Uniform Engineering Problem Set 5
Week 6 Spring 2009

Solutions

M10 (M6.1)

Given a statically determinate beam made of steel, we consider four cross-sections with different shapes, but of the same area of 15,000 mm².

First note that the resultants S(x) and M(x) are not affected by the cross-section. Thus, the problem points to look at the geometrical contribution of the cross-sections to the various cases.

(a) The deflection is determined via:

\[
\frac{M}{EI} = \frac{d^2w}{dx^2}
\]
Boundary conditions are not affected by cross-sections so in order to minimize \( w \), one must minimize \( \frac{N}{Ez} \). \( M(x) \) and \( E \) do not change with cross-section, so it is a matter of maximizing \( I \).

Consider each cross-section:

(1) The rectangular tube

This is symmetric about the center point so that is the centroid. The moment of inertia can be found by subtracting the amount of that of the inner material removed. This is because these are concentric geometries of the same shape.

\[
I_{\text{tube}} = I_{\text{outer}} - I_{\text{inner}}
\]
For a rectangle: \( I = \frac{6K^3}{12} \)

Thus:

\[
I_A = \frac{(100\text{mm}) (200\text{mm})^3}{12} - \frac{(50\text{mm}) (100\text{mm})^3}{12}
\]

\[
= \frac{1}{12} \left( 8 \times 10^6 \text{mm}^4 - 5 \times 10^7 \text{mm}^4 \right)
\]

\[
\Rightarrow I_A = 6.25 \times 10^7 \text{mm}^4
\]

(Note: can also obtain this result by working all the details of the parallel axis theorem with the four subsections of this cross-section.)

(3) Solid Rectangle

\[
\begin{array}{c}
\text{t} \\
150 \text{mm} \\
\end{array}
\]

\[
\begin{array}{c}
\text{y} \\
100 \text{mm} \\
\end{array}
\]
This corresponds for the expression for a rectangular cross-section:

$$I_{\text{c}} = \frac{bh^3}{12} = \frac{(100\text{mm}) (150\text{mm})^3}{12}$$

$$\Rightarrow I_{\text{c}} = 2.82 \times 10^7 \text{mm}^4$$

This cross-section does not have a symmetry about y-axis or ones parallel to it. It is therefore necessary to determine the location of the centroid and work from there.

Break the cross-section up into two rectangular sub-sections and place an initial axis system at some location.
(bottom of cross-section is chosen here) to get location of centroid via:

\[ z_c = \frac{\int z \, dA}{\int dA} \]

The \( z_c \) - integrals can be done piecewise. Furthermore, since each subsection of this is a rectangle, the centroid of each subsection is known (its center) and thus:

\[ \int z \, dA = z_c A = z_c bh \]

(Prove via: \[ \int z \, dy \, dz = b \int_{z=-h/2}^{z=+h/2} z \, dz \]

\[ = b \cdot \frac{z^2}{2} \bigg|_{z=-h/2}^{z=+h/2} \]

\[ = b \cdot \frac{z^2}{2} \]

\[ = z_c bh \]
\[ z_c = \frac{(150\text{mm})(100\text{mm})(100\text{mm}) + (50\text{mm})(50\text{mm})}{(100\text{mm})(100\text{mm}) + (50\text{mm})(50\text{mm})} \]

This is:

\[ z_c = \frac{z_{c0} b_0 h_0 + z_{c1} b_1 h_1}{b_0 h_0 + b_1 h_1} \]

From geometry:

\[ z_c = \frac{(1.5 \times 10^6 + 2.5 \times 10^5) \text{mm}^3}{(1 \times 10^4 + 5 \times 10^3) \text{mm}^2} \]

\[ \Rightarrow z_c = 117 \text{mm} \quad \text{from bottom} \]

Draw this and put the table together for the sub-sections with the axis system at the centroid:

![Diagram with axis system and dimensions]
(Note: \( A_{zc} \) of each of the subsection counter as they must)

Summing these items:

\[
I_0 = \sum (I_0 + A_{zc}^2)
\]

\[
= \left( (3.3 \times 10^5 + 1.09 \times 10^7) + (4.17 \times 10^6 + 2.24 \times 10^7) \right) mm^4
\]

\[
\Rightarrow I_0 = 4.58 \times 10^7 mm^4
\]

D "I"-beam
This is broken into three subsections - the 2 flanges and the web.

