UE Fluids F8 Solution S'09

Given: $V = 6\ m/s$, $\rho = 1.2\ kg/m^3$, $L = 5N$, $b = 2m$

Elliptical loading $\Gamma = \Gamma_0\sqrt{1-(2y/b)^2}$, so $L = \frac{\pi}{4}\rho V \Gamma_0 b$

a) Given wing planform: $C(y) = C_r + (C_t - C_r)\frac{2y}{b}$, $C_t = C_r \cdot \lambda$; $\lambda = 0.4$

$$C = C_r\left[1+(\lambda-1)\frac{2y}{b}\right], \quad S = 2\int_0^{b/2} c\, dy = C_r\left[b + b\tfrac{1}{2}(\lambda-1)\right] = C_r b \frac{1+\lambda}{2}$$

Required $C_L = \dfrac{L}{\frac{1}{2}\rho V^2 S} = 0.8 \rightarrow S = \dfrac{L}{\frac{1}{2}\rho V^2 C_L} = 0.289\ m^2$

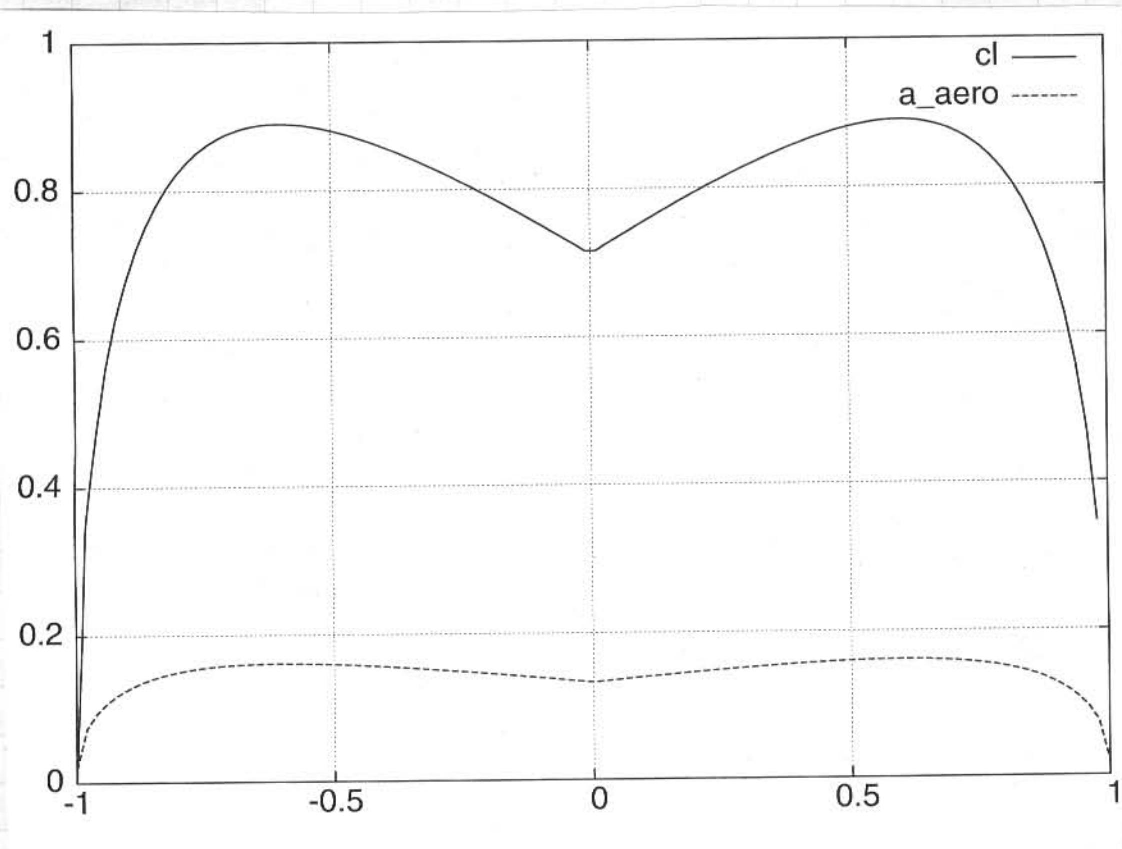
$$C_r = \frac{S}{b}\cdot\frac{2}{1+\lambda} = \frac{0.289\,m^2}{2m}\cdot\frac{2}{1.4} = 0.207\ m, \quad C_t = \lambda C_r = 0.0827\ m$$

