Fluids Psel 11 + 12 Solutions S'09

a) Isentropic relation between "∞" and airfoil point

$$\frac{T}{T_\infty} = \frac{P}{P_\infty}^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.6\times10^4}{4.35\times10^6}\right)^{\frac{1}{3.5}} = 0.94737 \rightarrow T = 0.94737\, T_\infty \; (245\text{K}) = 232.1\text{ K}$$

State eqn: $\rho = \frac{P}{RT} = \frac{3.6\times10^4 \text{ Pa}}{287 \text{ J/kgT} \cdot 232.1\text{K}} = 0.540 \text{ kg/m}^3$

b) $\frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1 \text{ atm}}{10 \text{ atm}}\right)^{\frac{1}{3.5}} = 0.518 \rightarrow T = 0.518 \cdot 500\text{K} = 259.0\text{K}$

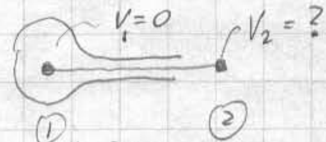
$p = 1 \text{ atm} = 1.013\times10^5 \text{ Pa}$, $\rho = \frac{P}{RT} = \frac{1.013\times10^5 \text{ Pa}}{287 \text{ J/kgK} \cdot 259\text{K}} = 1.363 \text{ kg/m}^3$

c) Convert to SI units: $V = 1300 \text{ ft/s} = \frac{1300 \text{ ft/s}}{3.28 \text{ ft/m}} = 396.34 \text{ m/s}$, $T = 480\text{R} = \frac{480\text{R}}{9/5 \text{ R/K}} = 266.7\text{K}$

$c_p = 1003 \text{ J/kgK}$: $h_0 = c_p T + \frac{1}{2}V^2 = 1003\cdot266.7 + \frac{1}{2}396.34^2 = 346009 \text{ m}^2/\text{s}^2$

or $h_0 = 346000 \text{ m}^2/\text{s}^2 \cdot (3.28 \text{ ft/m})^2 = 3.723\times10^6 \text{ ft}^2/\text{s}^2$

d) Adiabatic flow; $h_0 = \text{const}$ or $h_{0_1} = h_{0_2}$



$h_1 + \frac{1}{2}V_1^2 \,(=0) = h_2 + \frac{1}{2}V_2^2$, $h_1 = c_p T_1 = 1003 \text{ J/kgK}\cdot1000\text{ K} = 1.003\times10^6 \text{ m}^2/\text{s}^2$

$h_2 = c_p T_2 = 1003 \text{ J/kgK}\cdot600\text{K} = 0.602\times10^6 \text{ m}^2/\text{s}^2$

$V_2 = \sqrt{2(h_1 - h_2)} = 895.8 \text{ m/s}$

e) Isentropic (and adiabatic) flow between "∞" and airfoil point

$p_\infty = 0.61 \text{ atm} = 0.61 \text{ atm}\cdot1.013\times10^5 \text{ Pa/atm} = 0.618\times10^5 \text{ Pa}$, $T_\infty = \frac{p_\infty}{\rho_\infty R} = \frac{0.618\times10^5}{0.819\cdot287} = 262.9\text{ K}$

$\frac{T}{T_\infty} = \left(\frac{P}{P_\infty}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{0.50 \text{ atm}}{0.61 \text{ atm}}\right)^{\frac{1}{3.5}} = 0.9448 \rightarrow T = 0.9448\cdot T_\infty = 248.4\text{ K}$

Adiabatic: $h_0 = h_{0\infty} \rightarrow c_p T + \frac{1}{2}V^2 = c_p T_\infty + \frac{1}{2}V_\infty^2 \rightarrow V = \sqrt{2\left(c_p T_\infty + \frac{1}{2}V_\infty^2 - c_p T\right)} = 345.1 \text{ m/s}$

f) $\frac{T}{T_\infty} = \left(\frac{0.30 \text{ atm}}{0.61 \text{ atm}}\right)^{\frac{1}{3.5}} = 0.8165 \rightarrow T = 0.8165\, T_\infty = 214.7\text{ K}$

$V = \sqrt{2\left(c_p T_\infty + \frac{1}{2}V_\infty^2 - c_p T\right)} = 432.2 \text{ m/s}$