The I-beam is symmetric in y about its center point, so this is the centroid to work about and to use the parallel axis theorem as used before:

\[ I = I_0 + A z_c^2 \]

Assemble the table and work from there:

<table>
<thead>
<tr>
<th>Section</th>
<th>A [mm²]</th>
<th>( z_c [mm] )</th>
<th>( A z_c^2 [mm^4] )</th>
<th>( I_0 [mm^4] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100mm × 50mm = 5 × 10³</td>
<td>50 + 25 = 75</td>
<td>2.82 × 10⁷</td>
<td>((50mm) \times (50mm) \times (1/12) = 1.04 \times 10^6)</td>
</tr>
<tr>
<td>2</td>
<td>50mm × 100mm = 5 × 10³</td>
<td>0</td>
<td>0</td>
<td>((100mm) \times (50mm) \times (1/12) = 4.17 \times 10^6)</td>
</tr>
<tr>
<td>3</td>
<td>100mm × 50mm = 5 × 10³</td>
<td>-50 - 25 = -75</td>
<td>2.82 × 10⁷</td>
<td>((100mm) \times (50mm) \times (1/12) = 1.04 \times 10^6)</td>
</tr>
</tbody>
</table>

Now proceed to:

\[ I_D = \sum \left( I_0 + A z_c^2 \right) \]

\[ = \left[ (1.04 \times 10^6 + 2.82 \times 10^7) + (4.17 \times 10^6 + 0) \right. \]
\[ + \left. (1.04 \times 10^6 + 2.82 \times 10^7) \right] \text{ mm}^4 \]

\[ \Rightarrow I_D = 6.27 \times 10^7 \text{ mm}^4 \]
Summarizing for all 4 sections:

- $I_{A} = 6.25 \times 10^7 \text{ mm}^4$ rectangular tube
- $I_{B} = 2.82 \times 10^7 \text{ mm}^4$ solid rectangle
- $I_{C} = 4.58 \times 10^7 \text{ mm}^4$ T-beam
- $I_{D} = 6.27 \times 10^7 \text{ mm}^4$ I-beam

\textbf{Note:} Within calculating the I-beam and rectangular tube have the same $I$ about the $y$-axis. This is because their geometry is fundamentally the same about the $y$-axis with the "web" of the rectangular tube split into two equal sections summing to 50 mm total.

So, the smallest deflection occurs when the value of $I$ is maximized. This occurs for rectangular/cube (Section A) and I-beam (Section D).
(b) The axial stress is determined via:

\[ \sigma_{xx} = -\frac{Mz}{I} \]

Thus, the maximum magnitude (i.e. absolute value) will occur for maximizing \( \left| \frac{z}{I} \right| \)

where \( z \) is the distance from the centroid, \( I \) is a constant for any cross-section and the maximum value of \( M(z) \) is unattained by this.

So consider this for each case:

1. **Case A:** \( I_{A} = 6.25 \times 10^{7} \text{ mm}^4 \) \( |z_{\text{max}}| = 100 \text{ mm} \)

   \[ \Rightarrow \left| \frac{z}{I} \right|_{A} = 1.60 \times 10^{-6} \text{ mm}^{-3} \]

2. **Case B:** \( I_{B} = 2.82 \times 10^{7} \text{ mm}^4 \) \( |z_{\text{max}}| = 75 \text{ mm} \)

   \[ \Rightarrow \left| \frac{z}{I} \right|_{B} = 2.66 \times 10^{-6} \text{ mm}^{-3} \]

3. **Case C:** \( I_{C} = 4.58 \times 10^{7} \text{ mm}^4 \) \( |z_{\text{max}}| = 117 \text{ mm} \)

   \[ \Rightarrow \left| \frac{z}{I} \right|_{C} = 2.55 \times 10^{-6} \text{ mm}^{-3} \]
D: \[ I_0 = 6.27 \times 10^2 \text{ mm}^4 \quad \Rightarrow \quad \left| \frac{I}{I_0} \right| = 1.59 \times 10^{-6} \text{ mm}^{-3} \]

