2006

P1 SOLUTIONS BY WAITZ

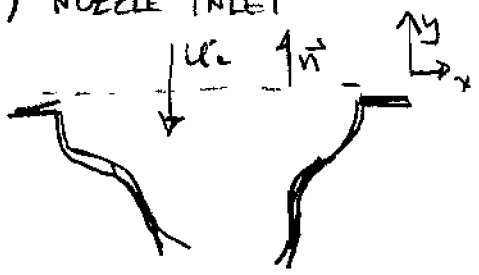
STARTING WITH THE INTEGRAL FORM OF THE MOMENTUM EQUATION FOR A CONTROL VOLUME OF FIXED MASS:

$$\underbrace{\sum \vec{F} - \vec{F}_b}_{\text{SUM OF FORCES ON C.V.}} = \underbrace{\int_V \frac{\partial(\rho\vec{u})}{\partial t} dV}_{\text{CHANGE OF MOM. OF MASS IN C.V. OVER TIME}} + \underbrace{\int_S \vec{u}(\rho\vec{u})\cdot\vec{n}\,ds}_{\text{MOM. FLUX ACROSS BOUNDARIES OF C.V.}}$$

FOR STEADY FLOW AND NO ACCELERATION OF THE REFERENCE FRAME OF THE C.V. THEN

$$\sum F_x = \int_S u_x(\rho\vec{u})\cdot\vec{n}\,ds \quad \& \quad \sum F_y = \int_S u_y(\rho\vec{u})\cdot\vec{n}\,ds$$

a) NOZZLE INLET

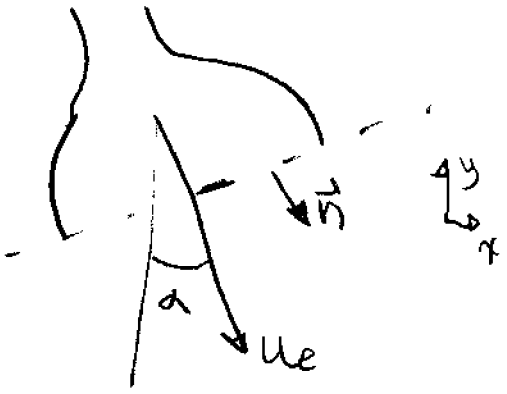


X-COMP. OF MOM. FLUX = 0

y-COMP. OF MOM. FLUX = $-u_i\rho(-u_i)A_i$

$= \rho u_i u_i A_i$

b) NOZZLE OUTLET



X-COMP OF MOM. FLUX = $(u_e \sin\alpha)\rho u_e A_e$

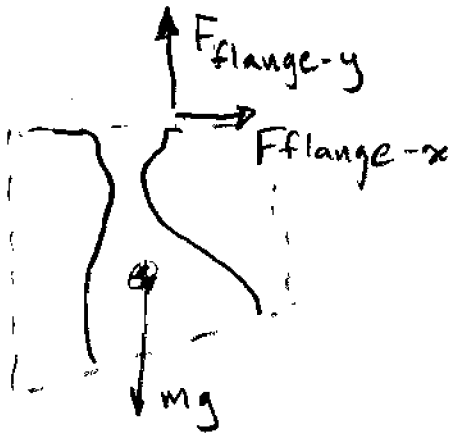
y-COMP OF MOM. FLUX = $(-u_e\cos\alpha)\rho u_e A_e$

c) PRESSURE ALWAYS ACTS NORMAL TO SURFACE

X-COMP = $-(p_e - p_0)\sin\alpha\, A_e$

y-COMP = $-(p_i - p_0)A_i + (p_e - p_0)\cos\alpha\, A_e$

d)



SUM UP x & y COMPONENTS

$$\sum F_x = \int_s u_x(\rho\vec{u})\cdot\vec{n}\,ds \Rightarrow F_{flange-x} + F_{pressure-x} = \text{X-COMP MOM FLUX}$$

$$F_{flange-x} - (p_e-p_o)\sin\alpha A_e = u_e\sin\alpha\rho u_e A_e$$

$$\boxed{F_{flange\,x} = u_e\sin\alpha\rho u_e A_e + (p_e-p_o)\sin\alpha A_e}$$

$$\sum F_y = \int_s u_y(\rho\vec{u})\cdot\vec{n}\,ds \longrightarrow F_{flange-y} + F_{pressure-y} + F_{\substack{NOZZLE\\WEIGHT}} = \text{MOM FLUX}_y$$

$$F_{flange-y} - (p_i-p_o)A_i + (p_e-p_o)\cos\alpha A_e \underbrace{-mg}_{\hat{}} = \rho u_i u_i A_i + (-u_e\cos\alpha)\rho u_e A_e$$

$$F_{flange-y} = \rho u_i u_i A_i - \rho u_e A_e(u_e\cos\alpha) + (p_i-p_o)A_i - (p_e-p_o)\cos\alpha A_e + mg$$

$$\boxed{F_{flange\,y} = mg + \dot{m}(u_i - u_e\cos\alpha) + (p_i-p_o)A_i - (p_e-p_o)\cos\alpha A_e}$$