Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
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16.003/16.003 Unified Engineering III, IV
Spring 2009

Problem Set 2

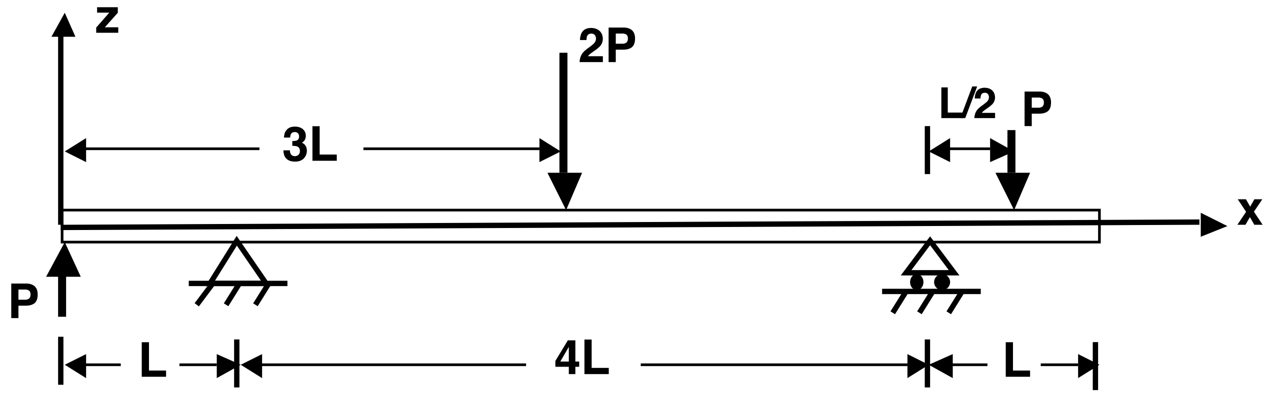
Name: ______________________

Due Date: 2/20/2009

	Time Spent (min)
M4	
M5	
T4	
T5	
F3	
Study Time	

Announcements:

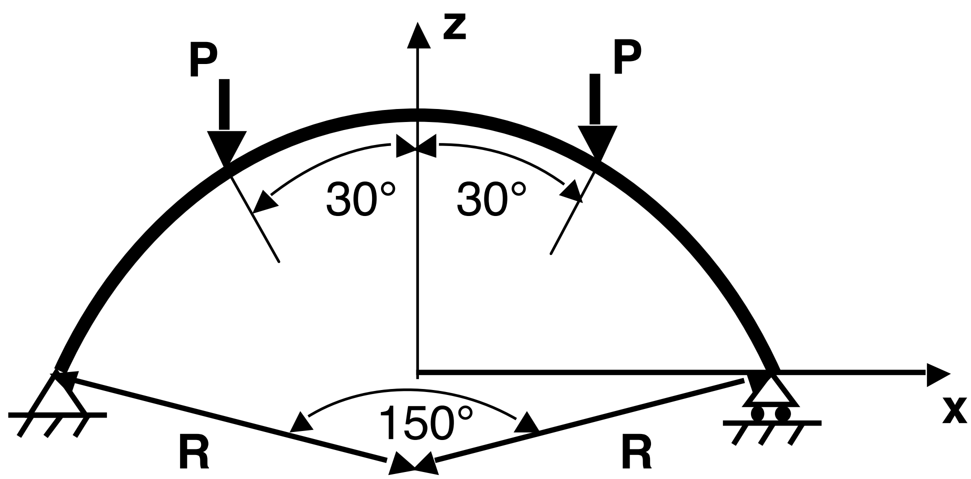
M4 (M3.1) ***(15 points)*** A beam of total length 6L is pinned after the first length, L, of the beam, and has a roller support inboard by a length L from the end of the beam. The beam is loaded by a concentrated upward load of magnitude P at the beam tip (x = 0), by a concentrated downward load of magnitude 2P at the half point (x = 3L), and by a concentrated downward load of magnitude P at a distance L/2 outboard of the roller support. This configuration is shown in the accompanying figure.



(a) Determine the reactions for this structural configuration.

(b) Determine the shear and moment diagrams.

M5 (M3.2) ***(15 points)*** Consider an arched beam that is a 150° arc piece of a circular geometry with a radius of R. The arch is simply-supported and is loaded at each point 30° from its peak by a downward load of magnitude P. The arch has a radius of R. This configuration is shown in the accompanying figure.



(a) Determine the reactions for this structural configuration.

(b) Determine the axial force, shear force, and bending moment along the beam as functions of the distance from the origin along the x-axis. Draw these as functions of x.
(NOTE: The axial and shear force resultants at any point act **parallel** and **perpendicular** to the tangent line of a beam at that point. This is because the "virtual cut" is perpendicular to the tangent line.)