Note that once again the rectangular tube and the I-beam yield the same result and...

the cross-sections with the smallest value of the maximum magnitude of \( \sigma_{xx} \) are the

- rectangular tube (section (A))
- I-beam (section (D))

(c) The shear stress is determined via:

\[ \sigma_{xz} = - \frac{SQ}{I_b} \]

\( S(x) \) is a constant (i.e., the same) for all cross-sections. To find the maximum magnitude of \( \sigma_{xz} \) for any case, it is necessary to maximize \( \left| \frac{Q}{I_b} \right| \) where
I is a constant but Q and b can change with z.
Consider for each of the cross-sections...

(A) Rectangular tube

By inspection, Q_{max} is at the centroid (due to symmetry) and this is the point of minimum width (minimum width of edges = 50 mm). So the maximum value of $\frac{Q}{b}$ is at $z = 0$. Find this.

Use: $Q = \int b z dz$

$Q(z = 0) = \int_0^{50 \text{ mm}} (50 \text{ mm}) z dz + \int_{50 \text{ mm}}^{100 \text{ mm}} (100 \text{ mm}) z dz$

$= \left\{ (50 \text{ mm}) \frac{z^2}{2} \right\}_0^{50 \text{ mm}} + \left\{ (100 \text{ mm}) \frac{z^2}{2} \right\}_{50 \text{ mm}}^{100 \text{ mm}}$

$= \left\{ 6.25 \times 10^{-4} \text{ mm}^3 + 5 \times 10^{-5} \text{ mm}^3 - 1.25 \times 10^{-5} \text{ mm}^3 \right\}$

$\Rightarrow Q(z = 0) = 4.375 \times 10^{-5} \text{ mm}^3$

So for the rectangular tube, the maximum $\left| \frac{Q}{Ib} \right|$ is at $z = 0$ with $\left| \frac{Q}{Ib} \right|_{\text{max}} = 1.40 \times 10^{-4} \text{ mm}^{-2}$.
(B) **Solid Rectangle**

Q = This is maximised at the center point (x = 0) which is the location of the centroid. The width b, is a constant so there is no need to consider that item. So:

\[ Q_{\text{max}} = \int_0^{75 \text{mm}} (100 \text{mm}) \cdot z \, dz \]

\[ = (100 \text{mm}) \frac{z^2}{2} \bigg|_0^{75 \text{mm}} \]

\[ = 2.25 \times 10^5 \text{mm}^3 \]

So:

\[ \left| \frac{Q}{I_b} \right|_{\text{max}} = 9.97 \times 10^{-5} \text{mm}^{-2} \]

(C) **T - Beam**

The width changes between the two sub-sections of the cross-section, so this needs to be considered for each cross-section.
for subsection 1, the maximum value of \( Q \) occurs at the centroid. So:

\[
Q_{\text{max}} = \int_0^{\delta_3/2} (100 \text{ mm})^2 \, dz = \frac{(100 \text{ mm})^2}{2} \int_0^{\delta_3/2} \, dz = 3.44 \times 10^{-5} \text{ mm}^3
\]

\[
\Rightarrow \left| \frac{Q}{5} \right|_{\text{max}} = \frac{3.44 \times 10^{-5} \text{ mm}^3}{100 \text{ mm}} = 3.44 \times 10^{-3} \text{ mm}^2
\]

for section 2, \( Q \) will be maximum nearest the centroid, thus where this subsection starts at \( z = -17 \text{ mm} \):

\[
Q_{\text{max}} = \int_{-17 \text{ mm}}^{-13 \text{ mm}} (50 \text{ mm})^2 \, dz = \frac{(50 \text{ mm})^2}{2} \int_{-17 \text{ mm}}^{-13 \text{ mm}} \, dz
\]

\[
= \frac{25 \text{ mm}^2 (2.89 \times 10^2 - 1.37 \times 10^2)}{2} = 3.35 \times 10^{-5} \text{ mm}^3
\]