b) For elliptical loading: $\alpha_i = \dfrac{L}{\frac{1}{2}\rho V^2 \pi b^2} = 0.0184\ rad = 1.055^\circ$

$$C_\ell = \frac{\Gamma}{\frac{1}{2}cV}, \quad \Gamma_0 = \frac{4}{\pi}\frac{L}{\rho V b} = 0.442\ m^2/s, \quad C_\ell = 0.712\cdot\frac{\sqrt{1-(2y/b)^2}}{1-0.6\,2y/b}$$

also. $C_\ell = 2\pi(\alpha_{aero} - \alpha_i + \cancel{\alpha})$

c) $$\alpha_{aero}(y) = \frac{C_\ell}{2\pi} + \alpha_i = \frac{0.712}{2\pi}\frac{\sqrt{1-(2y/b)^2}}{1-0.6\,2y/b} + 0.0184$$



UE Fluids F9 Solution S '09

a) $L' = \rho V_\infty \Gamma = \rho V_\infty^2 2b \left[A_1 \sin\theta + A_3 \sin 3\theta \right]$

Plot for i, ii, iii, using $\theta = \arccos\left(\frac{2y}{b}\right)$ (below)

b) $L = \frac{\pi}{2} \rho V_\infty^2 b^2 A_1$, $D_i = \pi b^2 \frac{1}{2} \rho V_\infty^2 \left[A_1^2 + 3A_3^2 \right]$, $e = \frac{1}{D_i} \cdot \frac{(L/b)^2}{\frac{1}{2}\rho V^2 \pi}$

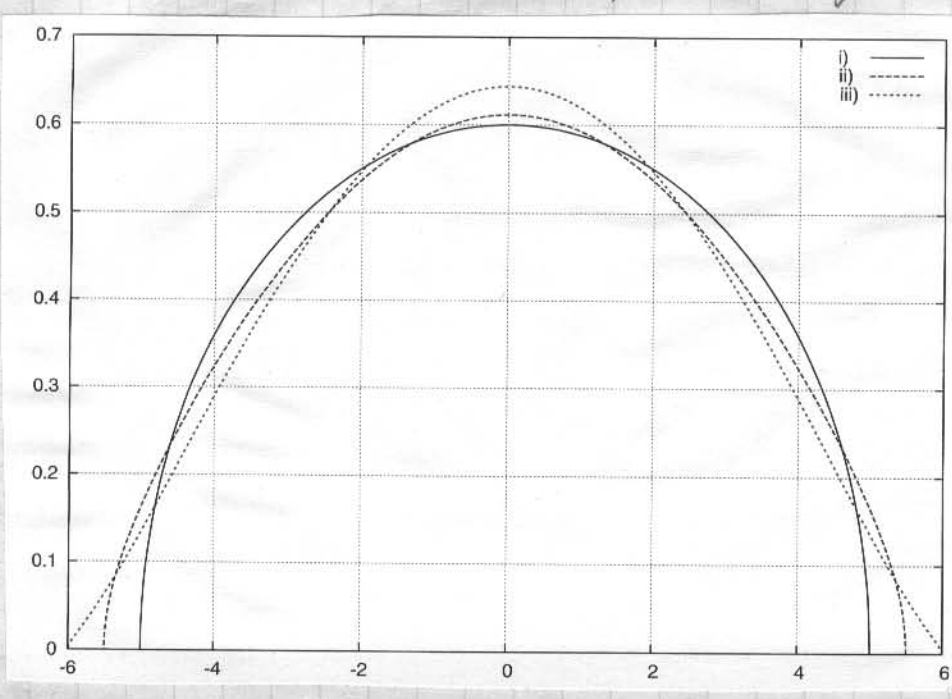
i) $L = 4.712$, $D_i = 0.1414$, $e = 1.000$

