Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.003/16.004 Unified Engineering III, IV
Spring 2009

Problem Set 9

Name: ________________________

Due Date: 4/17/2009

	Time Spent (min)
F11-12	
S6	
S7	
S8	
S9	
M15	
M16	
M17	
SPL10	
Study Time	

Announcements:

Compressible-flow calculation drills:

a) Anderson problem 7.4

b) Anderson problem 7.5

c) Anderson problem 7.7

d) Anderson problem 7.8

e) Anderson problem 7.9

f) Anderson problem 7.11

S6: (5pts)

a) Find the Laplace transform of te^{-at}.

b) Use the operational transform given by Eq. 12.23 to find the Laplace transform of $\frac{d}{dt}(te^{-at})$.

c) Check your result in part (b) by first differentiating and then transforming the resulting expression.

S7: (5pts)

a) Find $\mathscr{L}\left\{\int_{0-}^{t} e^{-ax}\, dx\right\}$.

b) Find $\mathscr{L}\left\{\int_{0-}^{t} y dy\right\}$.

c) Check the results of (a) and (b) by first integrating and then transforming.

S8: (5pts)

a) Find $\mathscr{L}\left\{\frac{d}{dt}\sin \omega t\right\}$.

b) Find $\mathscr{L}\left\{\frac{d}{dt}\cos \omega t\right\}$.

c) Find $\mathscr{L}\left\{\frac{d^3}{dt^3}t^2\right\}$.

d) Check the results of parts (a), (b), and (c) by first differentiating and then transforming.

S9: (5pts)

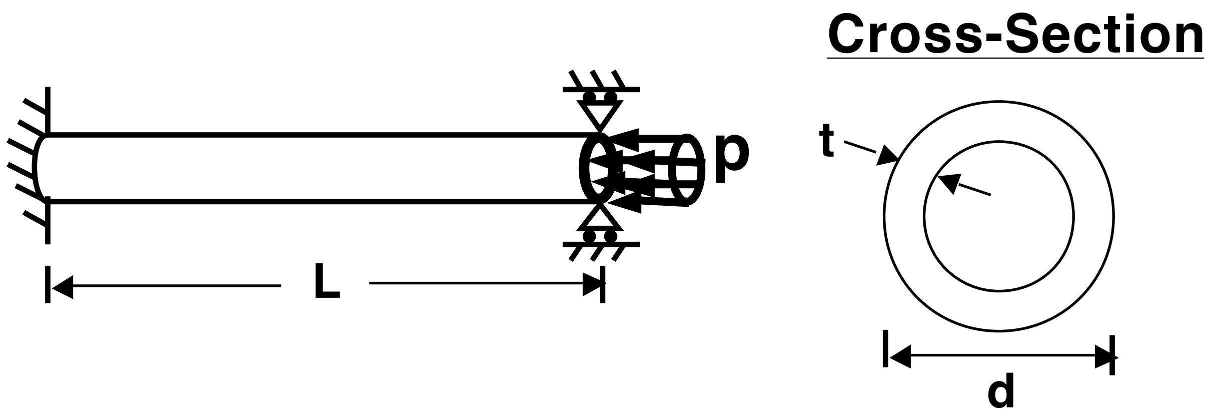
Find the Laplace transform of each of the following functions:

a) $f(t) = -20e^{-5(t-2)}u(t-2)$.

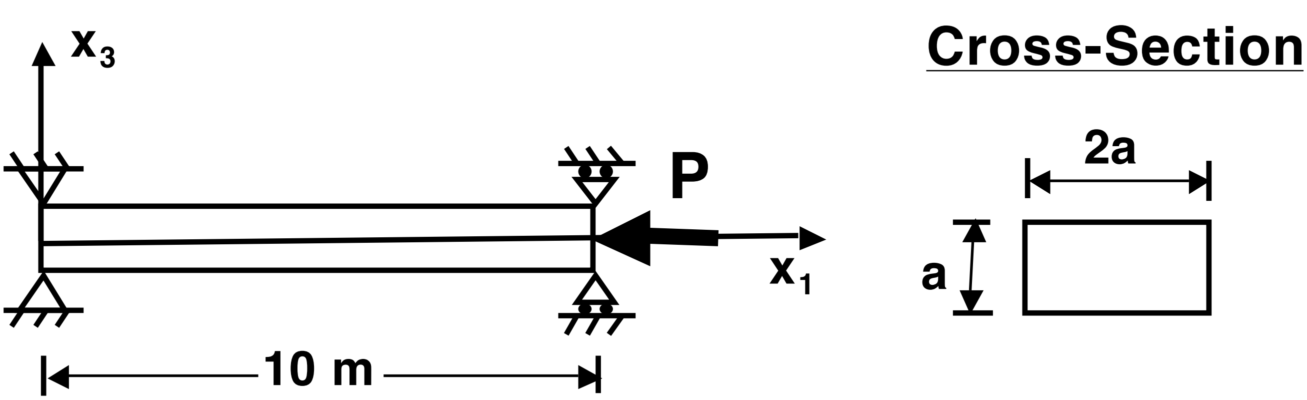
b) $f(t) = (8t-8)[u(t-1) - u(t-2)]$
$+ (24-8t)[u(t-2) - u(t-4)]$
$+ (8t-40)[u(t-4) - u(t-5)]$.

