UE Fluids Problem F3 Solution S'09

1a) $\boxed{A_0 = \frac{1}{\pi}\int_0^{\pi} \theta \, d\theta = \frac{1}{\pi}\left[\frac{1}{2}\theta^2\right]_0^{\pi} = \frac{\pi}{2}}$

$$A_n = -\frac{2}{\pi}\int_0^{\pi} \theta \cos n\theta \, d\theta = -\frac{2}{\pi}\left\{ \cancel{\frac{1}{n}\theta \sin n\theta \Big|_0^{\pi}}^{=0} - \frac{1}{n}\int_0^{\pi} \sin n\theta \, d\theta \right\}$$ (integrating by parts)

$$A_n = -\frac{2}{\pi}\frac{1}{n}\cdot\left(-\cos n\theta\Big|_0^{\pi}\right)$$

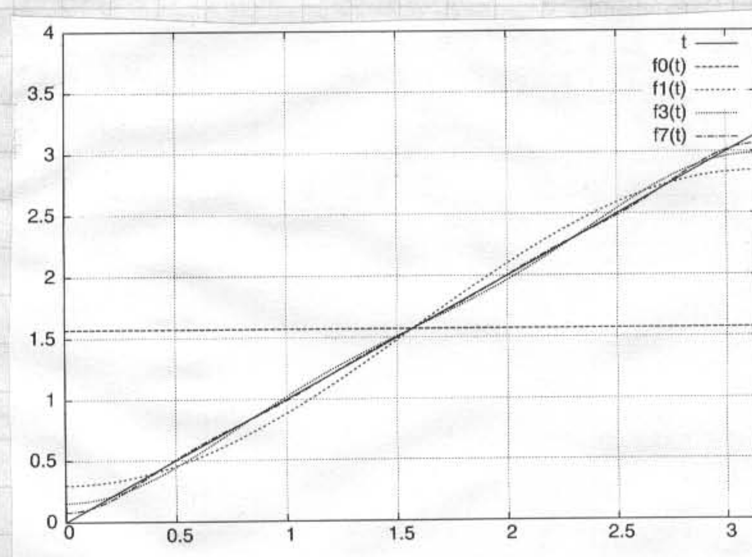
note: $\cos(0) = 1$

$$\cos(n\pi) = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$\boxed{A_n = \begin{cases} 0 & n \text{ even} \\ \dfrac{4}{\pi n^2} & n \text{ odd} \end{cases}}$$

$$\therefore\ f(\theta) \simeq \frac{\pi}{2} - \frac{4}{\pi}\left[\cos\theta + \frac{\cos 3\theta}{3^2} + \frac{\cos 5\theta}{5^2} + \dots\right]$$

1b) The plot clearly shows that the series approaches $f(\theta)$ as the number of terms N is increased. Accuracy improves as N is increased.



2a) $V_u(x) = V_\infty + \frac{1}{2}\gamma(x) \quad \rightarrow \quad V_u^2 = V_\infty^2 + V_\infty\gamma + \frac{1}{4}\gamma^2$

$V_\ell(x) = V_\infty - \frac{1}{2}\gamma(x) \quad \rightarrow \quad V_\ell^2 = V_\infty^2 - V_\infty\gamma + \frac{1}{4}\gamma^2$

$p_u + \frac{1}{2}\rho V_u^2 = p_{o\infty} \quad \rightarrow \quad \boxed{p_u = p_{o\infty} - \frac{1}{2}\rho\left[V_\infty^2 + V_\infty\gamma + \frac{1}{4}\gamma^2\right]}$

$p_\ell + \frac{1}{2}\rho V_\ell^2 = p_{o\infty} \quad \rightarrow \quad \boxed{p_\ell = p_{o\infty} - \frac{1}{2}\rho\left[V_\infty^2 - V_\infty\gamma + \frac{1}{4}\gamma^2\right]}$

$\boxed{L' = \int_0^c (p_\ell - p_u)\,dx = \int_0^c \rho V_\infty \gamma \, dx = \rho V_\infty \int_0^c \gamma\, dx}$ (all but $V_\infty\gamma$ terms cancel)

2b) $\Gamma = \int d\Gamma = \int_0^c \gamma\, dx$

$\boxed{L' = \rho V_\infty \Gamma = \rho V_\infty \int_0^c \gamma\, dx}$ same result as 2a) but much easier!