \left( 6.4 + 0.2 \sigma_{Al} \right) + \sqrt{0.16 \sigma_{Al}^2 + (0.2 \sigma_{Al} - 2.1)^2} \]  
(5')

\[ \text{from (6):} \quad 50 = \left( 6.4 + 0.2 \sigma_{Al} \right) - \sqrt{0.16 \sigma_{Al}^2 + (0.2 \sigma_{Al} - 2.1)^2} \]  
(6')

\[ \text{with } \sigma_{Al} = \sigma_{II} \text{ (applied, modified)}, \quad \sigma_{Al} = \sigma_{12} \text{ (applied, modified)} \]
Now apply different values of $\sigma_{\text{AL}}$ for each case and determine the values of $\sigma_{\text{AT}}$ that cause failure. Be sure to catch cases of each equal to zero. Then plot them.

Work the equations to use them: 

**From (4'')**:

$625 = 0.16 \sigma_{\text{AT}}^2 + (0.2\sigma_{\text{AL}} - 2.1)^2$

$\Rightarrow \sigma_{\text{AT}} = 2.5 \sqrt{625 - (0.2\sigma_{\text{AL}} - 2.1)^2}$

**From (5'')**:

$(43.6 - 0.2\sigma_{\text{AL}})^2 = 0.16 \sigma_{\text{AT}}^2 + (0.2\sigma_{\text{AL}} - 2.1)^2$

or

$(-56.4 - 0.2\sigma_{\text{AL}})^2 = 0.16 \sigma_{\text{AT}}^2 + (0.2\sigma_{\text{AL}} - 2.1)^2$

**Solve**:

$\sigma_{\text{AT}} = 2.5 \sqrt{(43.6 - 0.2\sigma_{\text{AL}})^2 - (0.2\sigma_{\text{AL}} - 2.1)^2}$

or

$\sigma_{\text{AT}} = 2.5 \sqrt{(-56.4 - 0.2\sigma_{\text{AL}})^2 - (0.2\sigma_{\text{AL}} - 2.1)^2}$

**From (6'')**:

$0.16 \sigma_{\text{AT}}^2 + (0.2\sigma_{\text{AL}} - 2.1)^2 = (-43.6 + 0.2\sigma_{\text{AL}})^2$

or

$= (56.4 + 0.2\sigma_{\text{AL}})^2$

**Solve**:

$\sigma_{\text{AT}} = 2.5 \sqrt{(-43.6 + 0.2\sigma_{\text{AL}})^2 - (0.2\sigma_{\text{AL}} - 2.1)^2}$

or

$\sigma_{\text{AT}} = 2.5 \sqrt{(56.4 + 0.2\sigma_{\text{AL}})^2 - (0.2\sigma_{\text{AL}} - 2.1)^2}$
To work these with all values in \((\text{ksi})\)

<table>
<thead>
<tr>
<th>(\sigma_{\text{H.L.}})</th>
<th>(\sigma_{12\text{A.T.}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\pm 62.3)</td>
</tr>
<tr>
<td>+10.5</td>
<td>(\pm 62.5)</td>
</tr>
<tr>
<td>+30, -30</td>
<td>(\pm 61.7, \pm 59.1)</td>
</tr>
<tr>
<td>+60, -60</td>
<td>(\pm 57.4, \pm 51.6)</td>
</tr>
<tr>
<td>+90, -90</td>
<td>(\pm 48.2, \pm 37.2)</td>
</tr>
<tr>
<td>+120, -</td>
<td>(\pm 30.1, -)</td>
</tr>
<tr>
<td>+135, -114</td>
<td>0</td>
</tr>
</tbody>
</table>

*From (4)*

<table>
<thead>
<tr>
<th>(\sigma_{\text{H.L.}})</th>
<th>(\sigma_{12\text{A.T.}}) (a)</th>
<th>(\sigma_{12\text{A.T.}}) (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\pm 10.9)</td>
<td>(\pm 140)</td>
</tr>
<tr>
<td>+30, -30</td>
<td>(\pm 93.5, \pm 122)</td>
<td>(\pm 156, \pm 164)</td>
</tr>
<tr>
<td>+60, -60</td>
<td>(\pm 75.0, \pm 134)</td>
<td>(\pm 169, \pm 165)</td>
</tr>
<tr>
<td>+90, -90</td>
<td>(\pm 50.1, \pm 146)</td>
<td>(\pm 181, \pm 181.8)</td>
</tr>
<tr>
<td>+114, -</td>
<td>0, -</td>
<td>higher, -</td>
</tr>
<tr>
<td>-136</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>
\[
\begin{array}{c|c|c}
\theta_{\text{A.L.}} & \sigma_{12, \text{A.I.}} (a) & \sigma_{12, \text{A.I.}} (b) \\
\hline
0 & \pm 10.9 & \pm 14.0 \\
+30, -30 & \pm 9.35, \pm 12.2 & \pm 15.6, \pm 12.4 \\
+60, -60 & \pm 7.50, \pm 13.4 & \pm 16.9, \pm 10.5 \\
+90, -90 & \pm 5.01, \pm 14.6 & \pm 18.1, \pm 8.18 \\
+114, - & 0 \cdot - & \text{higher} \cdot - \\
-136 & - \cdot \text{higher} & 0 \\
\end{array}
\]