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(Add a short summary of the concepts you are using to solve the problem)

Problem T4

An inventor thinks that he has found a clever way to improve the performance of a ramjet by carrying out the heat addition at a constant temperature, T_{max}. The ideal cycle representing this consists of isentropic compression from station 0 (far upstream) to station 1, isothermal heat addition (and expansion) from 1 to 2, isentropic expansion from 2 to 3 and constant pressure heat rejection to the atmosphere from 3 to 0. The cycle temperature T_{max} (=T_1 = T_2) is given by $T_{max}/T_0 = 10$, the ratio $T_2/T_3 = 2.5$, and the entropy change from 1 to 2 is given by $\Delta s_{1\text{-}2}/c_p = \ln(4) = 1.386$. The air can be considered to be an ideal gas with $\gamma = 1.4$ and $R = 287$ J/kg-K.

a) Sketch this cycle in an h-s diagram and label all states (static and stagnation).

b) What is the thermal efficiency of this cycle?

c) What would be the efficiency of a Brayton cycle with the same maximum temperature and this ratio of T_2/T_3? It might be helpful to draw the Brayton cycle on the same sketch as for part a).

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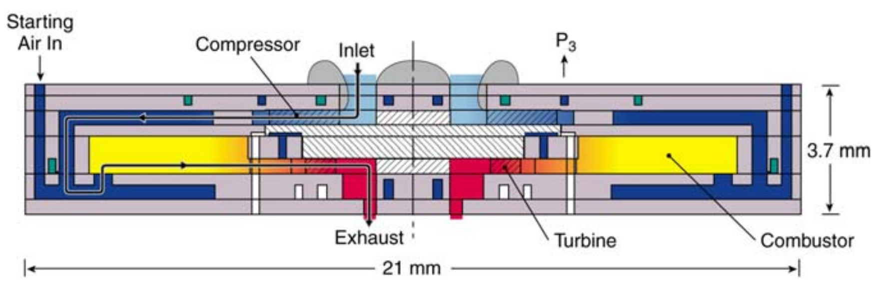
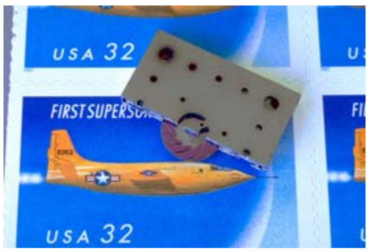
(Add a short summary of the concepts you are using to solve the problem)

Problem T5

Consider a "Micro-Engine", invented by Professor Epstein at the Gas Turbine Laboratory, which is a small (10mm in diameter or less) complete gas turbine engine made using micro-fabrication technology developed in the electronics industry. The first use of these will be as a source of power in small packages (the power and energy densities are more than an order of magnitude larger than current batteries) but one can also think of many propulsion uses for them.





A key feature of a successful micro-engine is a high enough thermal efficiency. This problem addresses the component technology needed to achieve this. The peak temperature in the cycle is limited by the material used and is 1500K. Suppose the cycle pressure ratio is 3:1 and the turbine has an adiabatic efficiency of 0.7. The inlet conditions can be assumed 300 K and 1 bar. Assume air to be a perfect gas with $\gamma = 1.4$ and R = 287 J/kg K.

a) Sketch the cycle in an h-s diagram.

b) How does the thermal efficiency vary with compressor efficiency? Derive an expression and sketch the trend.

c) At what value of compressor efficiency does the cycle produce net work?

d) If the inlet of the micro-engine has a diameter of 2 mm, and the inlet Mach number is 0.3, what sort of power can be delivered (via a micro power turbine downstream of the micro-engine) if the compressor efficiency is 0.55?

This first problem is intended as a Fourier analysis drill.

Any function over the interval $0 \leq \theta \leq \pi$ can be defined in terms of its Fourier cosine series coefficients A_n:

$$f(\theta) \;=\; A_0 \;-\; \sum_{n=1}^{N} A_n \,\cos n\theta$$

The negative sign is non-standard, but is used here to be consistent with the F3 notes.
1a) Determine the coefficients A_0, A_n for the particular case

$$f(\theta) \;=\; \theta$$

Normally, numerical integration can be used to compute A_n for any chosen n. But for this (simple) case you are required to perform the integration for A_n exactly, with n left unspecified. You can then plug in n values to get $A_1, A_2 \ldots$ as needed.
Correctness check: You should get $A_n = 0$ for $n = 2, 4, 6 \ldots$, and $A_n \neq 0$ for $n = 1, 3, 5 \ldots$.

1b) Plot $f(\theta)$, and its series approximations with $N = 0, 1, 3, 7$ (all on one plot). Does approximating $f(\theta)$ with its cosine series appear to be valid in this case?

This second problem is practice with alternative ways to calculate airfoil lift.

A vortex sheet on the x-axis extends over $0 \leq x \leq c$, and has some strength distribution $\gamma(x)$. There is also a freestream velocity V_∞ parallel to the x-axis. The fluid density is ρ.

2a) Determine the pressures below and above the sheet, $p_l(x)$ and $p_u(x)$.

2b) Obtain an expression for the lift/span L' as an integral of the pressure forces on the sheet.

2c) Obtain an expression for the lift/span L' by using the Kutta-Joukowsky theorem,
$L' = \rho V_\infty \Gamma$.