(Note: Can also obtain by subtracting \( \int_{-17 \text{ mm}}^{\delta_3/2} (100 \text{ mm})^2 \, dz \) from \( Q_{\text{max}} \) )
\[ \Rightarrow \left| \frac{Q}{b} \right|_{\text{max}} = \frac{3.35 \times 10^5 \text{ mm}^3}{50 \text{ mm}} = 6.70 \times 10^3 \text{ mm}^{-2} \]

So for the T-beam, the maximum \( \left| \frac{Q}{Ib} \right| \) is at \( z = -17 \text{ mm} \) is the second sub-section with the value:

\[ \left| \frac{Q}{Ib} \right|_{\text{max}} = 1.46 \times 10^{-4} \text{ mm}^{-2} \]

\( \odot \) **T-beam**

It is necessary to consider the Q's for the separate sections since b changes. However, one knows that \( Q \) will reach its maximum value at the centroid which is the center here. This is where the width is a minimum so \( \left| \frac{Q}{b} \right| \) being a maximum must be at this point \( (z = 0) \).

So calculate \( Q \) at this point:
\[ Q_{\text{max}} = \int_0^{50 \text{mm}} (100 \text{mm})^2 \, dz + \int_0^{50 \text{mm}} (50 \text{mm})^2 \, dz \\
= (100 \text{mm}) \frac{z^2}{2}\bigg|_0^{50 \text{mm}} + (50 \text{mm}) \frac{z^2}{2}\bigg|_0^{50 \text{mm}} \\
= 3.75 \times 10^5 \text{ mm}^3 + 6.25 \times 10^4 \text{ mm}^3 \\
Q_{\text{max}} = 4.38 \times 10^5 \text{ mm}^3 \\
\frac{Q}{b} \bigg|_{\text{max}} = 8.75 \times 10^3 \text{ mm}^2 \\
\]

So, for the T-beam, the maximum value is at \( z = 0 \) with the value of:

\[ \left| \frac{Q}{b} \right|_{\text{max}} = 1.40 \times 10^{-4} \text{ mm}^{-2} \]

Summarizing for the 4 cases:

(A) \( 1.40 \times 10^{-4} \text{ mm}^{-2} \) at \( z = 0 \) rectangular tube

(B) \( 9.97 \times 10^{-5} \text{ mm}^{-2} \) at \( z = 0 \) solid rectangle

(C) \( 1.46 \times 10^{-4} \text{ mm}^{-2} \) at \( z = -17 \text{mm} \) T-beam

(D) \( 1.40 \times 10^{-4} \text{ mm}^{-2} \) at \( z = 0 \) I-beam
So the cross-section with the smallest value of the maximum magnitude of $T_{xz}$ is the solid rectangle - section B. and this occurs at the center ($z = 0$)

(Notice: Once again the rectangular-tube and I-beam give the same result and due to the same reasons).

(d) Some items to note:

- An I-beam is generally the most efficient configuration with regard to bending stiffness and to get the minimum deflection. This shows using the most material as far from the centroid as possible results in greatest effectiveness. This also helps in minimizing stresses as the stresses can create a larger moment as they have a larger moment arm.
In this case, the rectangular tube and I-beam have the same geometry relative to the y-axis. Thus, these two cross-sections yield the same results for this particular case.

Shear stresses are lower in magnitude for "bulkyer" cross-sections such as the solid rectangle. Thus, if this becomes a key consideration, such factors must be come involved in the overall design.
This is the same as in last week's (Week 5) (Problem Set 4) problem set except that the far tip has a roller support as opposed to being free. Thus, the problem is now statically indeterminate and equations will need to be solved simultaneously. Much of what was determined last week, but the boundary conditions definitely change.