ii) $L = 4.712$, $D_i = \boxed{0.1219}$ $e = 0.9579$

iii) $L = 4.712$, $D_i = 0.1226$ $e = 0.8007$

The lowest total drag is case ii)

c) The span efficiency is only part of determining D_i.
Also important is the span: $D_i = \frac{(L/b)^2}{\frac{1}{2}\rho V^2 e}$
For fixed L and V, like in this case, what really counts is $e \cdot b^2$



UE Fluids F10 Solution S '09

a) $L = W = \frac{1}{2}\rho V^2 S C_L \rightarrow V = \sqrt{\frac{2W/S}{\rho C_L}}$

V_{min} occurs when $C_L = C_{L max} = 1.0$: $V_{min} = \sqrt{\frac{2W/S}{\rho \cdot 1.0}} = 5.44$ m/s

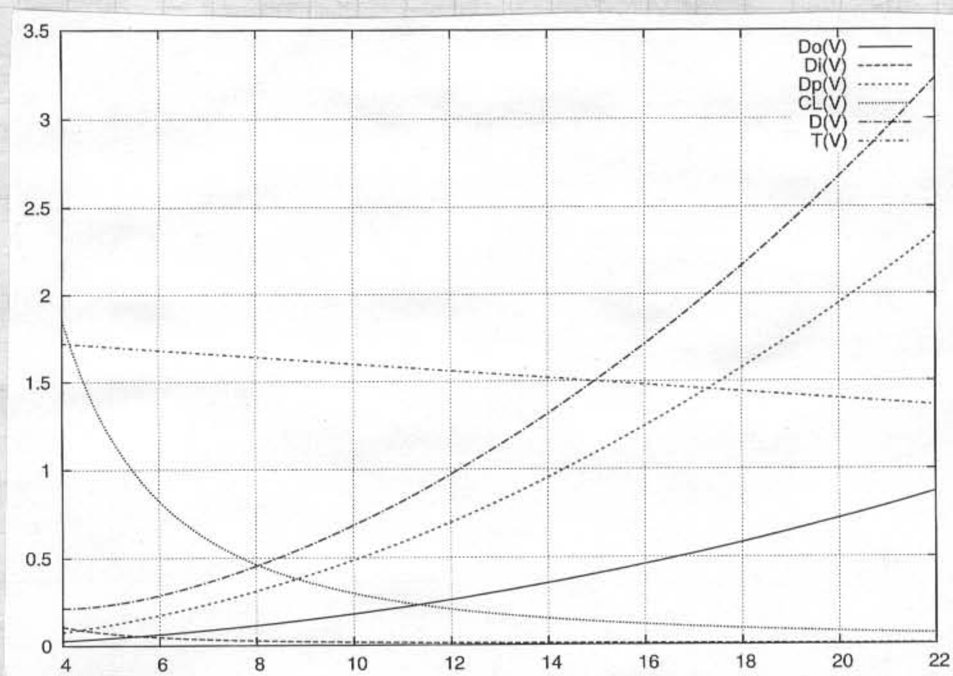
b) $D_o(V) = \frac{1}{2}\rho V^2 CDA_0$

$D_p(V) = \frac{1}{2}\rho V^2 C_d \cdot S$

$D_i(V) = \frac{W^2/b^2}{\frac{1}{2}\rho V^2 e}$

$\rightarrow D(V) = D_o(V) + D_p(V) + D_i(V)$

Plot all:



V_{max} occurs where $T = D$: $V_{max} = 15.0$ m/s (from plot)

At this speed, $C_L = \frac{W}{\frac{1}{2}\rho V_{max}^2 S} = 0.1317$