Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.003/16.003 Unified Engineering III, IV
Spring 2009

Problem Set 4

Name: ______________________

Due Date: 3/6/2009

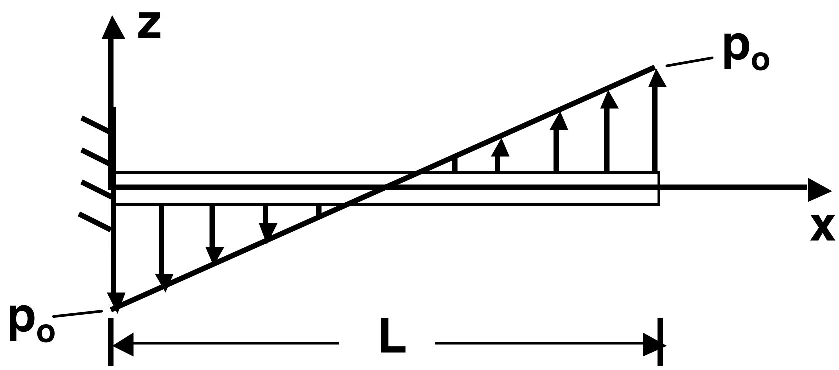
	Time Spent (min)
M8	
M9	
T7	
T8	
T9	
F6	
F7	
Lab 1	
Lab 2	
GE Part A	
Study Time	

Announcements:

Please record times spent on the labs so far this semester. Thanks!

Unified Engineering Problem Set 4 **Lectures: M9, M10**
Week 5 Spring, 2009 **Units: M4.4**

M8 **(M5.1)** ***(10 points)*** A beam of length L is clamped at one end in its cantilevered configuration. The beam has a constant cross-section with area of A and moment of inertia of I, and is made of a material with modulus of E and Poisson's ratio of ν. The beam is loaded by a linearly-increasing load acting downward over the first half of the beam with intensity of p_o at the root of the beam; and acting upward over the last half of the beam with the intensity of p_o at the tip. This cantilevered configuration is shown in the accompanying figure.

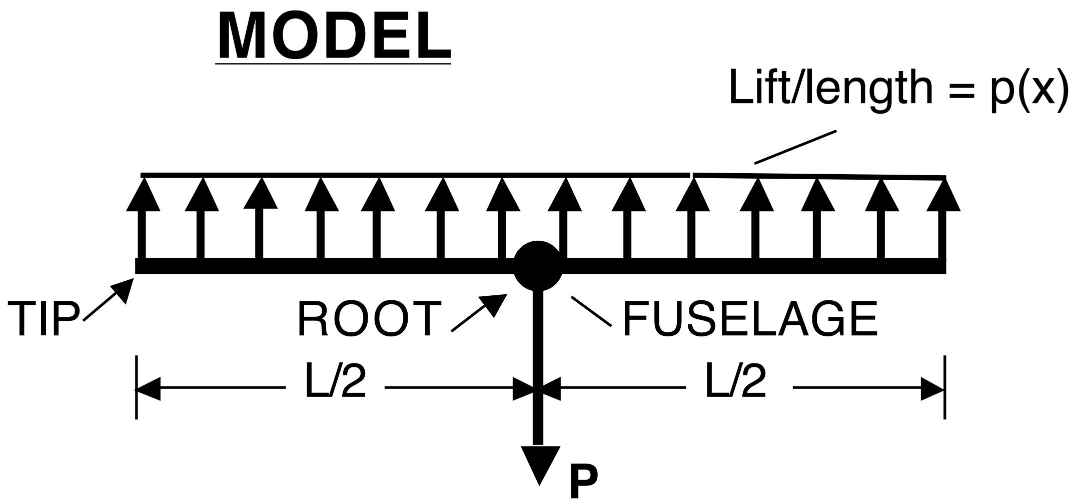


(a) Determine the maximum deflection of this beam and its location.

(b) Determine the maximum axial stress magnitude, σ_{xx}, and its location in the x-direction.

(c) Determine the maximum shear stress magnitude, σ_{xz}, and its location in the x-direction.