(a) Begin with the loading being the same. So:

\[ q(x) = p_0 \left( \frac{2x}{L} - 1 \right) \]
Now draw the Free Body Diagram, but this is a bit different:

→ Apply Equilibrium Equations to get expressions for the reactions:

\[ \sum F_x = 0 \Rightarrow H_A = 0 \]
\[ \sum F_y = 0 \Rightarrow V_A + V_B + \int_0^L q(x) \, dx = 0 \]
\[ \Rightarrow V_A + V_B = 0 \quad (1) \]

(Notice: Number of the equations that will need to be solved)

\[ \sum M_0 = 0 \Rightarrow -M_A + \int_0^L q(x) \, dx + V_B L = 0 \]
\[ \Rightarrow M_A - V_B L = \frac{P_0 L^2}{6} \quad (2) \]
Now work to get the shear and moment results in a similar way except the boundary conditions are different:

\[
\frac{dy}{dx} = g(x) \Rightarrow S(x) = \int g(x) \, dx
\]

as per last week's solution:

\[
S(x) = p_0 \left( \frac{x^2}{L} - x \right) + C_1
\]

and \( S(0) = V_A \) and their value is an unknown at this point. So:

\[
V_A = C_1
\]

\[
\Rightarrow S(x) = p_0 \left( \frac{x^2}{L} - x \right) + V_A \tag{3}
\]

Now progress to:

\[
\frac{dM}{dx} = S(x) \Rightarrow M(x) = \int S(x) \, dx
\]

\[
M(x) = \int \left( p_0 \left( \frac{x^2}{L} - x \right) + V_A \right) \, dx
\]

\[
= \frac{p_0 x^3}{3L} - \frac{p_0 x^2}{2} + V_A x + C_2
\]
Use the boundary condition that there is no moment at the roller:

\[ M(L) = 0 \]

\[ \Rightarrow \frac{P_0 L^2}{3} - \frac{P_0 L^2}{2} + V_A L + C_2 = 0 \]

\[ \Rightarrow C_2 = \frac{P_0 L^2}{6} - V_A L \]

Finally:

\[ M(x) = \frac{P_0 x^3}{3L} - \frac{P_0 x^2}{2} + V_A x + \frac{P_0 L^2}{6} - V_A L \] (A)

Now move on to get an expression for the deflection via the Moment-Curvature Relation

\[ M = EI \frac{d^2w}{dx^2} \]

\[ \Rightarrow \frac{d^2w}{dx^2} = \frac{1}{EI} \left( \frac{P_0 x^3}{3L} - \frac{P_0 x^2}{2} + V_A x + \frac{P_0 L^2}{6} - V_A L \right) \]

\[ V_A \text{ and } P_0 \text{ do not vary with } x, \text{ so their}\]

\[ \text{can be integrated:} \]

\[ \frac{dw}{dx} = \frac{1}{EI} \left( \frac{P_0 x^4}{12L} - \frac{P_0 x^3}{6} + V_A x^2 + \frac{P_0 L^2}{6} x - V_A L x \right) \] + C_3

Use the boundary condition that

\[ \frac{dw}{dx} = 0 \quad @ \text{ the root } (x = 0) \]
\[ \Rightarrow c_3 = 0 \]

Proceeding:

\[ w(x) = \frac{1}{EI} \left( \frac{P_0 x^5}{60 L} - \frac{P_0 x^4}{24} + \frac{P_0 L x^2}{12} - \frac{V_A x^3}{6} \right) + C_4 \]

Again use the boundary condition that \( w = 0 \) at \( x = 0 \):

So:

\[ w(x) = \frac{1}{EI} \left( \frac{P_0 x^5}{60 L} - \frac{P_0 x^4}{24} + \frac{P_0 L x^2}{12} - \frac{V_A x^3}{6} \right) \]

(5)

There is a third boundary condition on the deflection:

\[ w(L) = 0 \]

Using this in (5):

\[ 0 = \frac{1}{EI} \left( \frac{P_0 L^4}{60} - \frac{P_0 L^4}{24} + \frac{P_0 L^3}{12} - \frac{V_A L^3}{3} \right) \]

\[ V_A \left( \frac{3}{6} - \frac{1}{6} \right) = \frac{P_0 L}{120} \left( \frac{2 - 5 + 10}{120} \right) \]

\[ V_A \frac{1}{3} = \frac{7}{120} P_0 L \]

