Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
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16.003/16.003 Unified Engineering III, IV
Spring 2009

Problem Set 3

Name: ______________________

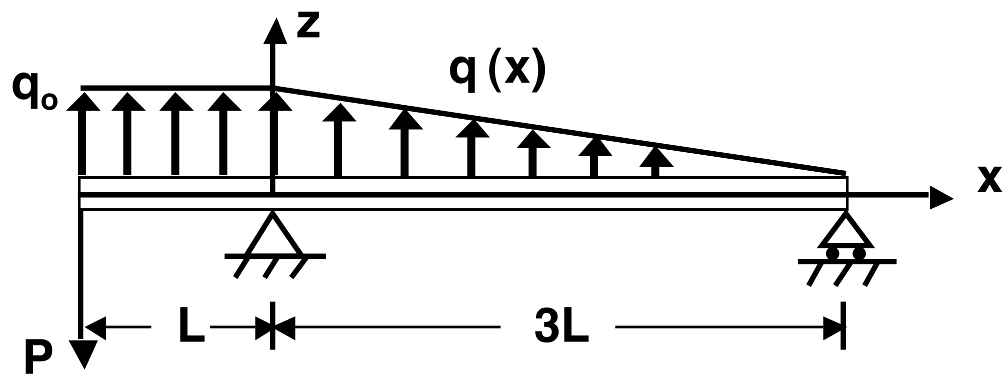
Due Date: 2/27/2009

	Time Spent (min)
M6	
M7	
T6	
F4	
F5	
Study Time	

Announcements:

Unified Engineering Problem Set 3 — **Lectures: M7, M8**
Week 4 Spring, 2009 — **Units: M4.3**

M6 **(M4.1)** ***(15 points)*** A beam of total length 4L has a roller support at the far end and is pinned at a distance L from the other end. The beam is loaded by a downward tip load of magnitude P at the near tip, by a constant distributed upward loading of magnitude q_o inboard to the pin support, and by a linearly-varying distributed upward loading of magnitude q(x) that tapers to zero at the roller support and reaches its peak value of q_o at the pin support. The integrated load due to the distributed loadings, q_o and q(x), along the beam is equal in magnitude to the value of the load, P.



(a) Determine the reactions for this structural configuration.

(b) Using the relationships between loading, shear, and moment, determine the loading, shear, and moment distributions. Draw these diagrams.

(c) Check the obtained values for these parameters at the mid-point in each of the two bays of the beam (i.e., @ x = - L/2, x = 3L/2).

M7 **(M4.2)** ***(15 points)*** Now that we have begun to learn about beams, we can use this to build on the simple models of airplanes that we worked on in Week 5 (Problem Set 4) of last term when we modeled the lift distribution on a wing. We first will further explore how wings carry load in level flight.

The wing of the airplane, as shown subsequently, can be modeled as a beam of total span L which has no supports. The beam has a concentrated load (the weight of the *fuselage*[1], its contents, and the *empennage*[2]) P at its center and a

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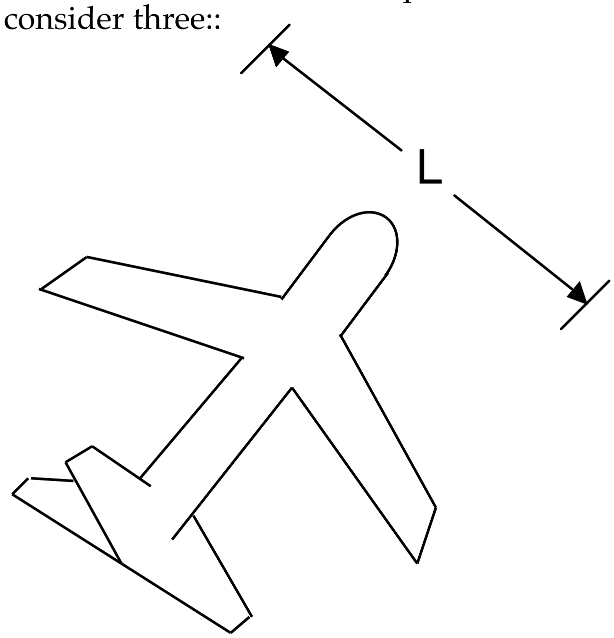
[1] The *fuselage* is that part of the airplane between the wings where the passengers and/or freight are carried.

