Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
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16.003/16.004 Unified Engineering III, IV
Spring 2009

Problem Set 11

Name: ________________________

Due Date: 5/1/2009

	Time Spent (min)
S13	
S14	
S15	
F13-14	
F15-16	
SPL12	
Study Time	

Announcements:

S13: (10 points)

Find $f(t)$ for each of the following functions.

a) $$F(s) = \frac{10s^2 + 85s + 95}{s^2 + 6s + 5}.$$

b) $$F(s) = \frac{5(s^2 + 8s + 5)}{s^2 + 4s + 5}.$$

S14: (10 points)

There is no energy stored in the circuit shown in Fig. P12.28 at the time the switch is opened.

a) Derive the integrodifferential equation that governs the behavior of the voltage v_o.

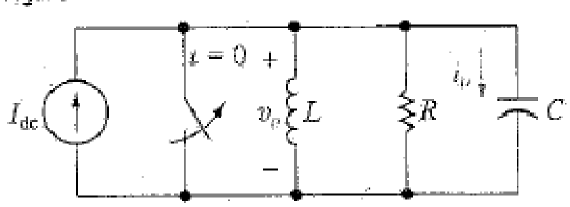
b) Show that

$$V_o(s) = \frac{I_{dc}/C}{s^2 + (1/RC)s + (1/LC)}.$$

c) Show that

$$I_o(s) = \frac{sI_{dc}}{s^2 + (1/RC)s + (1/LC)}.$$

Figure P12.28



S15: (10 points)

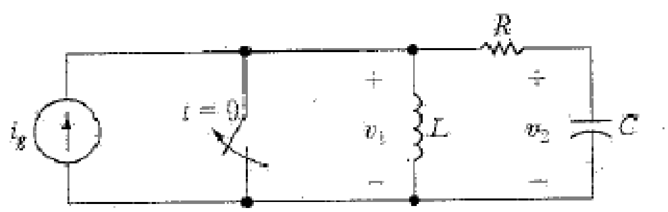
There is no energy stored in the circuit shown in Fig. P12.29 at the time the switch is opened.

a) Derive the integrodifferential equations that govern the behavior of the node voltages v_1 and v_2.

b) Show that

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}.$$

Figure P12.29



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Fluids Problem F13+F14 (worth 2 Pset Q's)

a) Express the quantity

$$1 - \frac{V^2}{2h_o}$$

in terms of the Mach number alone. Simplify as much as possible.

b) For the case $V^2 \ll h_o$ (low speed flow), simplify the compressible-flow isentropic pressure/velocity relation

$$p(V) \;=\; p_o \left(1 - \frac{V^2}{2h_o}\right)^{\frac{\gamma}{\gamma-1}}$$

using the following Taylor expansion we've employed previously in class.

$$(1+\epsilon)^b \;=\; 1 \,+\, b\epsilon \,+\, \text{h.o.t.}$$

Drop the h.o.t. bits and simplify as much as possible using the state equation. You will then have a low-speed approximation $p_{LS}(V)$ which should look familiar.

c) Derive expressions for $h(V)$, $\rho(V)$, and also the Mach number $M(V) \,=\, V/\sqrt{(\gamma-1)h}$. Using V as a dummy plotting parameter, plot p/p_o and p_{LS}/p_o versus M, both on the same plot. Pick a range of V which roughly gives $M = 0 \ldots 2$. For numerical evaluation, you can assume

$$(\gamma-1)h_o = 1 \qquad\qquad p_o = 1 \qquad\qquad \rho_o = \frac{\gamma}{\gamma - 1}\frac{p_o}{h_o} = \gamma = 1.4$$

although the dimensionless plots should be independent of these particular numerical values.

d) Plot the fractional pressure "error"

$$E = \frac{p_{LS} - p}{p}$$

which results from using the low-speed approximation, again versus M. Determine the range of Mach numbers where the low-speed approximation p_{LS} is valid to within 1%, or equivalently where $|E| < 0.01$

e) For low speed flows, we care more about the pressure *coefficient* rather than the absolute pressure. Hence, plot the pressure-coefficient error

$$E' = \frac{p_{LS} - p}{\frac{1}{2}\rho V^2}$$

again versus M. Determine the range of Mach numbers where the low-speed approximation p_{LS} is valid for obtaining the pressure coefficient to within 1%.

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Fluids Problem F15+F16 (worth 2 Pset Q's)

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a) Anderson problem 8.7.
Skip the last entropy calculation part, since in UE Fluids we don't explicitly work with entropy.

b) Anderson problem 8.8

c) Anderson problem 8.12

d) Anderson problem 8.15
Also calculate the stagnation pressure at the stagnation point of the SR-71.