S16:

a)

$$C_n = \frac{1}{T}\int_{-T/4}^{0} -V_m e^{-jn\omega_o t}\,dt + \frac{1}{T}\int_{0}^{T/4} V_m e^{-jn\omega_o t}\,dt$$

$$= \frac{-V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_{T/4}^{0}\right] + \frac{V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_{0}^{T/4}\right]$$

$$= -j\frac{V_m}{\pi n}\left(1-\cos\frac{n\pi}{2}\right)$$

$$v(t) = \sum_{n=-\infty}^{\infty} -j\frac{V_m}{\pi n}\left(1-\cos\frac{n\pi}{2}\right)e^{jn\omega_o t}$$

b)

$$c_0 = a_v = \left(\frac{1}{2}(\frac{T}{4})I_m(2)\right)\frac{1}{T} = \frac{I_m}{4}$$

$$c_n = \frac{1}{T}\int_{-T/4}^{0} -\frac{4I_m}{T}te^{-jn\omega_o t}\,dt + \frac{1}{T}\int_{0}^{T/4}\frac{4I_m}{T}te^{-jn\omega_o t}\,dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{-4I_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\Big|_{-T/4}^{0}\right]$$

$$= \frac{-I_m}{(n\pi)^2}\left[1 - e^{jn\pi/2}(-jn\pi/2+1)\right]$$

$$\text{Int2} = \frac{4I_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\Big|_{0}^{T/4}\right]$$

$$= \frac{I_m}{(n\pi)^2}\left[e^{-jn\pi/2}(jn\pi/2+1) - 1\right]$$

$$\therefore\quad c_n = \frac{I_m}{n^2\pi^2}[e^{-jn\pi/2}(1+jn\pi/2) - 1 + e^{jn\pi/2}(1-jn\pi/2) - 1]$$

$$= \frac{I_m}{n^2\pi^2}[2\cos(n\pi/2) + n\pi\sin(n\pi/2) - 2]$$

S17:

[a] $F(\omega) = j\frac{2A}{\omega_o}\omega \qquad -\frac{\omega_o}{2} \leq \omega \leq \frac{\omega_o}{2}$

$$f(t) = \frac{1}{2\pi}\int_{-\omega_o/2}^{\omega_o/2} \frac{j2A}{\omega_o}\omega e^{j\omega t}\, d\omega$$

$$= \frac{jA}{\pi\omega_o}\left[\frac{e^{jt\omega}}{-t^2}(jt\omega - 1)\Big|_{-\omega_o/2}^{\omega_o/2}\right]$$

$$= \frac{A}{\pi\omega_o t^2}[\omega_o t\cos(\omega_o t/2) - 2\sin(\omega_o t/2)]$$

[b] $f(t) = \frac{A}{\pi\omega_o}\left[\frac{\omega_o t\cos(\omega_o t/2) - 2\sin(\omega_o t/2)}{t^2}\right]$

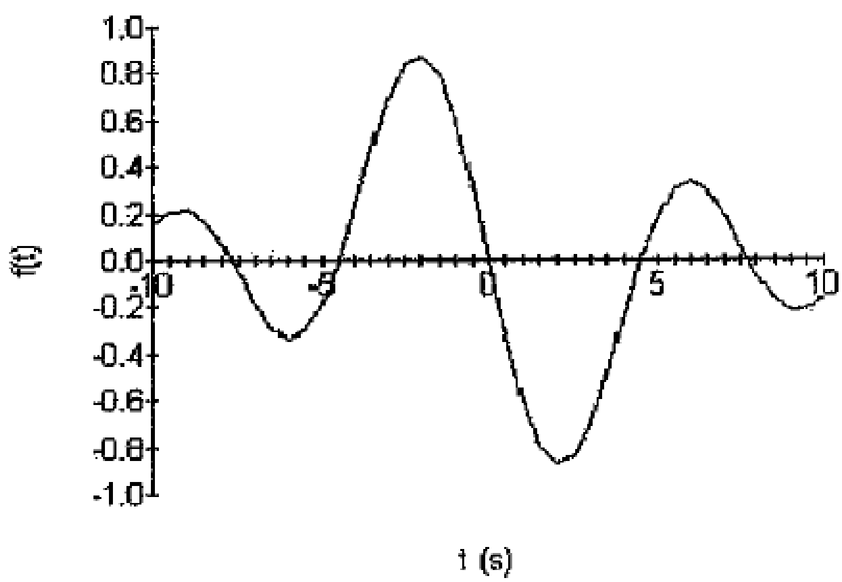
$$f(0) = \lim_{t\to 0}\left\{\frac{A}{\pi\omega_o}\left[\frac{\omega_o t(-\frac{\omega_o}{2}\sin\frac{\omega_o t}{2}) + \omega_o\cos\frac{\omega_o t}{2} - \omega_o\cos\frac{\omega_o t}{2}}{2t}\right]\right\}$$

$$= \lim_{t\to 0}\left\{\frac{A}{\pi\omega_o}\left[\frac{-\omega_o^2}{4}\sin\left(\frac{\omega_o t}{2}\right)\right]\right\} = 0$$

[c] When $A = 2\pi$ and $\omega_o = 2$ rad/s

$$f(t) = \frac{1}{t^2}[2t\cos t - 2\sin t]$$

$f(-t) = -f(t) \qquad$ odd function



S18:

[a] $F(s) = \mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \left[\frac{1}{(a+j\omega)^2}\right] + \left[\frac{1}{(a-j\omega)^2}\right]$$

$$= \frac{2(a^2-\omega^2)}{(a^2-\omega^2)^2 + 4a^2\omega^2} = \frac{2(a^2-\omega^2)}{(a^2+\omega^2)^2}$$

[b] $F(s) = \mathcal{L}\{t^3 e^{-at}\} = \dfrac{-6}{(s+a)^4}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-6}{(a+j\omega)^4} + \frac{-6}{(a-j\omega)^4} = -j48a\omega\frac{a^2-\omega^2}{(a^2+\omega^2)^4}$$

[c] $F(s) = \mathcal{L}\{e^{-at}\cos\omega_0 t\} = \dfrac{s+a}{(s+a)^2+\omega_0^2} = \dfrac{0.5}{(s+a)-j\omega_0} + \dfrac{0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{0.5}{(a+j\omega)-j\omega_0} + \frac{0.5}{(a+j\omega)+j\omega_0}$$

$$+ \frac{0.5}{(a-j\omega)-j\omega_0} + \frac{0.5}{(a-j\omega)+j\omega_0}$$

$$= \frac{a}{a^2+(\omega-\omega_0)^2} + \frac{a}{a^2+(\omega+\omega_0)^2}$$

[d] $F(s) = \mathcal{L}\{e^{-at}\sin\omega_0 t\} = \dfrac{\omega_0}{(s+a)^2+\omega_0^2} = \dfrac{-j0.5}{(s+a)-j\omega_0} + \dfrac{j0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-(\omega-\omega_0)}{a^2+(\omega-\omega_0)^2} + \frac{(\omega-\omega_0)}{a^2+(\omega+\omega_0)^2}$$

[e] $F(\omega) = \displaystyle\int_{-\infty}^{\infty} \delta(t-t_o)e^{-j\omega t}\,dt = e^{-j\omega t_o}$

(Use the sifting property of the Dirac delta function.)