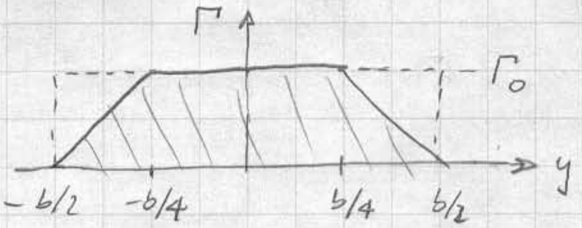
UE Fluids F6 Solution S'09

a)
$$\Gamma(y) = \begin{cases} \Gamma_o (2 - 4y/b), & b/4 \ldots b/2 \\ \Gamma_o & -b/4 \ldots b/4 \\ \Gamma_o (2 + 4y/b), & -b/2 \ldots -b/4 \end{cases}$$



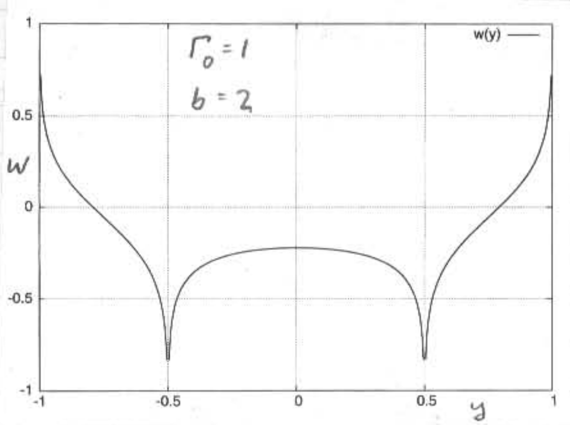
$$\boxed{L = \int_{-b/2}^{b/2} \rho V_\infty \Gamma \, dy = \frac{3}{4} \rho V_\infty \Gamma_o b} \quad \text{(graphically)}$$

b)
$$\boxed{\gamma(y) = -\frac{d\Gamma}{dy} = \begin{cases} 4\Gamma_o/b \\ 0 \\ -4\Gamma_o/b \end{cases}}$$

c)
$$w(y_o) = \frac{1}{4\pi} \int_{-b/2}^{b/2} \gamma \frac{dy}{y_o - y} = \frac{1}{4\pi} \frac{4\Gamma_o}{b} \left[-\int_{-b/2}^{-b/4} \frac{dy}{y_o - y} + \int_{b/4}^{b/2} \frac{dy}{y_o - y} \right]$$

$$w(y_o) = \frac{\Gamma_o}{\pi b} \left[\ln|y_o - y| \Big|_{-b/2}^{-b/4} - \ln|y_o - y| \Big|_{b/4}^{b/2} \right]$$

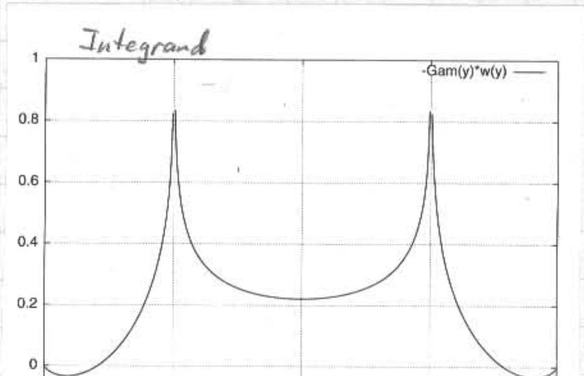
$$\boxed{w(y_o) = \frac{\Gamma_o}{\pi b} \left[\ln \frac{|y_o + b/4|}{|y_o + b/2|} - \ln \frac{|y_o - b/2|}{|y_o - b/4|} \right]}$$



d).
$$D_i = \int L' \alpha_i \, dy = \int \rho \cancel{V_\infty} \Gamma \left(\frac{-w}{\cancel{V_\infty}} \right) dy = \int_{-b/2}^{b/2} \rho \Gamma (-w) \, dy$$

Plug in piecewise $\Gamma(y)$ definition:

$$D_i = \frac{\rho \Gamma_o^2}{\pi b} \left\{ -\int_{b/4}^{b/2} \left(2 - \frac{4y}{b}\right) \left(-\ln \frac{|y + b/4|}{|y + b/2|} + \ln \frac{|y - b/2|}{|y - b/4|} \right) dy \right.$$
$$+ \int_{-b/4}^{b/4} \left(\quad '' \quad \right) dy$$
$$\left. + \int_{-b/2}^{-b/4} \left(2 + \frac{4y}{b}\right) \left(\quad '' \quad \right) dy \right\}$$



UE Fluids F7 Solution S'09

a) $\alpha_i(y_0) = \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \frac{d\Gamma}{dy} \frac{dy}{y_0 - y}$

Known result for elliptical $\Gamma(y)$: $\alpha_i = \frac{\Gamma_0}{2b V_\infty}$ (constant)

b) $\alpha_{eff} = \alpha - \alpha_i = \alpha - \frac{\Gamma_0}{2bV_\infty}$

c) $c_\ell = 2\pi(\alpha_{eff} - \alpha_{L=0}) = 2\pi\left(\alpha - \frac{\Gamma_0}{2bV_\infty} - \alpha_{L=0}\right)$ (constant)

d) $\Gamma = \frac{1}{2} V_\infty c \cdot c_\ell$

$$\underbrace{\Gamma_0}_{\text{constant}} \sqrt{1 - (2y/b)^2} = \underbrace{\frac{1}{2} V_\infty \cdot 2\pi \overbrace{\left(\alpha - \frac{\Gamma_0}{2bV_\infty} - \alpha_{L=0}\right)}^{\alpha_{eff}}}_{\text{constant}} \; c(y)$$

$c(y)$ must be elliptical:

$$c(y) = c_0 \sqrt{1 - (2y/b)^2} \quad ; \quad c_0 = \frac{\Gamma_0}{V_\infty \pi (\alpha_{eff} - \alpha_{L=0})}$$

e) Let's assume $\Gamma(y)$ will still be elliptical, but Γ_0 will change

With $c(y)$ staying fixed, we have from d),

$$\Gamma_0 = \frac{1}{2} V_\infty \cdot 2\pi \left(\alpha - \frac{\Gamma_0}{2bV_\infty} - \alpha_{L=0}\right) c_0$$

solve for Γ_0:

$$\Gamma_0 \left(1 + \frac{\pi c_0}{2b}\right) = V_\infty \pi (\alpha - \alpha_{L=0}) c_0$$

$$\Gamma_0 = V_\infty c_0 \pi \frac{(\alpha - \alpha_{L=0})}{1 + \frac{\pi c_0}{2b}}$$

This is valid for any α, giving $\boxed{\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}}$

<u>Elliptical $\Gamma(y)$</u> is consistent with $\alpha_i = \text{const}$, $c_\ell = \text{const}$.