Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.003/16.003 Unified Engineering III, IV
Spring 2009

Problem Set 8

Name: ________________________

Due Date: 4/10/2009

	Time Spent (min)
T18	
T19	
T20	
S3	
S4	
S5	
M12	
M13	
M14	
Study Time	

Announcements:

Unified Engineering Spring 2009
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T18

A double glazed window has two sheets of glass separated by a thin layer of air. The air gap is thin enough so that convection between the glass planes may be neglected. Determine the reduction in heat transfer when a single-glazed window is replaced by a double-glazed window. The window glass is 0.4 cm thick, the outside convection heat transfer coefficient h_o is 12 W/m^2-K, the inside heat transfer coefficient h_i is 6 W/m^2-K, and the air gap is 0.5 cm thick. The thermal conductivity of air is 0.026 W/m-K. Model the situation using a thermal resistance circuit.

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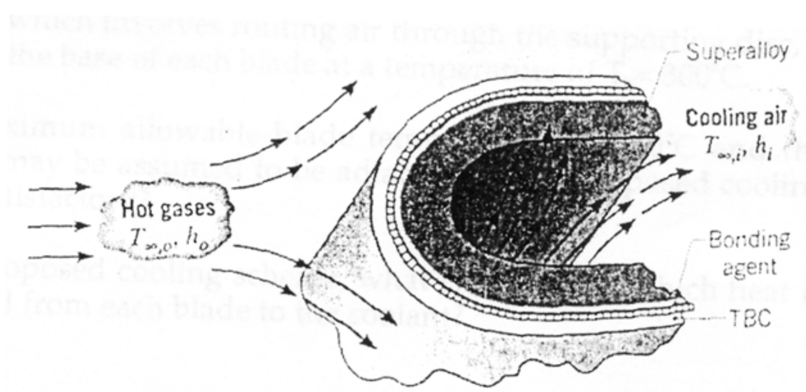
Problem T19

The performance of gas turbine engines may be improved by increasing the tolerance of the turbine blades to hot gases emerging from the combustor. The approach to achieving high operating temperatures involves application of a *thermal barrier coating* (TBC) to the exterior surface of a blade, while passing cooling air through the blade. Typically, the blade is made from a high-temperature superalloy, such as Inconel (k=20 W/mK), while a ceramic, such as Zirconia (k=1.3 W/mK), can be used as a TBC.



Consider conditions for which hot gases at $T_{\infty,o} = 1700K$ and cooling air at $T_{\infty,i} = 400K$ provide outer and inner surface convection coefficients of h_o=1000 W/m^{2}K and h_i=500 W/m^{2}K, respectively. Radiation effects may be neglected, and the turbine blade may be approximated as a plane wall. Let's first assume that TBC is not being used.

a) Setup the thermal resistance circuit for the heat transfer from the hot combustion gas to the cooling air.

b) Sketch the temperature distribution from the hot gas to the cooling air.

c) Determine the highest temperature in the blade material. Can the Inconel be maintained at a temperature that is below its maximum allowable value of 1250K?

Next, a 0.5 mm-thick Zirconia TBC is attached to a 5-mm thick Inconel blade wall by means of a metallic bonding agent, which provides an interfacial thermal resistance of $R_{t,c}$=10^{-4}m^{2}K/W.

d) Setup the thermal resistance circuit including the TBC and the metallic bonding agent.

e) Find the maximum blade metal temperature. Can the Inconel be maintained at a temperature that is below its maximum allowable value of 1250K?

f) Plot the temperature distribution with the TBC.

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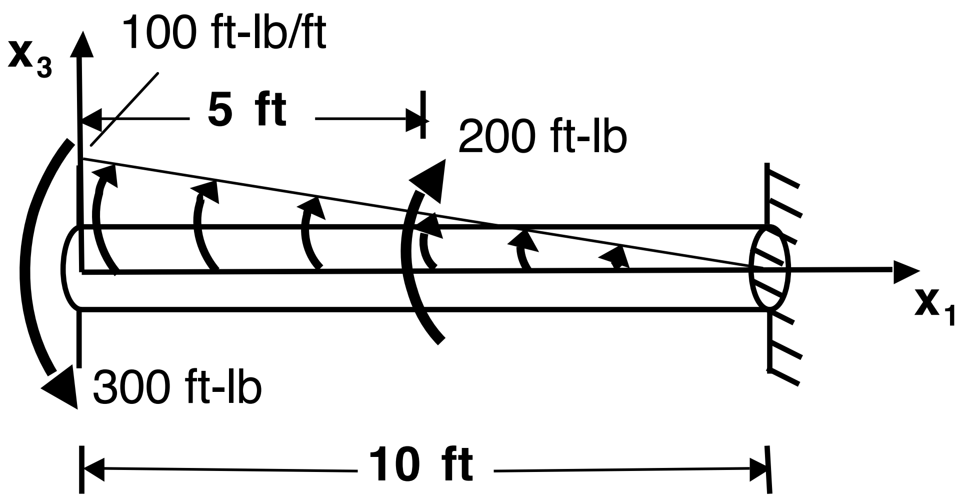
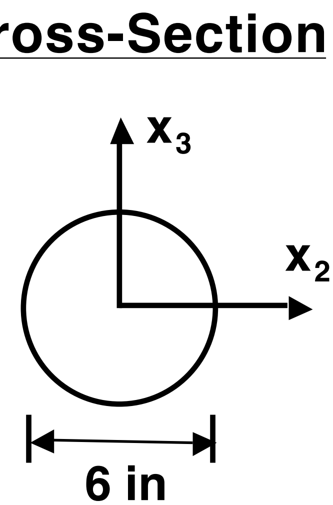
Problem T20

Atmospheric air is in parallel flow (c_{oo}=15 m/s, T_{oo}=15 C) over a flat heater surface that is to be maintained at a temperature of 140 C. The heater surface area is 0.25 m^2, and the airflow is known to induce a drag force of 0.25 N on the heater. What is the electrical power needed to maintain the prescribed surface temperature?

