UNIFIED ENGINEERING I, II, III, IV

16.001/16.002/16.003/16.004

2007-2008

SAMPLE PROBLEM AND SOLUTIONS

Problem 1.1A (Mechanics of Physics)

A rocket is projected vertically upward and achieves a burnout at time equal to zero. It reaches a height above burnout equal to $\boldsymbol{h}$ at time $\boldsymbol{t_1}$ going up and $\boldsymbol{t_2}$ going down. Determine $\boldsymbol{h}$ and the speed $\boldsymbol{v_o}$ at burnout.

SOLUTION

Paul Lagace Problem 1.1 A September 6, 2007

Given facts:

- burnout at time = 0 and velocity = v_o
- at time = t_1, height above burnout equals h on way up
- at time = t_2, height above burnout equals h on way down
- Air drag can be neglected

Other knowns:

- Force due to gravity (in negative direction)
- dv/dt = a
- dz/dt = v

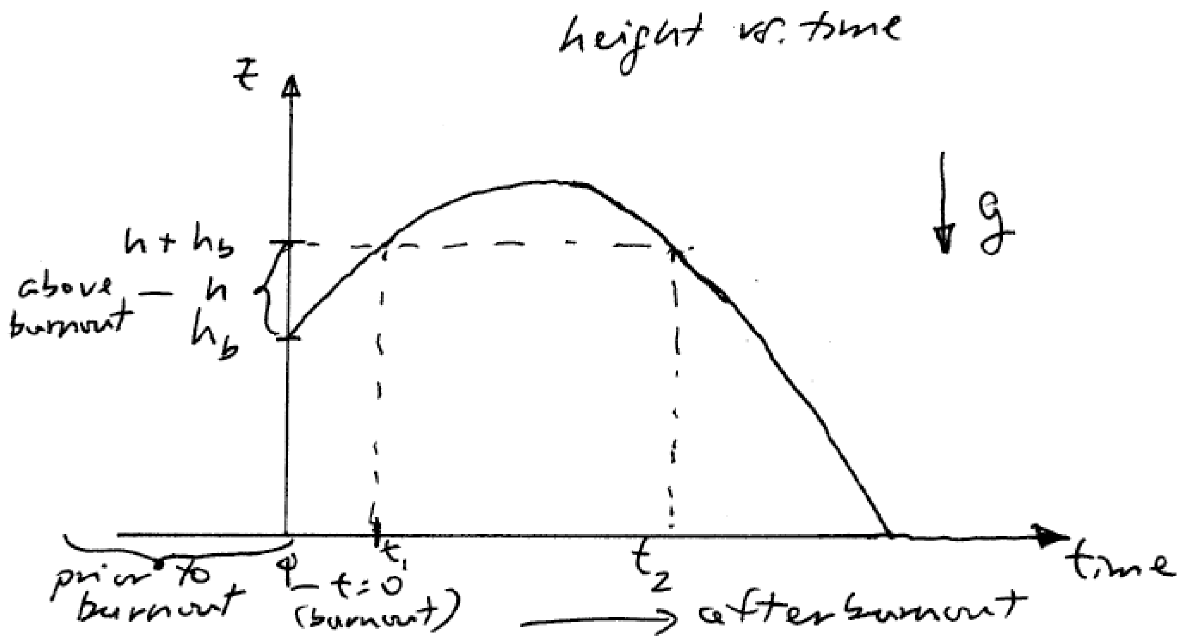
a = acceleration
v = velocity
z = height

Problem asks for:

h = height above burnout at defined times t_1 and t_2

v_o = velocity at burnout

Draw sketch to help view problem:



Define:

h_b = height at burnout

Approach:

→ Begin with general velocity-acceleration relationship:

$dv(t)/dt = a(t)$

Here $a(t) = -g$ (only the gravity)

Now integrate to get:

$$v(t) = \int a(t)\, dt$$
$$= -\int g\, dt$$
$$= -gt + C_1$$

We know that at burnout: $t = 0$, $v = v_o$

$$\Rightarrow v_o = C_1$$

giving: $v(t) = -gt + v_o$ (1)

→ Now use this in the height (distance)-velocity relationship:

$$dz(t)/dt = v(t) = -gt + + v_o$$

and integrating this to get:

$$z(t) = \int v(t)\,dt = -\int gt\,dt + \int v_o\,dt$$
$$= -1/2\,gt^2 + v_o t + C_2$$

We have defined the height at burnout to be h_b (with $t = 0$). Thus:

$$h_b = C_2$$

giving:

$$z(t) = h(t) = -1/2\,gt^2 + v_o t + h_b \qquad (2)$$

→ Now we use the given facts relating defined times and height/distance above burnout. Use these in the derived equation (2).

<u>First</u>: at $t = t_1$, $z(t) = h + h_b$

So: $h + \cancel{h_b} = -1/2\,g\,t_1^2 + v_o\,t_1 + \cancel{h_b}$

giving: $h = v_o\,t_1 - 1/2\,g\,t_1^2$ (3)

<u>Second</u>: at $t = t_2$, $z(t) = h + h_b$

So: $h + \cancel{h_b} = -1/2\,g\,t_2^2 + v_o\,t_2 + \cancel{h_b}$

giving: $h = v_o\,t_2 - 1/2\,g\,t_2^2$ (4)

Now equate the resulting equations (3) and (4) to get:

$$h = v_o\,t_1 - 1/2\,g\,t_1^2 = v_o\,t_2 - 1/2\,g\,t_2^2 \qquad (3)$$

→ Solve for v_o as asked for by the problem:

$$v_o\,(t_2 - t_1) = 1/2\,g\,(t_2^2 - t_1^2)$$

$$\Rightarrow \quad v_o = [\,1/2\,g\,(t_2 + t_1)\,\cancel{(t_2 - t_1)}\,] \,/\, \cancel{(t_2 - t_1)}$$

Finally:

$$\boxed{v_o = 1/2\,g\,(t_1 + t_2)}$$

check via units:

$$[\,L\,/\,T\,] = [\,L\,/\,T^2\,]\,[\,T\,\}\quad \surd$$

→ Continue using these facts and the derived relationship for height/distance above burnout at the defined times.

Now use the steps resulting in equations (3) and (4) and the solution for v_o in these equations.

In (3): $h = v_o t_1 - 1/2\, g\, t_1^2$

$$\Rightarrow \quad h = 1/2\, g\,(t_1 + t_2)\, t_1 - 1/2\, g\, t_1^2$$

expanding:

$$h = \cancel{1/2\, g\, t_1^2} + 1/2\, g\, t_1 t_2 - \cancel{1/2\, g\, t_1^2}$$

$$\Rightarrow \quad h = 1/2\, g\, t_1 t_2$$

Check by doing this........

In (4): $h = v_o t_2 - 1/2\, g\, t_2^2$

$$\Rightarrow \quad h = 1/2\, g\,(t_1 + t_2)\, t_2 - 1/2\, g\, t_2^2$$

expanding:

$$h = 1/2\, g\, t_1 t_2 + \cancel{1/2\, g\, t_2^2} - \cancel{1/2\, g\, t_2^2}$$

$$\Rightarrow \quad h = 1/2\, g\, t_1 t_2$$

Same as from (3). So height above burnout, h, asked for in problem is:

$$\boxed{h = 1/2\, g\, t_1 t_2}$$

final check via units:

$$[\,L\,] = [\,L\,/\,T^2\,]\,[\,T\,\}\,[\,T\,\}\quad \surd$$