UE Fluids — Problem 13 + 14 Solution — S'09

a) $\boxed{1-\frac{V^2}{2h_0}} = 1-\frac{V^2}{2h+V^2} = \frac{(2h+\cancel{V^2})-\cancel{V^2}}{2h+V^2} = \frac{1}{1+\frac{V^2}{2h}} = \frac{1}{1+\frac{\gamma-1}{2}\frac{V^2}{(\gamma-1)h}} = \frac{1}{1+\frac{\gamma-1}{2}\frac{V^2}{a^2}} = \boxed{\frac{1}{1+\frac{\gamma-1}{2}M^2}}$

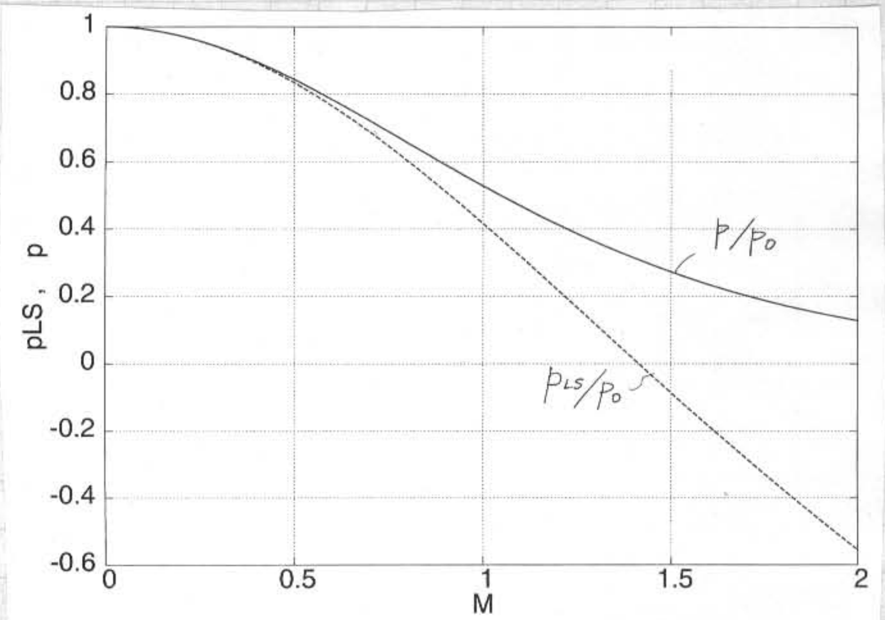
b) $p = p_0\left(1-\frac{V^2}{2h_0}\right)^{\frac{\gamma}{\gamma-1}} \simeq p_0 - \frac{1}{2}\frac{\gamma}{\gamma-1}\frac{p_0}{h_0}V^2$, state eqn: $\frac{\gamma}{\gamma-1}\frac{p_0}{h_0} = \rho_0$

$\therefore \boxed{p_{LS} = p_0 - \frac{1}{2}\rho_0 V^2}$ same as Bernoulli with $\rho = \rho_0 = \phi$

c) $h = h_0 - \frac{1}{2}V^2$, $\rho = \frac{\gamma}{\gamma-1}\frac{p}{h} = \rho_0\left(1-\frac{V^2}{2h_0}\right)^{\frac{1}{\gamma-1}}$

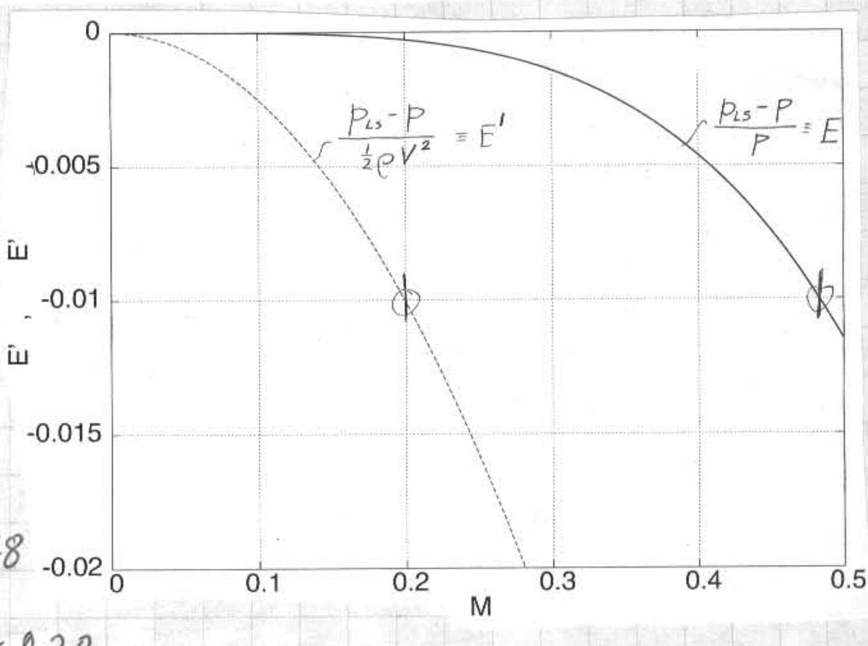
also, $M = \frac{V}{a} = \frac{V}{\sqrt{(\gamma-1)h}} = \frac{V}{\sqrt{(\gamma-1)(h_0-\frac{1}{2}V^2)}}$

Plot $\frac{p}{p_0}$, $\frac{p_{LS}}{p_0}$ vs M →



d), e)

Plot E, E' vs M →



For $\frac{p_{LS}}{p_0}$ within 1%, must have $M < 0.48$

For $\frac{p_{LS}-p}{\frac{1}{2}\rho V^2}$ within 1%, must have $M < 0.20$