S3:

[a] $\mathcal{L}\{t\} = \frac{1}{s^2}$; therefore $\mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$

[b] $\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{j2}$

Therefore

$$\mathcal{L}\{\sin \omega t\} = \left(\frac{1}{j2}\right)\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) = \left(\frac{1}{j2}\right)\left(\frac{2j\omega}{s^2+\omega^2}\right)$$
$$= \frac{\omega}{s^2+\omega^2}$$

[c] $\sin(\omega t + \theta) = (\sin \omega t \cos\theta + \cos \omega t \sin\theta)$

Therefore

$$\mathcal{L}\{\sin(\omega t+\theta)\} = \cos\theta\mathcal{L}\{\sin \omega t\} + \sin\theta\mathcal{L}\{\cos \omega t\}$$
$$= \frac{\omega\cos\theta + s\sin\theta}{s^2+\omega^2}$$

[d] $\mathcal{L}\{t\} = \int_0^\infty te^{-st}\,dt = \frac{e^{-st}}{s^2}(-st-1)\Big|_0^\infty = 0 - \frac{1}{s^2}(0-1) = \frac{1}{s^2}$

[e] $f(t) = \cosh t\cosh\theta + \sinh t \sinh\theta$

From Assessment Problem 12.1(a)

$$\mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

From Assessment Problem 12.1(b)

$$\mathcal{L}\{\sinh t\} = \frac{1}{s^2-1}$$

$$\therefore\ \mathcal{L}\{\cosh(t+\theta)\} = \cosh\theta\left[\frac{s}{(s^2-1)}\right] + \sinh\theta\left[\frac{1}{s^2-1}\right]$$
$$= \frac{\sinh\theta + s[\cosh\theta]}{(s^2-1)}$$

S4:

$$\mathcal{L}\{e^{-at}f(t)\} = \int_{0^-}^\infty [e^{-at}f(t)]e^{-st}\,dt = \int_{0^-}^\infty f(t)e^{-(s+a)t}\,dt = F(s+a)$$

S5:

$$\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(at)e^{-st}\,dt$$

Let $u = at, \qquad du = a\,dt, \qquad u = 0^- \quad \text{when} \quad t = 0^-$

and $u = \infty \quad \text{when} \quad t = \infty$

Therefore $\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(u)e^{-(u/a)s}\dfrac{du}{a} = \dfrac{1}{a}F(s/a)$