[2] The *empennage* is more commonly known as the tail.

distributed load (the lift of the wing minus its weight) along its span. You learn from Fluids that this distribution is often modeled as varying, with different possible variations, along the span. We examined two possible variations in the Fall term problem. We will add another, simpler one here (constant along the span) so that we consider three::

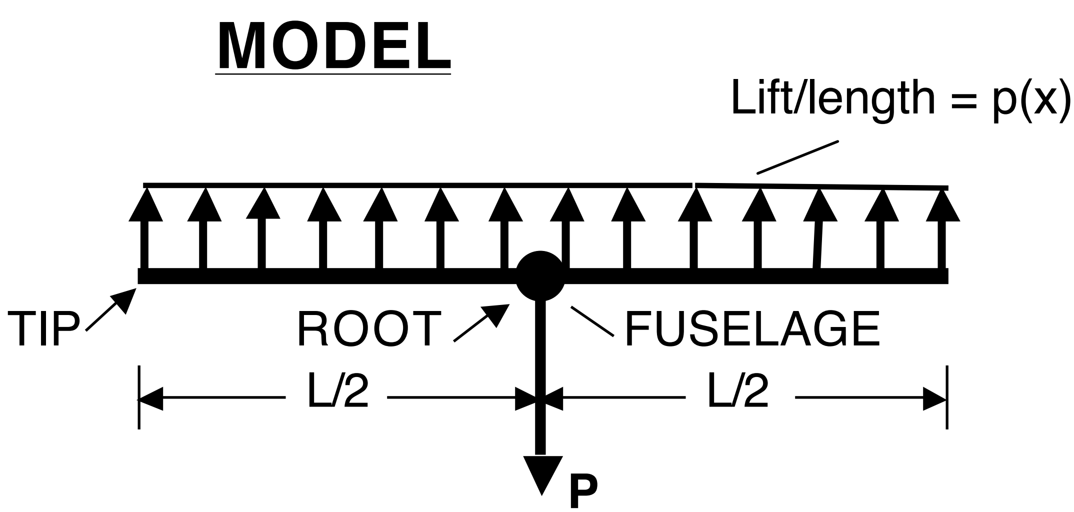


(1) constant along the *span* of the wing;

(2) linear variation along the *span* of the wing with maximum value at the *root* and with a value at the *tip* of half that at the *root*; and

(3) quadratic variation along the *span* of the wing (such that Lift = b - ax^2) with maximum value at the *root* and with a value of zero at the *tip*.

For simplicity, we ignore the weight of the wing in this work. The model is illustrated here for Case 1 (lift constant along the span).



For each case, perform analysis for parts (a) and (b), expressing results in terms of P, p_o, L, and the distance x from the root.

(a) Determine the reactions for the structural configuration.

(b) Determine the axial force, shear force, and bending moment as functions of the distance from the *root*[3] of the wing.

Subsequently, compare the results for the various lift models. In order to do this, it will be necessary to normalize the expressions. Do this for distance from the root by normalizing by L to get expressions in (x/L). Expressions for loading and loading resultants will need to be normalized using the "known" values of length, L, and known weight, P. This will be used with p_o to give expressions in terms of $[p_o(L)^n/P]$ where the value of the exponent n on the length, L, will depend upon the loading (resultant) being considered. With this established for each case, then:

(c) Directly compare the axial force, shear force, and bending moment as functions of normalized distance from the *root* for the various models by plotting each of these on a common plot.

(d) Using common sense arguments and the results from part (b), describe where it is likely that the wing is most highly loaded for each case.

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[3] The *root* of the wing is the location where the wing is joined to the fuselage.