Finally:

\[ V_A = \frac{7}{40} P_0 L \]
Use these in (1):

\[ V_A + V_B = 0 \]

\[ \Rightarrow V_B = -\frac{7}{40} p_0 L \]

And then these in (2):

\[ M_A = \frac{p_0 L^2}{6} + V_B L \]

\[ \Rightarrow M_A = \frac{p_0 L^2}{6} \left( \frac{1}{6} - \frac{7}{40} \right) \]

\[ = \frac{p_0 L^2}{6} \left( \frac{20 - 21}{120} \right) \]

\[ \Rightarrow M_A = \frac{1}{120} p_0 L^2 \]

**Note:** Reactions not needed to get \( w(x) \)

Return to (5) to find the expression for \( w(x) \) using the values for \( V_A \):

\[ w(x) = \frac{1}{EI} \left( \frac{p_0 x^5}{60 L} - \frac{p_0 x^4}{24} + \frac{7p_0 L x^3}{240} + \frac{p_0 L^2 x^2}{12} - \frac{7p_0 L^2 x^2}{80} \right) \]

and combine the last two terms:
\[ w(x) = \frac{P_0}{EI} \left( \frac{x^5}{60L} - \frac{x^4}{24} + \frac{7Lx^3}{240} - \frac{L^2x^2}{240} \right) \]

Check that the units are correct:

\[
[LG^3] \equiv \frac{[F/L]}{[F/L^2][L^4]} = [L] \checkmark
\]

So proceed to find the maximum magnitude of deflection and its location by taking the derivative and setting it to zero:

\[
\frac{dw}{dx} = \frac{P_0}{EI} \left( \frac{x^4}{12L} - \frac{x^3}{6} + \frac{7Lx^2}{80} - \frac{L^2x}{120} \right) = 0
\]

Either explicitly solve the quartic or try values \((0 < \frac{x}{L} < 1)\) in a nondimensionalized expression. Using the latter:

\[
0 = \frac{1}{12} \left( \frac{x}{L} \right)^4 - \frac{1}{6} \left( \frac{x}{L} \right)^3 + \frac{7}{80} \left( \frac{x}{L} \right)^2 - \frac{1}{120} \left( \frac{x}{L} \right)
\]

\[
\Rightarrow 0 = 10 \left( \frac{x}{L} \right)^4 - 20 \left( \frac{x}{L} \right)^3 + 105 \left( \frac{x}{L} \right)^2 - \left( \frac{x}{L} \right)
\]

Approximate the middle to begin and work from there (closer to the fixed end -- the roller).
Use \( \frac{x}{L} \) as a first value:

\[
\frac{x}{L} = 0.5 \Rightarrow 0.25
\]

Then go to \( \frac{x}{L} = 0.6 \Rightarrow 0.156 \)

On to \( \frac{x}{L} = 0.7 \Rightarrow -0.014 \)

So come back a bit to \( \frac{x}{L} = 0.69 \Rightarrow 0.006 \)

Use this as good enough.

Normalize the \( w(x) \) expression:

\[
w(x) = \frac{P_0 L^4}{EI} \left\{ \frac{1}{60} \left( \frac{x}{L} \right)^5 - \frac{1}{24} \left( \frac{x}{L} \right)^4 + \frac{7}{240} \left( \frac{x}{L} \right)^3 \right\} - \frac{1}{240} \left( \frac{x}{L} \right)^2
\]

So:

\[
W_{\text{max}} = \frac{P_0 L^4}{EI} (7.60 \times 10^{-4}) \odot \frac{x}{L} = 0.69
\]

As a sketch, the deflection is:
(b) To find the maximum magnitude of the axial stress, start with:

\[ \sigma_{xx} = -\frac{M x}{I} \]

and I do not vary w.r.t. \( x \), so look for the maximum of

\[ \frac{\sigma_{xx}}{(x/I)} = -M(x) \]

