2.21. (a) The desired convolution is

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= \beta^n \sum_{k=0}^{n} (\alpha/\beta)^k \text{ for } n \geq 0 \\ &= [\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}]u[n] \text{ for } \alpha \neq \beta. \end{aligned}$$

(b) From (a),

$$y[n] = \alpha^n \left[\sum_{k=0}^{n} 1 \right] u[n] = (n+1)\alpha^n u[n].$$

(c) For $n \leq 6$,

$$y[n] = 4^n \left\{ \sum_{k=0}^{\infty} (-\frac{1}{8})^k - \sum_{k=0}^{3} (-\frac{1}{8})^k \right\}.$$

For $n > 6$,

$$y[n] = 4^n \left\{ \sum_{k=0}^{\infty} (-\frac{1}{8})^k - \sum_{k=0}^{n-1} (-\frac{1}{8})^k \right\}.$$

Therefore,

$$y[n] = \begin{cases} (8/9)(-1/8)^4 4^n, & n \leq 6 \\ (8/9)(-1/2)^n, & n > 6 \end{cases}$$

(d) The desired convolution is

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3] + x[4]h[n-4] \\ &= h[n] + h[n-1] + h[n-2] + h[n-3] + h[n-4]. \end{aligned}$$

This is as shown in Figure S2.21.

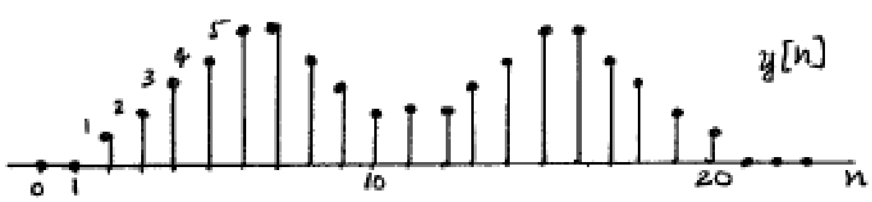

Figure S2.21

2.22. (a) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \\ &= \int_0^t e^{-\alpha\tau} e^{-\beta(t-\tau)}\,d\tau, \quad t \geq 0 \end{aligned}$$

Then

$$y(t) = \begin{cases} \frac{e^{-\beta t}[e^{-(\alpha-\beta)t}-1]}{\beta-\alpha} u(t) & \alpha \neq \beta \\ t e^{-\beta t} u(t) & \alpha = \beta \end{cases}.$$

(b) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \\ &= \int_0^2 h(t-\tau)\,d\tau - \int_2^5 h(t-\tau)\,d\tau. \end{aligned}$$

This may be written as

$$y(t) = \begin{cases} \int_0^2 e^{2(t-\tau)}\,d\tau - \int_2^5 e^{2(t-\tau)}\,d\tau, & t \leq 1 \\ \int_{t-1}^2 e^{2(t-\tau)}\,d\tau - \int_2^5 e^{2(t-\tau)}\,d\tau, & 1 \leq t \leq 3 \\ -\int_{t-1}^5 e^{2(t-\tau)}\,d\tau, & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$

Therefore,

$$y(t) = \begin{cases} (1/2)[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}], & t < 1 \\ (1/2)[e^{2} + e^{2(t-5)} - 2e^{2(t-2)}], & 1 \leq t \leq 3 \\ (1/2)[e^{2(t-5)} - e^{2}], & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$

(c) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_0^2 \sin(\pi\tau)h(t-\tau)d\tau. \end{aligned}$$

This gives us

$$y(t) = \begin{cases} 0, & t<1 \\ (2/\pi)[1-\cos\{\pi(t-1)\}], & 1<t<3 \\ (2/\pi)[\cos\{\pi(t-3)\}-1], & 3<t<5 \\ 0, & 5<t \end{cases}$$

(d) Let

$$h(t) = h_1(t) - \frac{1}{3}\delta(t-2),$$

where

$$h_1(t) = \begin{cases} 4/3, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

Now,

$$y(t) = h(t) * x(t) = [h_1(t) * x(t)] - \frac{1}{3}x(t-2).$$

We have

$$h_1(t) * x(t) = \int_{t-1}^{t} \frac{4}{3}(a\tau + b)d\tau = \frac{4}{3}[\frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 + bt - b(t-1)].$$

Therefore,

$$y(t) = \frac{4}{3}[\frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 + bt - b(t-1)] - \frac{1}{3}[a(t-2)+b] = at + b = x(t).$$

(e) $x(t)$ periodic implies $y(t)$ periodic. ∴ determine 1 period only. We have

$$y(t) = \begin{cases} \int_{t-1}^{-\frac{1}{2}}(t-\tau-1)d\tau + \int_{-\frac{1}{2}}^{t}(1-t+\tau)d\tau = \frac{1}{4}+t-t^2, & -\frac{1}{2}<t<\frac{1}{2} \\ \int_{t-1}^{\frac{1}{2}}(1-t+\tau)d\tau + \int_{\frac{1}{2}}^{t}(t-1-\tau)d\tau = t^2-3t+7/4, & \frac{1}{2}<t<\frac{3}{2} \end{cases}$$

The period of $y(t)$ is 2.

... different values of T.

2.26. (a) We have

$$\begin{aligned} y_1[n] = x_1[n] * x_2[n] &= \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k] \\ &= \sum_{k=0}^{\infty}(0.5)^k u[n+3-k]. \end{aligned}$$

This evaluates to

$$y_1[n] = x_1[n] * x_2[n] = \begin{cases} 2\{1-(1/2)^{n+4}\}, & n \ge -3 \\ 0, & \text{otherwise} \end{cases}.$$

(b) Now,

$$y[n] = x_3[n] * y_1[n] = y_1[n] - y_1[n-1].$$

Therefore,

$$y[n] = \begin{cases} 2\left\{1 - (1/2)^{n+3}\right\} + 2\left\{1 - (1/2)^{n+4}\right\} = (1/2)^{n+3}, & n \geq -2 \\ 1, & n = -3 \\ 0, & \text{otherwise} \end{cases}.$$

Therefore, $y[n] = (1/2)^{n+3} u[n+3]$.

(c) We have

$$y_2[n] = x_2[n] * x_3[n] = u[n+3] - u[n+2] = \delta[n+3].$$

(d) From the result of part (c), we get

$$y[n] = y_2[n] * x_1[n] = x_1[n+3] = (1/2)^{n+3} u[n+3].$$