M9 **(M5.2)** ***(10 points)*** Let's continue exploring our simple model of how wings carry load in level flight. We now expand this to consider stress distributions and deflections. The three models of the load configuration continue to be considered here. The model for the constant value load configuration is again shown here. Use the results from the solution for the problem set for Week 4 (Problem Set 3) for the axial force, shear force, and bending moment as appropriate. Assume that the wing has constant cross-sectional properties of I and A, and is made of an isotropic material with a modulus of E and Poisson's ratio of ν.



Do the following for each case and compare the results:

(a) Determine and sketch the distribution of the axial stress, σ_{xx}, along the wing; and find the location of the maximum magnitude along the wing.

(b) Determine and sketch the distribution of the shear stress, σ_{xz}, along the wing; and find the location of the maximum magnitude along the wing.

(c) Determine and sketch the deflection of the wing, w; and find the location of the maximum magnitude along the wing.

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Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

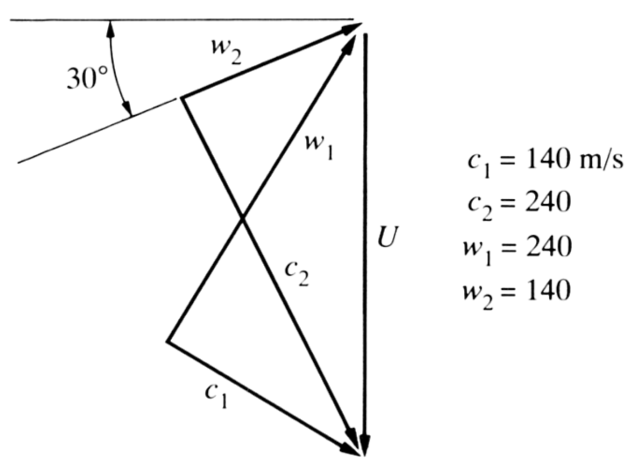
Problem T7

The velocity triangles of the second stage of a high-pressure-ratio axial compressor are shown below. The subscripts 1 and 2 denote entrance to the rotor and stator, respectively. Station 3 (not shown) is at the exit of the stator. The stage is designed such that the absolute velocity vector at the exit of the stator, c_3, is equivalent to the absolute velocity vector entering the rotor, c_1. This is called a repeating stage. The static temperature at the entrance to the rotor is 300 K and the adiabatic stage efficiency is 0.85. You can assume an ideal rotor and that all of the loss occurs in the stator. Air can be modeled as an ideal gas with constant specific heats, γ = 1.4 and R = 287 J/kg-K.

a) Draw an *h*-*s* diagram and sketch the static and the stagnation states through the compressor stage (rotor and stator, label all stations and indicate the work transfer where applicable).

b) What is the stagnation pressure ratio of this stage?

c) What is the shaft power of this stage if the mass flow through the machine is 10 kg/s?



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(Add a short summary of the concepts you are using to solve the problem)

Problem T8

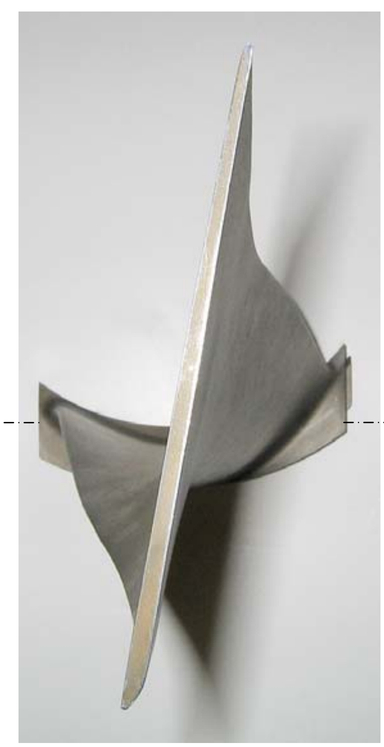
Consider the last stage of a low-pressure steam turbine. I have such a turbine rotor blade in my office (see pictures below) and we wish to estimate its performance. You are asked to reconstruct the velocity triangles near the hub using the blade geometry shown in the picture. The following assumptions can be made. The steam power plant operates in Europe where the frequency of the alternating current is 50 Hz (the turbine thus rotates at 50 Hz). In the relative frame at the tip, the flow leaves the rotor blade at the speed of sound. Downstream of the rotor the flow in the absolute frame has zero swirl at all radial stations. The axial velocity can be assumed constant throughout, that is $c_{x1}(r) = c_{x2}(r) = c_x$. At exit conditions the speed of sound of steam is 470 m/s and the density of steam is 0.59 kg/m^3 respectively.

a) Consider the flow in the rotor frame leaving the blade tip (use the photo to estimate the blade tip angle). Determine the blade tip radius r_{tip} and the axial velocity c_x.