From equation (8):

\[ M(x) = \frac{P_0 x^3}{3L} - \frac{P_0 x^2}{2} + \frac{P_0 L^2}{6} - V_A L \]

with \( V_A = \frac{7}{40} P_0 L \)

\[ \Rightarrow M(x) = \frac{P_0 x^3}{3L} - \frac{P_0 x^2}{2} + \frac{7P_0 L x}{40} + \frac{P_0 L^2}{6} - \frac{7P_0 L^2}{40} \]

manipulation gives

\[ M(x) = P_0 L^2 \left\{ \frac{1}{3} \left( \frac{x}{L} \right)^3 - \frac{1}{2} \left( \frac{x}{L} \right)^2 + \frac{7}{40} \left( \frac{x}{L} \right) - \frac{1}{120} \right\} \]

Check units:

\[ [F \cdot L] \equiv [F/L] \cdot [L^2] = [N/m] \cdot [m] \]

First check to see if there is a maximum magnitude within the \( 0 < x < L \)
by taking the derivative and setting it to zero: \( \frac{dM(x)}{dx} = 0 \)

\[
\begin{align*}
\text{First:} \quad \frac{dM(x)}{dx} & = p_0 L \left\{ \left( \frac{x}{L} \right)^2 - \left( \frac{x}{L} \right) + \frac{7}{40} \right\} \\
\text{solving this quadratic equation:} \\
\frac{x}{L} & = \frac{1 \pm \sqrt{1 - 0.7}}{2} = \frac{1 \pm 0.3}{2} \\
& = \frac{1 \pm 0.548}{2} \\
& = 0.774, \ 0.226
\end{align*}
\]

To check values at these two locations and at the endpoints:

\[
\begin{align*}
\frac{x}{L} = 0, \quad M(x) & = -\frac{p_0 L^2}{120} = (-8.33 \times 10^{-3}) p_0 L^2 \\
\frac{x}{L} = 1, \quad M(x) & = 0 \\
\frac{x}{L} = 0.226, \quad M(x) & = (4.53 \times 10^{-3}) p_0 L^2 \\
\frac{x}{L} = 0.774, \quad M(x) & = (-1.78 \times 10^{-2}) p_0 L^2
\end{align*}
\]
maximum magnitude of the moment and thus $\sigma_{xx}$ occurs at $\frac{x}{L} = 0.774$ with a value of

$$|\sigma_{xx\text{max}}| = (1.76 \times 10^{-2}) \frac{P_0 L^2 Z_{\text{max}}}{I}$$

(c) To find the maximum magnitude of the shear stress, start with:

$$\sigma_{xt} = -\frac{5Q}{I_b}$$

As for the case of $\sigma_{xx}$, $Q$, $I$ and $b$ do not vary with $x$. Thus, look for the maximum magnitude of:

$$\left| \frac{\sigma_{xt}}{(Q/I_b)} \right| = | -S(x)|$$

From Equation (3):

$$S(x) = P_0 \left( \frac{x^2}{L} - x \right) + V_A$$
end with \( V_A = \frac{2}{40} P_0 L \)

\[ \Rightarrow S(x) = P_0 L \left\{ \left( \frac{x}{L} \right)^2 - \left( \frac{x}{L} \right) + \frac{7}{40} \right\} \]

Again, take the derivative and set it to zero to check for locations within the beam. And also look at the end points.

\[ \frac{dS(x)}{dx} = P_0 \left\{ 2 \left( \frac{x}{L} \right) - 1 \right\} \]

Setting to 0:

\[ 2 \left( \frac{x}{L} \right) - 1 = 0 \]

\[ \Rightarrow \left( \frac{x}{L} \right) = \frac{1}{2} \]

Now check values:

\[ \frac{x}{L} = 0, \quad S(x) = \frac{7}{40} P_0 L = 0.175 P_0 L \]

\[ \frac{x}{L} = 1, \quad S(x) = \frac{7}{40} P_0 L = 0.175 P_0 L \]

\[ \frac{x}{L} = \frac{1}{2}, \quad S(x) = -0.075 P_0 L \]
maximum magnitude of the shear and tensile occurs at each end \((x = 0, L)\) with value of

\[
\left| \sigma_{x,z\text{max}} \right| = (0.175) \frac{P_0 L}{I} \left| \frac{Q}{b} \right|_{\text{max}}
\]