Unified Engineering Problem Set 8 **Lectures: M13, M14, M15, M16**
Week 10 Spring, 2009 **Units: M4.6, M4.7**

M12 **(M10.1)** ***(20 points)*** A steel rod has a solid circular cross-section with a 6-inch diameter and is 10 feet in length. The rod is clamped to a solid wall at one end, and is subjected to a distributed negative torque of linearly decreasing intensity from 100 ft-lb/ft at the tip to 0 at the wall. There is also a concentrated positive torque of 300 ft-lb at the tip and a concentrated negative torque of 200 ft-lb at the midpoint of the rod. The configuration is shown below. The modulus of steel is 30 Msi and the Poisson's ratio is 0.3.





(a) Determine the torque distribution in the rod structural configuration and sketch this as a function of x_1.

(b) Determine the twist at the free tip of the rod.

(c) Determine the maximum shear stress in the rod and its location.

(d) If the rod were a hollow tube with the same outer radius and a wall with a thickness of 1/2 inch, how would these answers change?

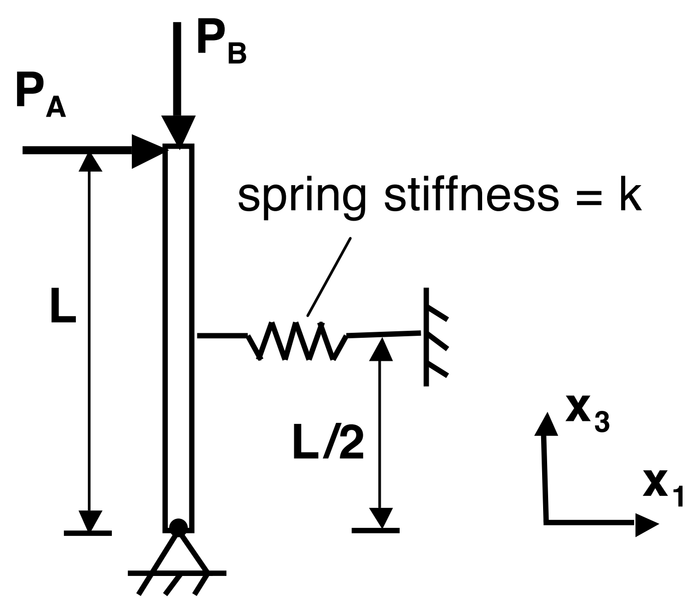
(e) Evaluate how the answers change for a hollow tube with wall thickness of t, and plot the results. If possible, assess the applicability of this solution as the value of t varies.

M13 **(M10.2)** ***(10 points)*** You are asked to serve as a consultant for a company designing a device to measure torque. Their current preliminary design entails a circular rod clamped at one end with a pointer at the other end which will be calibrated to a linear angular scale. There will be a gear placed along the shaft that will be connected by a chain to allow the proper torque measurement.

The designers want to maximize the sensitivity of the rod. The variables they have are the material of the rod, the rod cross-section and its size (maintaining the circular constraint), and the location of the attachment gear. The length is relatively constrained.

Please comment on the issues that must be considered. Use equations as needed/helpful. A hand sketch of a model of the configuration would be helpful. The answer should be on the order of one or two paragraphs (hand-written is acceptable) to the company giving your recommendations and reasoning. An appendix showing the derivation of any pertinent equations can be included as deemed necessary.

M14 **(M10.3)** ***(10 points)*** A column in the x_1- x_3 plane is supported by a pin at one end and is loaded by two tip loads, one in the positive x_1-direction, P_A, and one in the negative x_3-direction, P_B. These two loads stay with the tip of the column and point in their original orientation as the column moves. The column is of length L and is also supported by a linear spring at its mid-point. The spring has a spring constant of magnitude, k. This overall configuration is shown below.



(NOTE: Assume that any change in the orientation of the spring is sufficiently small such that it can be ignored.)

(a) Determine the exact equation governing the angular motion of this rigid column.

(b) Using a small angle assumption, determine the equation governing the stability of this configuration and determine the stability condition.

(c) Plot a figure showing the angular deformation of the rod as a function of the applied load in the x_3-direction, P_B, for various values of the load in the x_1-direction, P_A. Use the spring constant and the length of the column as normalizing parameters.

S3 (20 points)

Find the Laplace Transform of each of the following functions:

a) $f(t) = te^{-at}$

b) $f(t) = \sin \omega t$

c) $f(t) = \sin(\omega t + \theta)$

d) $f(t) = t$

e) $f(t) = \cosh(t + \theta)$

S4 (5 points)

Show that:

$£\{e^{-at} f(t)\} = F(s+a)$

S5 (5 points)

Show that

$£\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$