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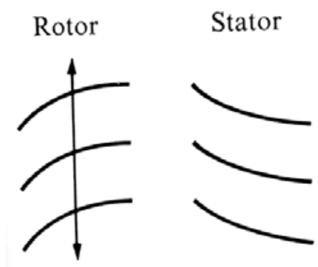
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

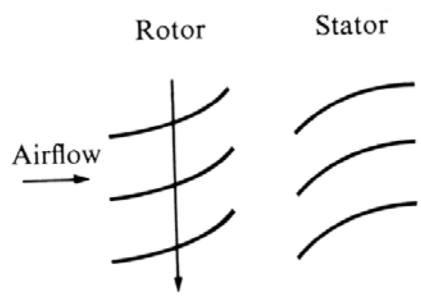
Problem T6

Consider the blading of a single-stage axial turbomachine shown below. What kind of machine (compressor or turbine) is represented by cases 1 and 2? What would happen in case 3? Draw the velocity vector diagrams and explain.

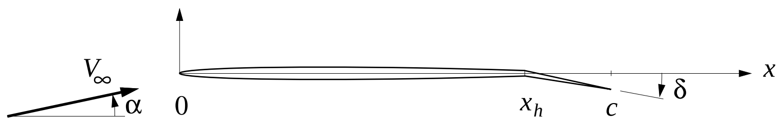


Case	Rotation	Inlet flow direction
1	↓	→
2	↑	←
3	↓	←

Now consider the blading below. Would it be desirable to build a compressor stage according to this drawing? Why or why not?



A symmetric airfoil has a trailing edge flap, with the hinge at $x_h/c = 0.8$, with the flap set at some small downward deflection angle δ.



a) Define and sketch the camberline-slope dZ/dx, both versus x and versus θ.

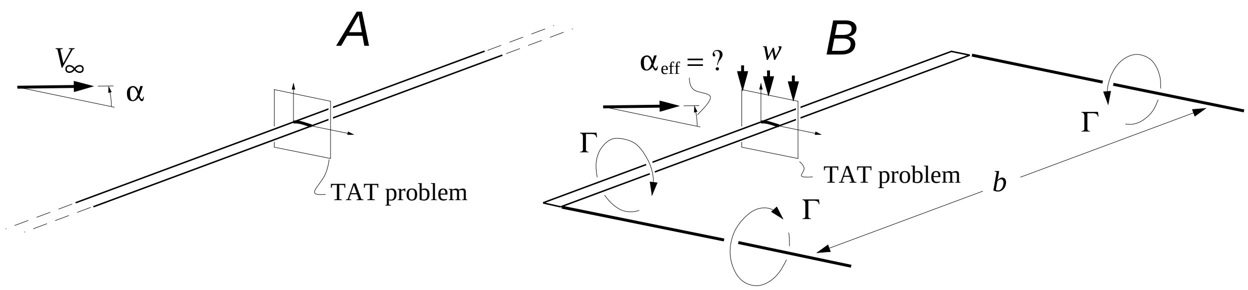
b) Use Thin Airfoil Theory to determine the airfoil's c_ℓ and $c_{m,c/4}$, as functions of α and δ. You may use either analytical or numerical integration for the necessary Fourier analysis.

c) Important quantities for an airplane designer are the angle of attack's force derivatives, and the flap's *control derivatives*.

$$\frac{\partial c_\ell}{\partial \alpha} \;,\quad \frac{\partial c_{m,c/4}}{\partial \alpha} \qquad , \qquad \frac{\partial c_\ell}{\partial \delta} \;,\quad \frac{\partial c_{m,c/4}}{\partial \delta}$$

The latter two indicate how much the c_ℓ and c_m will change in response to a control deflection, and thus ultimately determine how an airplane will respond to a control deflection. Determine all four derivatives for the present flapped airfoil.

Figure A shows a standard TAT problem for a 2D (infinitely-long 3D) wing. The freestream speed is some known V_∞ at some known angle of attack α. Figure B shows the same situation, except that the wing has been clipped to a span b, which results in the two semi-infinite vortices trailing from the wingtips. The circulation about each tip vortex is the same Γ as on the wing, in the directions shown for upward lift.



a) Let's assume that the chord is small, $c \ll b$, so that the TAT "box" can be treated as one point on the wing. Determine the vertical velocity w due to the two new tip vortices, at the TAT box midway between the two tips.
Hint: You may find it helpful to do a brief look-ahead to lecture F6, although this is not essential.

b) Determine the new effective freestream angle of attack α_{eff} seen by the TAT problem in B, as a result of the presence of the tip vortices.

c) Relate α and α_{eff} in this case to the α_{2D} and α_{3D} angles you dealt with in Lab 2.