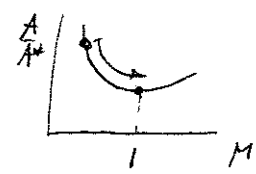
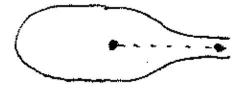
UE Fluids — Problem F19 Solution — S'07 (1)

a) The sonic ($M=1$) condition can only occur at a throat, or area minimum. This can be seen from the A/A^* versus M plot. Since the flow starts at $M \ll 1$ in the bottle, and A monotonically decreases, then M must monotonically increase up to $M=1$ at the nozzle.





b) Stagnation conditions in the bottle, which is in effect a reservoir:

$$p_0 = 4\times10^5 \text{ Pa (given)}, \quad T_0 = 300\text{ K (given)}, \quad \rho_0 = \frac{p_0}{RT_0} = 4.65 \text{ kg/m}^3$$

At the throat, where $M=1$, we have

$$p = p^* = p_0\left[1+\frac{\gamma-1}{2}\right]^{-\frac{\gamma}{\gamma-1}} = 2.11\times10^5 \text{ Pa}$$

$$T = T^* = T_0\left[1+\frac{\gamma-1}{2}\right]^{-1} = 250 \text{ K}$$

$$\rho = \rho^* = \rho_0\left(1+\frac{\gamma-1}{2}\right)^{-\frac{1}{\gamma-1}} = 2.95 \text{ kg/m}^3$$

$$u = a^* = \sqrt{\gamma R T^*} = 316.9 \text{ m/s}$$

c) Initial $\dot{m} = \rho u A = 2.95 \text{ kg/m}^3 \cdot 316.9 \text{ m/s} \cdot 0.0004 \text{ m}^2 = 0.374 \text{ kg/s}$

Initial mass in bottle: $m = \rho_0 \mathcal{V} = 3.318 \text{ kg/m}^3 \cdot 2\ell \, \frac{1}{1000\ell/\text{m}^3} = 0.0093 \text{ kg}$

If the initial $\dot{m}$ is sustained, the mass will flow out in $\boxed{\Delta t \simeq \frac{m}{\dot{m}} \simeq 0.025\text{ s}}$ (minimum)

In reality, $\dot{m}$ will decrease as the bottle empties so the actual time will be longer.