S10: (10 points)

[a] $F(s) = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$

$$K_1 = \frac{18 - 66 + 54}{(1)(2)} = 3; \qquad K_2 = \frac{72 - 132 + 54}{(-1)(1)} = 6$$

$$K_3 = \frac{162 - 198 + 54}{(-2)(-1)} = 9$$

$$\therefore \quad f(t) = [3e^{-t} + 6e^{-2t} + 9e^{-3t}]u(t)$$

[c] $s_{1,2} = -6 \pm \sqrt{36 - 100} = -6 \pm j8$

$$F(s) = \frac{11s^2 + 172s + 700}{(s+2)(s+6-j8)(s+6+j8)}$$

$$= \frac{K_1}{s+2} + \frac{K_2}{s+6-j8} + \frac{K_2^*}{s+6+j8}$$

$$K_1 = \frac{44 - 344 + 700}{4 - 24 + 100} = 5$$

$$K_2 = \frac{11(-6+j8)^2 + 172(-6+j8) + 700}{(-4+j8)j16}$$

$$= 3 - j4 = 5\underline{/-53.13^\circ}$$

$$\therefore \quad f(t) = [5e^{-2t} + 10e^{-6t}\cos(8t - 53.13^\circ)]u(t)$$

S11: (10 points)

[b] $$F(s) = \frac{K_1}{s} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2}$$

$$K_1 = \frac{10(4)}{4} = 10; \qquad K_2 = \frac{10(12-8+4)}{-2} = -40$$

$$K_3 = \frac{d}{ds}\left\{\frac{10(3s^2+4s+4)}{s}\right\}\Bigg|_{s=-2}$$

$$= \frac{10[(s)(6s+4) - (3s^2+4s+4)]}{s^2}\Bigg|_{s=-2} = 20$$

$$F(s) = \frac{10}{s} - \frac{40}{(s+2)^2} + \frac{20}{s+2}$$

$$f(t) = [10 - 40te^{-2t} + 20e^{-2t}]u(t)$$

[c] $s_{1,2} = -2 \pm \sqrt{4-5} = -2 \pm j1$

$$F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+2-j1} + \frac{K_3^*}{s+2+j1}$$

$$K_1 = \frac{50}{5} = 10$$

$$K_2 = \frac{d}{ds}\left\{\frac{s^3 - 6s^2 + 15s + 50}{s^2 + 4s + 5}\right\}\Big|_{s=0}$$

$$= \frac{(s^2+4s+5)(3s^2-12s+15) - (s^3-6s^2+15s+50)(2s+4)}{(s^2+4s+5)^2}\Big|_{s=0}$$

$$= \frac{5(15) - 50(4)}{25} = -5$$

$$K_3 = \frac{s^3 - 6s^2 + 15s + 50}{s^2(s+2+j1)}\Big|_{s=-2+j1}$$

$$(-2+j1)^3 = -2 + j11; \qquad (-2+j1)^2 = 3 - j4$$

$$K_3 = \frac{-2 + j11 - 6(3-j4) + 15(-2+j1) + 50}{(3-j4)(j2)}$$

$$= 3 + j4 = 5\underline{/53.13^\circ}$$

$$F(s) = \frac{10}{s^2} - \frac{5}{s} + \frac{5\underline{/53.13^\circ}}{s+2-j1} + \frac{5\underline{/-53.13^\circ}}{s+2+j1}$$

$$f(t) = [10t - 5 + 10e^{-2t}\cos(t + 53.13^\circ)]u(t)$$

S12: (10 points)

[d] $F(s) = \frac{K_1}{(s+2)^3} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2}$

$$K_1 = s^2 + 6s + 5 \Big|_{s=-2} = -3$$

$$K_2 = \frac{d}{ds}\{s^2 + 6s + 5\}\Big|_{s=-2} = 2s + 6\Big|_{s=-2} = 2$$

$$2K_3 = \frac{d}{ds}(2s+6)\Big|_{s=-2} = 2; \qquad K_3 = 1$$

$$F(s) = \frac{-3}{(s+2)^3} + \frac{2}{(s+2)^2} + \frac{1}{s+2}$$

$$f(t) = -\frac{3t^2e^{-2t}}{2} + 2te^{-2t} + e^{-2t} = [(2t - 1.5t^2 + 1)e^{-2t}]u(t)$$

[c]

$$F(s) = \begin{array}{r|l} & \quad s+5 \\ \hline s+20 & s^2 + 25s + 150 \\ & s^2 + 20s \\ \hline & \quad 5s + 150 \\ & \quad 5s + 100 \\ \hline & \qquad 50 \end{array}$$

$$F(s) = s + 5 + \frac{50}{(s+20)} = s + 5 + \frac{50}{s+20}$$

$$f(t) = \delta'(t) + 5\delta(t) + 50e^{-20t}u(t)$$