a) The sonic $(M=1)$ condition can only occur at a **throat**, or area minimum. This can be seen from the $A/A^*$ versus $M$ plot. Since the flow starts at $M<<1$ in the bottle, and $A$ monotonically decreases, then $M$ must monotonically increase up to $M=1$ at the nozzle.

b) Stagnation conditions in the bottle, which is in effect a reservoir:

\[ P_0 = 4 \times 10^5 \text{ Pa (given)} \]
\[ T_0 = 300 \text{ K (given)} \]
\[ \rho_0 = \frac{P_0}{RT_0} = 4.65 \text{ kg/m}^3 \]

At the throat, where $M=1$, we have

\[ P = P^* = P_0 \left[1 + \frac{\gamma}{2} \right]^{-\frac{\gamma}{\gamma-1}} = 2.11 \times 10^5 \text{ Pa} \]
\[ T = T^* = T_0 \left[1 + \frac{\gamma}{2} \right]^{-1} = 250 \text{ K} \]
\[ \rho = \rho^* = \rho_0 \left[1 + \frac{\gamma}{2} \right]^{-\frac{1}{\gamma-1}} = 2.95 \text{ kg/m}^3 \]
\[ u = a^* = \sqrt{\frac{\gamma}{RT_0}} = 316.9 \text{ m/s} \]

c) Initial \( \dot{m} = \rho u A = 2.95 \text{ kg/s} \cdot 316.9 \text{ m/s} \cdot 0.0004 \text{ m}^2 = 0.374 \text{ kg/s} \)

Initial mass in bottle: \( m = (\rho_0) \cdot v = 3.318 \text{ kg/m}^3 \cdot 2 \cdot \frac{1}{1000 \text{ m}^3} = 0.0033 \text{ kg} \)

If the initial \( \dot{m} \) is sustained, the mass will flow out in

\[ \Delta t = \frac{m}{\dot{m}} = 0.025 \text{ s} \]

In reality, \( \dot{m} \) will decrease as the bottle empties so the actual time will be longer.

\[ m(t) \]