UE Fluids — Problem F19-20 Solution — S'08

a) Supersonic expansion: $A_e^* = A_t^* = A_t \quad \therefore \frac{A_e}{A_e^*} = \frac{A_e}{A_t} = 2.193$ as given

For $\frac{A}{A^*} = 2.193$: $\boxed{M_e = 2.3}$, $\frac{p_{oe}}{p_e} = 12.5$, $\frac{\rho_{oe}}{\rho_e} = 6.076$, $\frac{T_{oe}}{T_e} = 2.058$

Isentropic flow all the way from reservoir, $\therefore$ $\boxed{p_{o_e} = p_r = 5 \text{ atm}}$ $\boxed{T_{o_e} = T_r = 520R}$

$\boxed{p_e = \frac{p_{oe}}{12.5} = 0.4 \text{ atm}}$, $\boxed{T_e = \frac{T_{oe}}{2.058} = 252.7\ R}$ $\left(p_e = 4.052 \times 10^4\ Pa,\ T_e = 140.4K \right)$

$\boxed{\rho_e = \frac{p_e}{RT_e} = 1.006\ kg/m^3}$ $\quad a_e = \sqrt{\gamma R T_e} = 237.5\ m/s$

$\boxed{u_e = M_e a_e = 546.3\ m/s}$

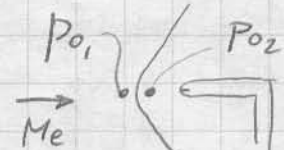
b) Supersonic expansion: $p_{oe} = p_r = 1$ atm

At exit, $\frac{p_{oe}}{p_e} = \left[1 + \frac{\gamma-1}{2} M_e^2\right]^{\frac{\gamma}{\gamma-1}} = \frac{1 \text{ atm}}{0.3143 \text{ atm}} = 3.18$

$$M_e = \left[\frac{2}{\gamma-1}\left\{\left(\frac{p_{oe}}{p_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}\right]^{1/2} = \left[\frac{2}{\gamma-1}\left((3.18)^{\frac{\gamma-1}{\gamma}} - 1\right)\right]^{1/2} = 1.4$$

For $M_e = 1.4$, $\boxed{A_e/A^* = 1.115}$ (Appendix A)

c) Supersonic nozzle: $p_{oe} = p_r = 2.02 \times 10^5\ Pa = p_{o_1}$



Pitot pressure is $p_{pitot} = 8.92 \times 10^4\ Pa = p_{o_2}$

For $\frac{p_{o_2}}{p_{o_1}} = \frac{8.92 \times 10^4}{2.02 \times 10^5} = 0.4416$, $M_e = 2.65$ (Appendix B)

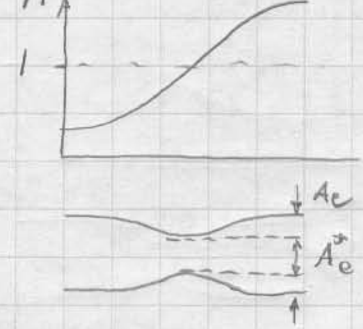
For $M_e = 2.65$, $\boxed{\frac{A_e}{A_e^*} = 3.036}$

d) First, determine p_e for <u>special cases</u>, starting with lowest p_e

A. <u>Ideally expanded</u>. No shock, $\therefore p_{oe} = p_r$, $A_e^* = A_t$

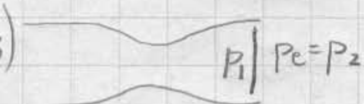
Supersonic case for $\frac{A_e}{A_e^*} = 1.53$: $M_e = 1.88$, $\frac{p_{oe}}{p_e} = 6.497$

$$p_e = \frac{p_{oe}}{p_{oe}/p_e} = \frac{p_r}{p_{oe}/p_e} = \frac{1\text{ atm}}{6.497} = 0.1539 \text{ atm}$$

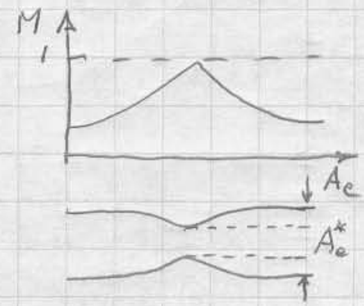


B. Shock just inside inlet. Use normal-shock relations.

From case A: $M_1 = 1.88$, $p_1 = 0.1539$ atm, $\frac{p_2}{p_1} = 3.957$ (App B)



Using normal-shock relations: $p_e = p_1\left(\frac{p_2}{p_1}\right) = 0.1539 \cdot 3.957 = 0.609$ atm

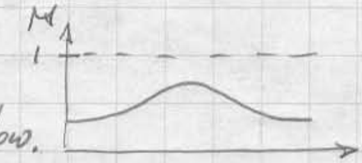


C. Shock pushed all the way to throat. No loss, $p_{oe} = p_r$

Subsonic case for $\frac{A_e}{A_e^*} = 1.53$: $M_e = 0.42$, $\frac{p_{oe}}{p_e} = 1.129$

$$p_e = \frac{p_{oe}}{p_{oe}/p_e} = \frac{p_r}{p_{oe}/p_e} = \frac{1\text{ atm}}{1.129} = 0.886 \text{ atm}$$

Now look at specified p_e values.

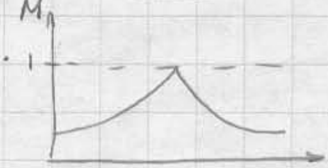


a) $p_e = 0.94$ atm $>$ 0.886 atm for case C. Subsonic flow.

$$p_{oe} = p_r = p_e\left[1 + \frac{\gamma-1}{2}M_e^2\right]^{\frac{\gamma}{\gamma-1}} \rightarrow M_e = \left[\left[\left(\frac{p_r}{p_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{2}{\gamma-1}\right]^{1/2} = 0.3$$

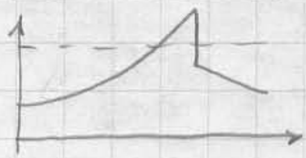
Could also have used App. A for $\frac{p_o}{p} = \frac{p_{oe}}{p_e} = \frac{1\text{ atm}}{0.94\text{ atm}} = 1.0638 \rightarrow M_e = 0.3$

b) $p_e = 0.886$ atm. This is case C. Incipient choking.



$M_e = 0.42$ from above

c) $p_e = 0.75$ atm. Between cases B, C. Shock inside.



M_e = ? (unknown p_{oe}, requires method in F20)

d) $p_e = 0.154$ atm. This is case A. Matched exit.

$M_e = 1.88$ from above.