Massachusetts Institute of Technology
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16.003/16.004 Unified Engineering III, IV
Spring 2009

Problem Set 10

Name: ______________________

Due Date: 4/24/2009

	Time Spent (min)
S10	
S11	
S12	
M18	
M19	
SPL11	
Study Time	

Announcements:

S10: (10 points)

Find f(t) for each of the following functions:

$$F(s) = \frac{18s^2 + 66s + 54}{(s + 1)(s + 2)(s + 3)}.$$

$$F(s) = \frac{11s^2 + 172s + 700}{(s + 2)(s^2 + 12s + 100)}.$$

S11: (10 points)

Find f(t) for each of the following functions:

$$F(s) = \frac{10(3s^2 + 4s + 4)}{s(s + 2)^2}.$$

$$F(s) = \frac{s^3 - 6s^2 + 15s + 50}{s^2(s^2 + 4s + 5)}.$$

S12: (10 points)

Find f(t) for each of the following functions:

$$F(s) = \frac{s^2 + 6s + 5}{(s + 2)^3}.$$

$$F(s) = \frac{s^2 + 25s + 150}{s + 20}.$$

M18 (M12.1) ***(10 points)*** A specially-assembled design team is considering the design of regions of the wing carry-through box of a transport aircraft. Multiple loading conditions must be held. They have identified some key locations in the box where the stresses are greatest and expressed these as proportional to some loading characteristic, q. This loading characteristic is related to the overall structural loading parameters, but it is not expressed directly in terms of such due to *certain design concerns*. The four stress conditions are:

Condition A:	$\sigma_{11} = 8q$	$\sigma_{12} = 4q$
	$\sigma_{22} = 2q$	$\sigma_{13} = 0$
	$\sigma_{33} = q$	$\sigma_{23} = 0$
Condition B:	$\sigma_{11} = -2q$	$\sigma_{12} = 0$
	$\sigma_{22} = -2q$	$\sigma_{13} = 0$
	$\sigma_{33} = -2q$	$\sigma_{23} = 0$
Condition C:	$\sigma_{11} = q$	$\sigma_{12} = 0$
	$\sigma_{22} = 0.5q$	$\sigma_{13} = 0$
	$\sigma_{33} = 2q$	$\sigma_{23} = 0$
Condition D:	$\sigma_{11} = -q$	$\sigma_{12} = 0$
	$\sigma_{22} = 2q$	$\sigma_{13} = 0$
	$\sigma_{33} = 0.5q$	$\sigma_{23} = 0$

The designers are currently considering a titanium with a yield stress of 98.0 ksi.

(a) Using Tresca's yield criterion, calculate the value of the loading characteristic, q, for the onset of yielding and the associated plane on which yield would occur for each.

(b) Repeat this calculation using the von Mises criterion.

(c) Comment on the overall results.

M19 (M12.2) ***(10 points)*** Much of an airplane fuselage can be modeled to first order as a pressurized cylinder with superposed longitudinal and torsional loads due to the empennage. The values of these longitudinal and torsional loads due to the empennage loading are independent. Consider a mid-sized airplane with a fuselage radius of 8 feet and a skin thickness of 0.045 inches. Given the "skeleton" construction of the fuselage that includes longerons and frames, assume that the skin only carries 40% of the applied loads due to pressure and from the empennage. For such a "thin shell" construction, the stresses due to pressure can be shown to be proportional to the pressure differential, p, and radius, R, and inversely proportional to the thickness, t. The "hoop stress" is in the circumferential direction and is:

$$\sigma_{hoop} = pR/t$$

while the stress in the longitudinal direction of the cylinder is:

$$\sigma_{long} = pR/2t$$

The limit condition for flight is 10 psi differential between the cabin pressure and the exterior pressure.

(a) Consider a piece of material on the fuselage that is stressed as noted (including the 40% factor). The fuselage is made of 2024 aluminum that has a modulus of 10.1 Msi and a yield stress of 50 ksi. Using limit condition and the Tresca failure criterion, determine an "operating stress envelope" for this piece of material with axes of applied longitudinal stress and applied torsional stress (due to the applied longitudinal and torsional loads *separate* from the pressure-induced stresses).

(b) A *damage tolerant* approach is now taken such that the material must tolerate a through-crack of 0.30 inches in length that can be detected nondestructively in scheduled inspection intervals. The fracture toughness of the 2024 aluminum is 31 ksi $in^{1/2}$. In applying the fracture mechanics criterion **(*look ahead:* see M5.4, pp. 5-9)**, ignore all stresses except that perpendicular to the crack. Determine the "operating stress envelope" for this piece of the same piece of material using limit condition and the damage tolerant approach.

(c) Compare the two results and make relevant comments.