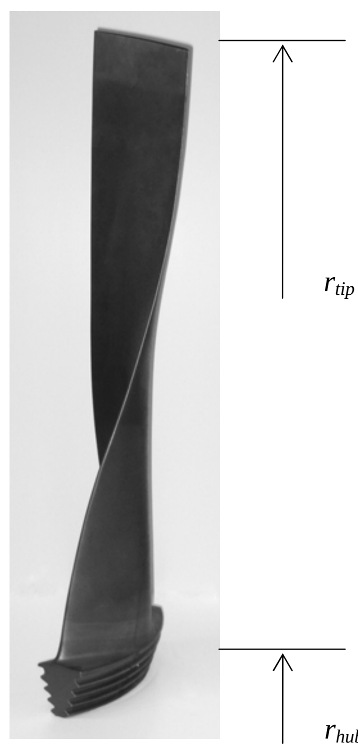
b) Draw the velocity triangles at the hub section (use the photo to estimate the blade metal angles). Determine the blade hub radius r_{hub} and the steam mass flow through the stage.

c) Find the shaft power of the turbine stage (you can assume that the specific shaft work is constant along the blade span).

Axis of rotation



Top view – flow is from left to right



Side view

(Add a short summary of the concepts you are using to solve the problem)

Problem T9

Water contained in a piston-cylinder assembly undergoes two processes in series from an initial state where the pressure is 10 bars and the temperature is 400°C.

Process 1-2: The water is cooled as it is compressed at constant pressure to the saturated vapor state at 10 bars.

Process 2-3: The water is cooled at constant volume to 150°C.

a) Sketch both processes on T-s and P-v diagrams.

b) For the overall process determine the work, in kJ/kg.

c) For the overall process determine the heat transfer in kJ/kg.

Unified Engineering Spring 2009
Fluids Problem 6

A wing has the following piecewise-linear circulation distribution:

$$\Gamma(y) \;=\; \begin{cases} \Gamma_0(2 - 4y/b) & , \quad b/4 < y < b/2 \\ \Gamma_0 & , \quad -b/4 < y < b/4 \\ \Gamma_0(2 + 4y/b) & , \quad -b/2 < y < -b/4 \end{cases}$$

a) The wing is operating at velocity V_∞ and air density ρ. Determine the lift L of this wing.

b) Determine the strength $\gamma(y)$ of the vortex sheet shed by this wing.

c) Determine the downwash velocity $w(y)$ produced by the $\gamma(y)$ distribution. Sketch $\Gamma(y)$, $\gamma(y)$, and $w(y)$. You may assume $\Gamma_0 = 1$ and $b = 2$ for plotting purposes.

d) Write down the spanwise integral for the the induced drag D_i of this wing (but don't bother to evaluate it — this would normally be done numerically). Plot the integrand versus y to see how D_i is distributed along the span for this particular $\Gamma(y)$.

A particular wing is operating in a freestream velocity V_∞ at an angle of attack α. The wing is flat, so that the wing's airfoil at every spanwise location is oriented at this same α.

The wing's airfoil 2D lift curve is

$$c_\ell(\alpha_{2D}) \;=\; 2\pi(\alpha_{2D} - \alpha_{L=0})$$

where α_{2D} is constant across the span. This is what the c_ℓ would be if the wing was 2D (infinite span).

But out actual wing has a finite span b. We will assume that it has an elliptical circulation distribution

$$\Gamma(y) = \Gamma_0\sqrt{1 - (2y/b)^2}$$

a) Determine the induced angle $\alpha_i(y)$ along the span of this wing.

b) Determine the effective angle of attack $\alpha_{\text{eff}}(y)$ along the span of the wing.

c) Determine the section lift coefficient $c_\ell(y)$ along the span of the wing.

d) What must be the chord distribution $c(y)$ (i.e. the wing planform) to be consistent with the assumed elliptical circulation distribution?

e) How will the shape of $\Gamma(y)$ change if the overall geometric wing α is changed?