Note that (5°) and (6°) produce the same results.

Plot these using the lower values of the (a) and (b) to fit the final envelope.
Operating stress envelope for fuselage via Tresca condition for supercooler stress with limit pressure $\sigma_{\text{L},A,L.}^\text{ksi}$ already accounted.

$\sigma_{12A,T.}$

---

From (5') & (6')

From (4')

---

---

III = Operating Envelope
(5) With the "damage tolerant" approach, use the basic fracture mechanics equation:

\[ \sigma_f = \frac{K_{IC}}{\sqrt{\pi a}} \]

Here: \( 2a = 0.30 \text{ in} \Rightarrow a = 0.150 \text{ in} \)

\( K_{IC} = 31 \text{ ksi} / \sqrt{\text{in}} \) for the 2024 aluminum

\[ \Rightarrow \sigma_f = \frac{31 \text{ ksi} / \sqrt{\text{in}}}{\sqrt{\pi (0.150 \text{ in})}} \]

\[ \Rightarrow \sigma_f = 45.2 \text{ ksi} \]

Thus, if the stress perpendicular to the crack exceeds 45.2, there is failure. The crack can be oriented in any direction, so one must find the principal stresses (i.e., the maximum extensional stresses) and then the related direction for the worst case.

\[ \rightarrow \text{The principal stresses were found in part (a)} \]
\[
\sigma_I = 6.4 + 0.2 \sigma_{AL} + \sqrt{0.16 \sigma_{AT}^2 + (0.2 \sigma_{AL} - 2.1)^2}
\]

\[
\sigma_{II} = 6.4 + 0.2 \sigma_{AL} - \sqrt{0.16 \sigma_{AT}^2 + (0.2 \sigma_{AL} - 2.1)^2}
\]

VALUES IN [ksi]

→ In the case of fracture mechanics, only tensile values are considered and only the lower value needs to be considered.

Thus, use \( \sigma_I \) and set this to the determined value of \( \sigma_I \) VALUES IN [ksi]

\[
\Rightarrow 75.2 = 6.4 + 0.2 \sigma_{AL} + \sqrt{0.16 \sigma_{AT}^2 + (0.2 \sigma_{AL} - 2.1)^2}
\]

working:

\[
(38.8 - 0.2 \sigma_{AL})^2 = 0.16 \sigma_{AT}^2 + (0.2 \sigma_{AL} - 2.1)^2
\]

→ Again, apply different values of \( \sigma_{AL} \) and get resulting values of \( \sigma_{AT} \). Then plot these.

\[
\sigma_{AT} = \pm 2.5 \sqrt{(38.8 - 0.2 \sigma_{AL})^2 - (0.2 \sigma_{AL} - 2.1)^2}
\]
<table>
<thead>
<tr>
<th>$\sigma_{11, \text{A.L.}}$</th>
<th>$\sigma_{12, \text{A.T.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\pm 96.9$</td>
</tr>
<tr>
<td>$+30$, $-30$</td>
<td>$\pm 81.4$, $\pm 110$</td>
</tr>
<tr>
<td>$+60$, $-60$</td>
<td>$\pm 62.3$, $\pm 122$</td>
</tr>
<tr>
<td>$+90$, $-90$</td>
<td>$\pm 33.5$, $\pm 133$</td>
</tr>
<tr>
<td>$+102$</td>
<td>0</td>
</tr>
</tbody>
</table>

all values in [km]\(^{-1}\)

$\rightarrow$ Plot where as before
"Operating stress envelope" for fuselage via damage tolerant approach with limit pressure already accounted
(c) Each of these approaches are different criteria and the plots do look substantially different as may well be expected. The Tresca criterion gives the yield point, while the damage tolerant approach gives the stress at which a crack will critically propagate. This latter case occurs only for tensile stresses and thus has a large operating area for a compressive longitudinal stress. The Tresca criterion considers compressive stress condition as well and thus closer off the operating envelope for that combination. In the tensile regime, the operating stress begins in a similar area as may be expected for a good design.