Unified Engineering Problem Set 9 **Lectures: M17, M18, M19**
Week 11 Spring, 2009 **Units: M4.7, M4.8**

M15 (M11.1) ***(10 points)*** A column of length L is clamped at one end and is loaded at the other end that is supported by roller supports (think of these as a ring of roller balls). The load is applied via a pressure of magnitude p that is equally distributed over the area of the cross-section. The column is a circular tube with an outer diameter, d, and a thickness of the walls, t, and is made of a material with modulus of E. Determine the expressions to find the buckling load and the buckling mode in terms of the applied pressure.



M16 (M11.2) ***(10 points)*** A series of 10-meter long columns is to be produced using aluminum (E = 70 GPa, ν = 0.3, σ_{cy} = 370 MPa, σ_{cu} = 425 MPa). These columns will be simply-supported and have rectangular cross-sections of various side lengths with an aspect ratio of 2. A compressive load will be applied at the column end with the roller supports. The supports at each end are attached totally around the cross-section edges. You are asked to determine a design chart showing the maximum load-carrying capability of these products as a function of the side length, a. Be sure to clearly indicate your reasoning and "important points" on the design chart.

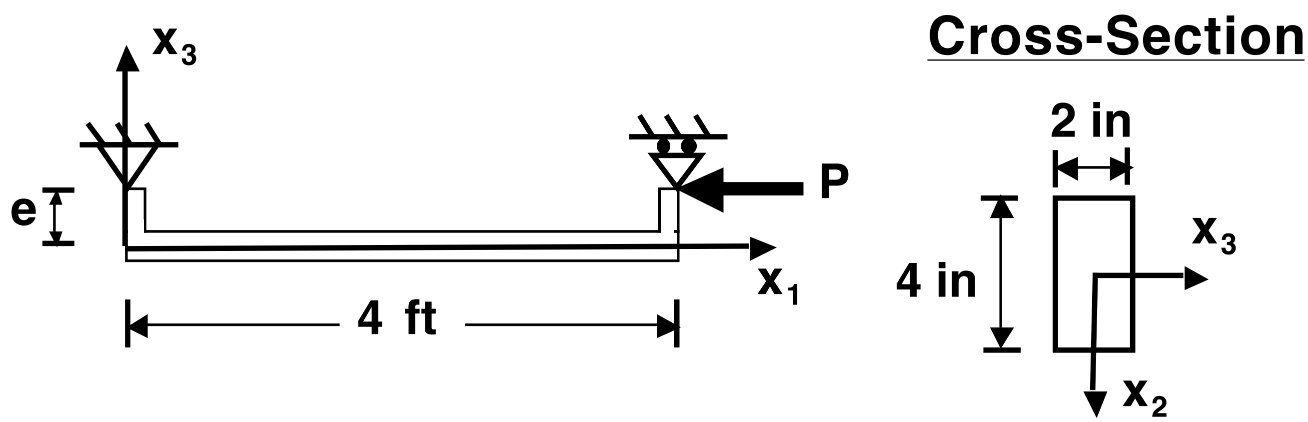


(a) Determine the buckling load for this configuration as a function of the side length of the aluminum columns.

(b) Determine the squashing load for this configuration as a function of the side length of the aluminum columns.

(c) Determine and sketch the design chart.

M17 **(M11.3)** ***(10 points)*** A titanium ($E = 18.5$ Msi, $\nu = 0.3$, $\sigma_{tu} = 150$ ksi, $\sigma_{cy} = 98$ ksi) column is 4 feet long and has a rectangular cross-section of 2 inches by 4 inches. The column is assumed simply-supported and is used to support a compressive load of magnitude P. Assume that the load is applied off the centerline of the column by an eccentricity of value e. Consider five cases of eccentricity normalized by the total length of the column: 0, 0.01, 0.02, 0.05, 0.1.



(a) For each case, determine the maximum load the column can carry.

(b) For each case, determine a normalized relationship between the applied load and the lateral deflection of the center of the column. Normalize the center deflection of the column by the specimen length and normalize the column load, P, by the critical buckling load, P_{cr}.

(c) Plot the normalized center deflection of the column versus the normalized load for the five cases of eccentricity.